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Research Article

On α-Continuity in Intuitionistic Topological Spaces

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Abstract. The aim of this paper is to explore intuitionistic α-continuity and obtain their characteristics.

Keywords. α-Continuity; Intuitionistic Topological Spaces

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1. Introduction

Atanassov introduced the concept of "Intuitionistic fuzzy sets" as a generalization of fuzzy sets. Later, Coker introduced the concept of "intuitionistic sets" in 1996 [3]. He also introduced the concept of intuitionistic topological spaces with intuitionistic sets and investigated basic properties of continuous functions and compactness. Further some more researchers are currently working in this field. In this paper we obtain Iα-continuity in intuitionistic topological spaces.

2. Preliminaries

Definition 2.1 ([3])**.** An intuitionistic set A is an object having the form $\langle X, A_1, A_2\rangle$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of members of A, while A_2 is called the set of nonmembers of A. Furthermore, let $\{A_i : i \in I\}$ be an arbitrary family of intuitionistic sets in X, where $A_i = \langle X, A_i^1, A_i^2\rangle$ then

(i) $\phi_\sim = \langle X, \phi, X\rangle$, $X_\sim = \langle X, X, \phi\rangle$

(ii) $A \subseteq B$ if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$

(iii) $\bar{A} = \langle X, A_2, A_1\rangle$

(iv) $A - B = A \cap \bar{B}$

(v) $[\ ]A = \langle X, A_1, A_1^c\rangle$

(vi) $\langle\ \rangle A = \langle X, A_2^c, A_2\rangle$

(vii) $\cap A_i = \langle X, \cap A_i^1, \cup A_i^2\rangle$ and $\cup A_i = \langle X, \cup A_i^1, \cap A_i^2\rangle$.

Definition 2.2 ([3])**.** An intuitionistic topological space on a nonempty set X is a family τ of intuitionistic sets in X satisfying the following axioms:

(i) $\phi_\sim, X \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$

Definition 2.3 ([7])**.** Let (X,τ) be an ITS(X). An intuitionistic set A of X is said to be

(i) Intuitionistic semiopen if $A \subseteq \mathrm{Icl}(\mathrm{Iint}(A))$

(ii) Intuitionistic preopen if $A \subseteq \mathrm{Iint}(\mathrm{Icl}(A))$

(iii) Intuitionistic regular open if $A = \mathrm{Iint}(\mathrm{Icl}(A))$

The family of all intuitionistic preopen and intuitionistic regular open of (X,τ) are denoted by IPOS and IROS, respectively.

Definition 2.4 ([5])**.** (i) If $B = \langle Y, B_1, B_2\rangle$ is an intuitionistic set in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the intuitionistic set in X defined by $f^{-1}(B) = \langle X, f^{-1}(B_1), f^{-1}(B_2)\rangle$

(ii) If $A = \langle X, f(A_1), f(A_2)\rangle$ is an intuitionistic set in X, then the image of A under f, denoted by $f(A)$ is the intuitionistic set in Y defined by $f(A) = \langle Y, f(A_1), f_-(A_2))\rangle$ where $f_-(A_2) = Y - (f(X - A_2))$.

Definition 2.5 ([2])**.** Let A and B be two intuitionistic sets on X and Y respectively.Then the product intuitionistic set of A and B on $X \times Y$ is defined by $U \times V = \langle (X,Y), A_1 \times B_1, A_2^c \times B_2^c\rangle$, where $A = \langle X, A_1, A_2\rangle$ and $B = \langle X, B_1, B_2\rangle$.

If (X,τ) and (Y,ϕ) are intuitionistic topological spaces then the product topology $\tau \times \phi$ on $X \times Y$ is the intuitionistic topology generated by the base $B = \{A \times B : A \in \tau, B \in \phi\}$.

Definition 2.6 ([2]). Let X, Y be two nonempty sets and $f : X \to Y$ a function. The graph of f, denoted by GR(f) is defined as the following intuitionistic set in $X \times Y$:

$$\mathrm{GR}(f) = \langle (x,y), \{(x,f(x)) : x \in X\}, \{(x,f(x)) : x \in X\}^c \rangle.$$

Definition 2.7 ([7]). Let (X,τ) be an intuitionistic topological space. (X,τ) is said to be T_2 if and only if for every $x, y \in X$ $(x \neq y)$ there exists $U, V \in \tau$ such that $x \in U$, $y \in V$ and $U \cap V = \phi$.

3. On Intuitionistic α-Continuity

Definition 3.1. In an intuitionistic topological space (X,τ) an intuitionistic set S is intuitionistic α-closed if X-$S \in I\alpha\mathrm{OS}(X)$.

Lemma 3.2. *Let A be a subset of an intuitionistic topological space (X,τ). Then the following are equivalent:*

(i) *A is intuitionistic α-closed*

(ii) $\mathrm{Icl}(\mathrm{Iint}(\mathrm{Icl}(A))) \subset A$

Proof. A is intuitionistic α closed

$$\Leftrightarrow X - A \in I\alpha\mathrm{OS}(X)$$

$$\Leftrightarrow X - A \subset \mathrm{Iint}(\mathrm{Icl}(\mathrm{Iint}(X - A)))$$

$$\Leftrightarrow A \supset X - [\mathrm{Iint}(\mathrm{Icl}(\mathrm{Iint}(X - A)))]$$

$$\Leftrightarrow \mathrm{IclIint}(\mathrm{Icl}(A))] \subset A$$

□

Definition 3.3. The function f is said to be intuitionistic α-continuous if $f^{-1}(V)$ is intuitionistic α-open in (X,τ) for every intuitionistic open set V of (Y,σ).

Theorem 3.4. *Every intuitionistic continuous map is intuitionistic α-continuous.*

Proof. Let $f : (X,\tau) \to (Y,\sigma)$ be intuitionistic continuous. Let V be an intuitionistic closed set of (Y,σ). Then $f^{-1}(V)$ is intuitionistic closed since f is intuitionistic continuous. Then $f^{-1}(V)$ is intuitionistic α-closed since every intuitionistic closed set is intuitionistic α-closed. Therefore f is intuitionistic α continuous. □

Example 3.5. $X = \{a,b,c\}$, $\tau = \{\phi, X, A, B\}$, $A = \langle X, \{c\}, \{a\}\rangle$, $B = \langle X, \phi, \{a\}\rangle$, $Y = \{\phi, Y, D, E\}$, $D = \langle Y, \phi, \{1,2\}\rangle$, $E = \langle Y, \{1\}, \{2\}\rangle$. Then $f : X \to Y$ given by $f(a) = 1$, $f(b) = 2$, $f(c) = 3$.

Then f is intuitionistic α-continuous but not intuitionistic continuous (since $\overline{(E)} = \langle Y, \{2\}, \{1\}\rangle$ is closed in Y but $f^{-1}\overline{(E)} = \langle X, \{b\}, \{a\}\rangle$ is intuitionistic α-closed but not intuitionistic closed in X and so we get that $f^{-1}\overline{(E)}$ is intuitionistic α-continuous but not intuitionistic continuous).

Theorem 3.6. *Let $f : (X,\tau) \to (Y,\sigma)$ be an intuitionistic function. Then the following are equivalent:*

(i) *f is intuitionistic α-continuous.*

(ii) *For any intuitionistic point $x \in X$ and any intuitionistic open set V of Y containing $f(x)$, there exists an intuitionistic α-open set U in X such that $x \in U$ and $f(U) \subset V$.*

(iii) *The inverse image of each intuitionistic closed subset of Y is an intuitionistic α-closed set in X.*

(iv) $\mathrm{Icl}(\mathrm{Iint}(\mathrm{Icl}(f^{-1}(B)))) \subset f^{-1}(\mathrm{Icl}(B))$ *for each subset B of Y.*

(v) $f[\mathrm{Icl}(\mathrm{Iint}(\mathrm{Icl}(A)))] \subset \mathrm{Icl}(f(A))$ *for each subset A of X.*

Proof. (i)⇔(iii): Let f be intuitionistic α-continuous, then from definition of intuitionistic α-continuous for every intuitionistic open set V of Y the preimage $f^{-1}(V)$ is an intuitionistic α-open in X.

(i)⇔(ii): *Necessity*: Let V be an intuitionistic open set in Y and $f(x) \in V$. Then by hypothesis, $f^{-1}(V)$ is intuitionistic α-open in X and $x \in f^{-1}(V)$. Let $U = f(x) \in V$, then $x \in U$ and $f(U) \subset V$.

Sufficiency: Let V be an intuitionistic open subset of Y and $x \in f^{-1}(V)$, then $f(x) \in V$. Therefore by hypothesis,there exists intuitionistic α-open set U in X such that $x \in U$ and $f(U) \subset V$. Then, $x \in U \subset f^{-1}(V)$. This implies $f^{-1}(V)$ is the union of intuitionistic α-open sets in X and since arbitrary union of intuitionistic α-open sets is α-open $f^{-1}(V)$ is α-open. Therefore f is intuitionistic α-continuous.

(i)⇒(iv): Let f be intuitionistic α-continuous and B be any intuitionistic subset of Y. Then $f^{-1}(\mathrm{Icl}(B))$ is an intuitionistic α-closed set in X. For any intuitionistic set B we have, $B \subset \mathrm{Icl}(B)$. By previous lemma, $f^{-1}(\mathrm{Icl}(B)) \supset \mathrm{Icl}(\mathrm{Iint}(\mathrm{Icl}[f^{-1}(\mathrm{Icl}(B)]])) \supset \mathrm{Icl}(\mathrm{Iint}(\mathrm{Icl}[f^{-1}(B)]]))$.

(iv)⇒(v): Suppose that $\mathrm{Icl}(\mathrm{Iint}(\mathrm{Icl}[f^{-1}(B)])) \subset f^{-1}(\mathrm{Icl}(B))$ for each B of Y. Let A be an intuitionistic subset of X. Put $B = f(A)$ then $A \subset f^{-1}(B)$. Therefore by hypothesis,we have $\mathrm{Icl}(\mathrm{Iint}(\mathrm{Icl}[A])) \subset \mathrm{Icl}(\mathrm{Iint}(\mathrm{Icl}[f^{-1}(B)])) \subset f^{-1}(\mathrm{Icl}(B))$. $f(\mathrm{Icl}(\mathrm{Iint}(\mathrm{Icl}[A]))) \subset \mathrm{Icl}[f(A)]$.

(v)⇒(iii): Suppose that, $f(\mathrm{Icl}(\mathrm{Iint}(\mathrm{Icl}[A]))) \subset \mathrm{Icl}[f(A)]$ for each subset A of X. This implies $\mathrm{Icl}(\mathrm{Iint}(\mathrm{Icl}[A])) \subset f^{-1}(\mathrm{Icl}[f(A)])$. Let F be any intuitionistic closed subset of Y and $A = f^{-1}(F)$ then $f(A) \subset F$. Therefore by hypothesis, $f(\mathrm{Icl}(\mathrm{Iint}(\mathrm{Icl}[A]))) \subset \mathrm{Icl}[f(A)] \subset \mathrm{Icl}(F) = F$. Therefore $\mathrm{Icl}(\mathrm{Iint}(\mathrm{Icl}[f^{-1}(F)])) \subset f^{-1}(F)$.

Hence $f^{-1}(F)$ is intuitionistic α-closed in X. □

Theorem 3.7. *Let $f : (X,\tau) \to (Y,\sigma)$ is intuitionistic α-continuous and $g : (Y,\sigma) \to (Z,\delta)$ is intuitionistic continuous then $g \circ f$ is intuitionistic α-continuous.*

Proof. Let U be an intuitionistic open set in Z then $g^{-1}(U)$ is intuitionistic open in Y (as g is intuitionistic continuous). Since f is intuitionistic α-continuous, $f^{-1}(g^{-1}(U)) =$

$(g\circ f)^{-1}(U)$ where $(g\circ f)^{-1}(U)$ is intuitionistic α-open in X. Consequently gof is intuitionistic α-continuous. □

Corollary 3.8. *Let $\{X_\beta/\beta\in\tau\}$ be any family of intuitionistic topological spaces. If $f:X\to\prod X_\beta$ is intuitionistic α-continuous then $p_\beta\circ f:X\to X_\beta$ is intuitionistic α-continuous for each $\beta\in\tau$, where p_β is the projection of $\prod X_\beta$ onto X_β.*

Proof. For each $\beta\in\tau$, p_β is continuous. Hence the above theorem is applicable. □

Theorem 3.9. *Let $f:(X,\tau)\to(Y,\sigma)$ is intuitionistic α-continuous and A is intuitionistic α-closed in X, then the restriction $f|A:A\to Y$ is intuitionistic α-continuous.*

Proof. From hypothesis, f is intuitionistic α-continuous and so for any intuitionistic closed set V in Y, $f^{-1}(V)\in I\alpha CS(X)$ and A is intuitionistic α-closed in X. Therefore $f^{-1}(V)\cap A\in I\alpha CS(X)$. But $(f|A)^{-1}(V)=f^{-1}(V)\cap A\in I\alpha CS(A)$. This implies $f|A$ is intuitionistic α-continuous. □

Theorem 3.10. *Let $f:(X,\tau)\to(Y,\sigma)$ be two intuitionistic topological spaces. If A is intuitionistic α-open in X and B is intuitionistic α-open in Y then $A\times B\in\alpha(X\times Y)$.*

Proof. From hypothesis, $A\subset \text{Iint}(\text{Icl}(\text{Iint}(A)))$, $B\subset \text{Iint}(\text{Icl}(\text{Iint}(B)))$. Then $A\times B\subset \text{Iint}(\text{Icl}(\text{Iint}(A)))\times\text{Iint}(\text{Icl}(\text{Iint}(B)))=\text{Iint}[(\text{Icl}(\text{Iint}(A)))\times\text{Icl}(\text{Iint}(B))]=IintIcl[\text{Iint}(A)\times\text{Iint}(B)]=\text{Iint}(\text{Icl}(\text{Iint}(A\times B)))$.

Therefore $A\times B\in\alpha(X\times Y)$. □

Theorem 3.11. *If $f:(X,\tau)\to(Y,\sigma)$ and $g:(X,\tau)\to(Y,\sigma)$ are intuitionistic α-continuous, where (Y,σ) is an Hausdorff space then the intuitionistic set $A=\{x/f(x)=g(x)\}$ is an intuitionistic α-closed set in (X,τ).*

Proof. Let $y\in X\text{-}A$.Then $f(y)\neq g(y)$. Since Y is Hausdorff there exists disjoint intuitionistic open sets U and V such that $f(y)\in U$ and $g(y)\in U$. Therefore $f^{-1}(U)$, $g^{-1}(V)\in I\alpha\text{OS}(X)$ [as f and g are intuitionistic α-continuous]. Let $B=f^{-1}(U)\cap g^{-1}(V)$ then $y\in B\in I\alpha\text{OS}(X)$. Moreover, $A\cap B=\phi$ otherwise we get contradiction to the fact that U and V are not disjoint. Consequently, $y\in B\subset X\text{-}A$. Therefore, $X\text{-}A$ is the union of intuitionistic α-open sets in X. Hence $X\text{-}A\in I\alpha\text{OS}(X)$. Therefore A is intuitionistic α-closed in (X,τ). □

Corollary 3.12. *If f is intuitionistic α-continuous of a Hausdorff space (X,τ) into itself then the set $\{x/f(x)=x\}$ is intuitionistic α-closed set.*

Proof. Let g be an identity mapping on (X,τ), which being continuous is intuitionistic α-continuous.
Hence the result follows from previous theorem. □

Theorem 3.13. *If $f_i : X_i \to X_i^*$ be intuitionistic α-continuous for $i = 1,2$. Let $f : X_1 \times X_2 \to X_1^* \times X_2^*$ be defined by $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$. Then f is intuitionistic α-continuous.*

Proof. Let $M_1^* \times M_2^* \subset X_1^* \times X_2^*$ where M_i^* is intuitionistic open in X_i^* for $i = 1,2$. Then $f^{-1}(M_1^* \times M_2^*) = f_1^{-1}(M_1^*) \times f_2^{-1}(M_2^*)$. By hypothesis, f_1 and f_2 are intuitionistic α-continuous then, $f_1^{-1}(M_1^*) \in I\alpha\mathrm{OS}(X_1)$ and $f_1^{-1}(M_1^*) \in I\alpha\mathrm{OS}(X_2)$. Therefore by Theorem 3.11, $f_1^{-1}(M_1^*) \times f_2^{-1}(M_2^*) \in \alpha(X_1 \times X_2)$. Now if M^* is any intuitionistic open in $X_1^* \times X_2^*$ then $f^{-1}(M^*) = f^{-1}(\cup M_\beta^*)$ where M_β^* is of the form, $M_{\beta_1}^* \times M_{\beta_2}^*$. Then $f^{-1}(M^*) = \cup f^{-1}(M_\beta^*) \in \alpha(X_1 \times X_2)$ where $f^{-1}(M_\beta^*)$ is intuitionistic α-open in $X_1 \times X_2$. Therefore f is intuitionistic α-continuous. □

Remark 3.14. In the above theorem f is the product mapping of f_1 and f_2. It is denoted by $f_1 \times f_2$.

Theorem 3.15. *If $f : (X,\tau) \to (Y,\sigma)$ is intuitionistic α-continuous and Y is a Hausdorff space then the set $\{(x_1,x_2)/f(x_1) = f(x_2)\}$ is an intuitionistic α-closed set of the product space $X \times X$.*

Proof. Let Δ be the diagonal of $Y \times Y$. Since Y is Hausdorff, Δ is intuitionistic closed subset of $Y \times Y$. Since f is intuitionistic α-continuous, $f \times f : X \times X \to Y \times Y$ is intuitionistic α-continuous. Therefore $(f \times f)^{-1}(\Delta)$ is intuitionistic α-closed set of $X \times X$. It follows, $(f \times f)^{-1}(\Delta) = \{(x_1,x_2)/f(x_1) = f(x_2)\}$. □

Definition 3.16. An intuitionistic space (X,τ) is said to be an R_1-space if for each pair of points x and y such that $\mathrm{Icl}(x) \neq \mathrm{Icl}(y)$ there exists intuitionistic open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \phi$.

Theorem 3.17. *If f and g are intuitionistic α-continuous from an intuitionistic topological space (X,τ) into R_1-space (Y,σ) then the set $A = \{x/\mathrm{Icl}[f(x)] = \mathrm{Icl}[g(x)]\}$ is intuitionistic α-closed in X.*

Proof. Let $y \in X\text{-}A$ then $\mathrm{Icl}[f(y)] \neq \mathrm{Icl}[g(y)]$. Now Y being R_1-space, there exists disjoint intuitionistic open set U and V such that $f(y) \in U$ and $g(y) \in V$. Since f and g are intuitionistic α-continuous, $f^{-1}(U)$ and $g^{-1}(V)$ are intuitionistic α-open in X. Therefore $f^{-1}(U) \cap g^{-1}(V)$ is an intuitionistic α-open set in X containing y and disjoint from A. If $p \in A \cap (f^{-1}(U) \cap g^{-1}(V))$ then $f(p) \in U$, $g(p) \in V$ and Y being R_1-space, $\mathrm{Icl}[f(p)] \subset U$ and $\mathrm{Icl}[g(p)] \subset V$, which together with $p \in A$ implies $U \cap V \neq \phi$. This is a contradiction to the fact that U and V are disjoint. Hence $y \in f^{-1}(U) \cap g^{-1}(V) \subset X\text{-}A$. Therefore $X\text{-}A$ is the union of intuitionistic α-open sets in X. Consequently, since arbitrary union of intuitionistic α-open sets is α-open we have $X\text{-}A \in I\alpha\mathrm{OS}(X)$. Therefore A is intuitionistic α-closed in X. □

Theorem 3.18. *If $f : (X,\tau) \to (Y,\sigma)$ and $g : (X,\tau) \to (X,\tau) \times (Y,\sigma)$ given by $g(x) = (x, f(x))$ be the graph mapping of f. Then f is intuitionistic α-continuous iff g is intuitionistic α-continuous.*

Proof. Necessity: Suppose f is intuitionistic α-continuous. Let $x \in X$ and W be intuitionistic open set of $X \times Y$ containing $g(x) = (x, f(x))$. Then there exists intuitionistic open sets, $G \subset X$ and $V \subset Y$ such that $(x, f(x)) \in G \times V \subset W$. Since f is intuitionistic α-continuous there exists an intuitionistic α-open set in X such that $x \in A$ and $f(A) \subset V$. Let $U = A \cap G$ then $x \in U \in I\alpha\mathrm{OS}(X)$. Further, $g(U) \subset G \times V \subset W$.
Hence g is intuitionistic α-continuous.

Sufficiency: Suppose g is intuitionistic α-continuous. Let $x \in X$ and V is intuitionistic open in Y containing $f(x)$. Then $X \times V$ is intuitionistic open in $X \times V$ containing $g(x)$. Since g is intuitionistic α-continuous, there exists $U \in I\alpha\mathrm{OS}(X)$ such that $x \in U$ and $g(U) \subset X \times V$. Therefore $f(U) \subset V$. Hence f is intuitionistic α-continuous. □

Theorem 3.19. *A mapping $f : (X,\tau) \to (Y,\sigma)$ is intuitionistic α continuous if and only if the preimage of every intuitionistic open set of Y is intuitionistic α-open in X.*

Proof. Let $A = \langle Y, A_1, A_2 \rangle$ be an intuitionistic open set in Y.

$$f^{-1}(\bar{A}) = f^{-1}\langle Y, A_2, A_1 \rangle = \langle X, f^{-1}(A_2), f^{-1}(A_1) \rangle .$$

Also $\overline{f^{-1}(A)} = f^{-1}\overline{\langle Y, A_1, A_2 \rangle} = \langle X, f^{-1}(A_2), f^{-1}(A_1) \rangle$.

Since $f^{-1}\overline{(A)} = \overline{f^{-1}(A)}$ for every intuitionistic set A of Y

$f^{-1}\overline{(A)}$ is intuitionistic α-closed set in X. So $f^{-1}(A)$ is intuitionistic α-open in X. Hence f is intuitionistic α continuous. Converse follows directly from the definition. □

Theorem 3.20. *Let (X,τ) and (Y,σ) be two intuitionistic topological spaces. If $f : (X,\tau_0,1) \to (Y,\sigma_0,1)$ and $f : (X,\tau_0,2) \to (Y,\sigma_0,2)$ are intuitionistic α-continuous, then $f : (X,\tau) \to (Y,\sigma)$ is intuitionistic α-continuous.*

Proof. Let $A = \langle Y, A_1, A_2 \rangle$ be an intuitionistic open set in Y. By hypothesis, $f : (X,\tau_0,1) \to (Y,\sigma_0,1)$ and $f : (X,\tau_0,2) \to (Y,\sigma_0,2)$ are $I\alpha$-continuous. So there exists an intuitionistic α-open sets $f^{-1}(\langle Y, A_1, \bar{A_1} \rangle) = \langle X, f^{-1}(A_1), f^{-1}(\bar{A_1}) \rangle$ in $(X,\tau_0,1)$, $f^{-1}(\langle Y, \bar{A_2}, A_2 \rangle) = \langle X, f^{-1}(\bar{A_2}), f^{-1}(A_2), \rangle$ in $(X,\tau_0,2)$. Since $A_2 \subset \bar{A_1}$ and $A_1 \subset \bar{A_2}$, $\langle X, f^{-1}(A_1), f^{-1}(A_2) \rangle \subseteq \langle X, f^{-1}(A_1), f^{-1}(\bar{A_1}) \rangle$ and $\langle X, f^{-1}(\bar{A_2}), f^{-1}(A_2) \rangle$. Hence $\langle X, f^{-1}(A_1), f^{-1}(A_2) \rangle$ is intuitionistic α-open in X and so $f : (X,\tau) \to (Y,\sigma)$ is intuitionistic α-continuous. □

Theorem 3.21. *Let (X,τ) and (Y,σ) be two intuitionistic topological spaces. If $f : (X,\tau_1) \to (Y,\sigma_1)$ and $f : (X,\tau_2) \to (Y,\sigma_2)$ are intuitionistic α-continuous, then $f : (X,\tau) \to (Y,\sigma)$ is intuitionistic α-continuous.*

Proof is similar to the above theorem.

Theorem 3.22. *Let $f : (X,\tau) \to (Y,\sigma)$ be intuitionistic α-continuous. Then $f(I\alpha\mathrm{cl}(A)) \subset \mathrm{Icl}(f(A))$ for every intuitionistic subset A of X.*

Proof. Let A be intuitionistic subset A of X. Then $\mathrm{Icl}(f(A))$ is intuitionistic closed in (Y,σ). Since f is intuitionistic α-continuous, $f^{-1}(\mathrm{Icl}(f(A)))$ is intuitionistic α-closed in X and $A \subset f^{-1}(f(A)) \subset f^{-1}(\mathrm{Icl}(f(A)))$. This implies $I\alpha\mathrm{cl}(A) \subset I\alpha\mathrm{cl}(f^{-1}(\mathrm{Icl}(f(A)))f^{-1}(\mathrm{Icl}(f(A)))$. Hence $f(I\alpha\mathrm{cl}(A)) \subset \mathrm{Icl}(f(A))$. □

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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