Journal of Informatics and Mathematical Sciences
Vol. 11, Nos. 3-4, pp. 421–431, 2019
ISSN 0975-5748 (online); 0974-875X (print)
Published by RGN Publications
http://www.rgnpublications.com
DOI: 10.26713/jims.v11i3-4.600

Research Article

Some Algebraic Polynomials and Topological Indices of Octagonal Network

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Abstract. M-polynomial of different molecular structures helps to calculate many topological indices. A topological index of graph G is a numerical parameter related to G which characterizes its molecular topology and is usually graph invariant. In the field of *quantitative structure-activity* (QSAR), *quantitative structure-activity structure-property* (QSPR) research, theoretical properties of the chemical compounds and their molecular topological indices such as the Zagreb indices, Randic index, Symmetric division index, Harmonic index, Inverse sum index, Augmented Zagreb index, multiple Zagreb indices etc. are correlated. In this report, we compute closed forms of M-polynomial, first Zagreb polynomial and second Zagreb polynomial of octagonal network. From the M-polynomial we recover some degree-based topological indices for octagonal network.

Keywords. M-polynomial; Zagreb polynomial; Topological index; Network

MSC. 05C12

Received: March 1, 2017 **Accepted:** November 27, 2018

1. Introduction

Chemical reaction network theory is an area of applied mathematics that attempts to model the behavior of real-world chemical systems. Since its foundation in the 1960s, it has attracted a growing research community, mainly due to its applications in biochemistry and theoretical chemistry. It has also attracted interest from pure mathematicians due to the problems that arise from the mathematical structures.

Cheminformatics is an emerging field in which *quantitative structure-activity* (QSAR) and *Structure-property* (QSPR) relationships predict the biological activities and properties of nanomaterial (see [1, 6, 27, 29]). In these studies, some physcio-chemical properties and topological indices are used to predict bioactivity of the chemical compounds (see [9, 10, 28]).

The branch of chemistry which deals with the chemical structures with the help of mathematical tools is called mathematical chemistry. Chemical graph theory is that branch of mathematical chemistry which applies graph theory to mathematical modeling of chemical phenomena. In chemical graph theory a molecular graph is a simple graph (having no loops and multiple edges) in which atoms and chemical bonds between them are represented by vertices and edges respectively. A graph $G(V,E)$ with vertex set $V(G)$ and edge set $E(G)$ is connected, if there exist a connection between any pair of vertices in G.

A network is simply a connected graph having no multiple edges and loops. A chemical graph is a graph whose vertices denote atoms and edges denote bonds between those atoms of any underlying chemical structure. The degree of a vertex is the number of vertices which are connected to that fixed vertex by the edges. In a chemical graph the degree of any vertex is at most 4. The distance between two vertices u and v is denoted as $d(u,v) = d_G(u,v)$ and is the length of shortest path between u and v in graph G. The number of vertices of G, adjacent to a given vertex v, is the "degree" of this vertex, and will be denoted by d_v. The concept of degree in graph theory is closely related (but not identical) to the concept of valence in chemistry. For details on basics of graph theory, any standard text such as [29] can be of great help.

Several algebraic polynomials have useful applications in chemistry such as Hosoya Polynomial (also called *Wiener polynomial*) [13] that plays a vital role in determining distance-based topological indices. Among other algebraic polynomials, M-polynomial [6], introduced in 2015 plays the same role in determining many degree-based topological indices.

Definition 1.1. Let G be a simple connected graph. The M-polynomial of G is defined as:

$$M(G,x,y) = \sum_{\delta \le i \le j \le \Delta} m_{ij}(G) x^i y^j,$$

where $\delta = \min\{d_v | v \in V(G)\}$, $\Delta = \max\{d_v | v \in V(G)\}$, and $m_{ij}(G)$ is the edge $vu \in E(G)$ such that $\{d_v, d_u\} = \{i, j\}$.

This polynomial is one of the key areas of interest in computational aspects of materials. From this M-polynomial, we can calculate many topological indices. M-polynomial of different molecular structures has been computed in [15, 20–22]. The topological index of a molecule structure can be considered as a non-empirical numerical quantity which quantifies the molecular structure and its branching pattern in many ways. In this point of view, the topological index can be regarded as a score function which maps each molecular structure to a real number and is used as a descriptor of the molecule under testing [4–6, 24, 27]. Topological indices give good predictions of the variety of physico-chemical properties of chemical compounds containing boiling point, the heat of evaporation, heat of formation, chromatographic retention times, surface tension, vapor pressure etc. Since the 1970s, two degree based graph invariants have been extensively studied. These are the first Zagreb index M_1 and the second Zagreb index M_2, introduced by Gutman and Trinajstic [12] and are defined as:

$$M_1(G) = \sum_{v \in V(G)} (d_v)^2 \quad \text{and} \quad M_2(G) = \sum_{uv \in E(G)} d_u d_v .$$

Results obtained in the theory of Zagreb indices are summarized in the review [11].

Second modified Zagreb index is defined as:

$$^{m}M_2(G) = \sum_{uv \in E(G)} \frac{1}{d_u d_v}.$$

In 1998, working independently, Bollobas and Erdos [3], and Amic *et al*. [2] proposed general Randic index. It has been extensively studied by both mathematicians and theoretical chemists (see, for example, [16, 18]). The Randic index is defined as:

$$R_\alpha(G) = \sum_{uv \in E(G)} (d_u d_v)^\alpha,$$

where α is an arbitrary real number.

Symmetric division index is defined as:

$$SDD(G) = \sum_{uv \in E(G)} \left\{ \frac{\min(d_u, d_v)}{\max(d_u, d_v)} + \frac{\max(d_u, d_v)}{\min(d_u, d_v)} \right\}.$$

Another variant of Randic index is the harmonic index defined as:

$$H(G) = \sum_{vu \in E(G)} \frac{2}{d_u + d_v}.$$

The Inverse sum index is defined as:

$$I(G) = \sum_{vu \in E(G)} \frac{d_u d_v}{d_u + d_v}.$$

The augmented Zagreb index is defined as:

$$A(G) = \sum_{vu \in E(G)} \left\{ \frac{d_u d_v}{d_u + d_v - 2} \right\}^3$$

and it is useful for computing heat of formation of alkanes [7, 14].

These topological indices can be recovered from M-polynomial [6] (see Table 1).

Table 1. Derivation of some degree-based topological indices from M-polynomial

Topological Index	Derivation from $M(G;x,y)$
First Zagreb	$(D_x + D_y)(M(G;x,y))\|_{x=y=1}$
Second Zagreb	$(D_x D_y)(M(G;x,y))\|_{x=y=1}$
Second Modified Zagreb	$(S_x S_y)(M(G;x,y))\|_{x=y=1}$
General Randić $\alpha \in \mathbb{N}$	$(D_x^\alpha D_y^\alpha)(M(G;x,y))\|_{x=y=1}$
Symmetric Division Index	$(D_x S_y + S_x D_y)(M(G;x,y))\|_{x=y=1}$
Harmonic Index	$2S_x J(M(G;x,y))_{x=1}$
Inverse sum Index	$S_x J D_x D_y (M(G;x,y))_{x=1}$
Augmented Zagreb Index	$S_x^3 Q_{-2} J D_x^3 D_y^3 (M(G;x,y))_{x=1}$

where $D_x = x\frac{\partial(f(x,y))}{\partial x}$, $D_y = y\frac{\partial(f(x,y))}{\partial y}$, $S_x = \int_0^x \frac{f(t,y)}{t}dt$, $S_y = \int_0^y \frac{f(x,t)}{t}dt$, $J(f(x,y)) = f(x,x)$, $Q_\alpha(f(x,y)) = x^\alpha f(x,y)$

In 2013, Shirdel *et al.* in [26] proposed "hyper-Zagreb index" which is also degree based index.

Definition 1.2. Let G be a simple connected graph. Then the hyper-Zagreb index of G is defined as

$$HM(G) = \sum_{uv \in E(G)} [d_u + d_v]^2.$$

In 2012 Ghorbani and Azimi [8] proposed two new variants of Zagreb indices.

Definition 1.3. Let G be a simple connected graph. Then the first multiple Zagreb index of G is defined as

$$PM_1(G) = \prod_{uv \in E(G)} [d_u + d_v].$$

We refer [19] to the readers for more detail.

Definition 1.4. Let G be a simple connected graph. Then the second multiple Zagreb index of G is defined as

$$PM_2(G) = \prod_{uv \in E(G)} [d_u + d_v].$$

Definition 1.5. Let G be a simple connected graph. Then the first Zagreb polynomial of G is defined as

$$M_1(G, x) = \sum_{uv \in E(G)} x^{[d_u + d_v]}.$$

Definition 1.6. Let G be a simple connected graph. Then second Zagreb polynomial of G is defined as

$$M_2(G, x) = \sum_{uv \in E(G)} x^{[d_u + d_v]}.$$

In this article, we compute M-polynomial, first Zagreb polynomial and second Zagreb polynomial of Octagonal network shown in Figure 1. We also compute some degree-based topological indices.

We denote octagonal network by $O_{\{n,m\}}$ for $n, m \geq 2$. The planer representation of $O_{\{n,m\}}$ is shown in Figure 1 with m rows and n columns of octagonal. Let V is the vertex set and E is the edge set of $O_{\{n,m\}}$. Then

$$V = \{x_i^j; 1 \leq i \leq 2n-1, i \text{ is odd and } 1 \leq j \leq 3m+1\}$$
$$\cup \{x_i^{3j-2}; 1 \leq i \leq 2n, i \text{ is even and } 1 \leq j \leq m+1\} \cup \{x_{2n}^{3j-1}, x_{2n}^{3j}; 1 \leq j \leq m\}$$

and

$$E = \{x_i^j x_i^{j+1}; 1 \leq i \leq 2n-1, i \text{ is odd and } 1 \leq j \leq 3m\}$$
$$\cup \{x_i^{3j-2} x_{i+1}^{3j-2}; 1 \leq i \leq 2n-1, i \text{ is odd and } 1 \leq j \leq m+1\}$$
$$\cup \{x_i^{3j-2} x_{i+1}^{3j-1}; 1 \leq i \leq 2n-2, i \text{ is even and } 1 \leq j \leq m\}$$

$$\cup\{x_i^{3j}x_{i-1}^{3j+1}; 3 \le i \le 2n-1, i \text{ is odd and } 1 \le j \le m\} \cup \{x_{2n}^{j}x_{2n}^{j+1}; 1 \le j \le 3m\}$$

The number of vertices in octagonal network is $(4m+2)n+2m$ and number of edges in an octagonal network is $(6m+1)n+m$.

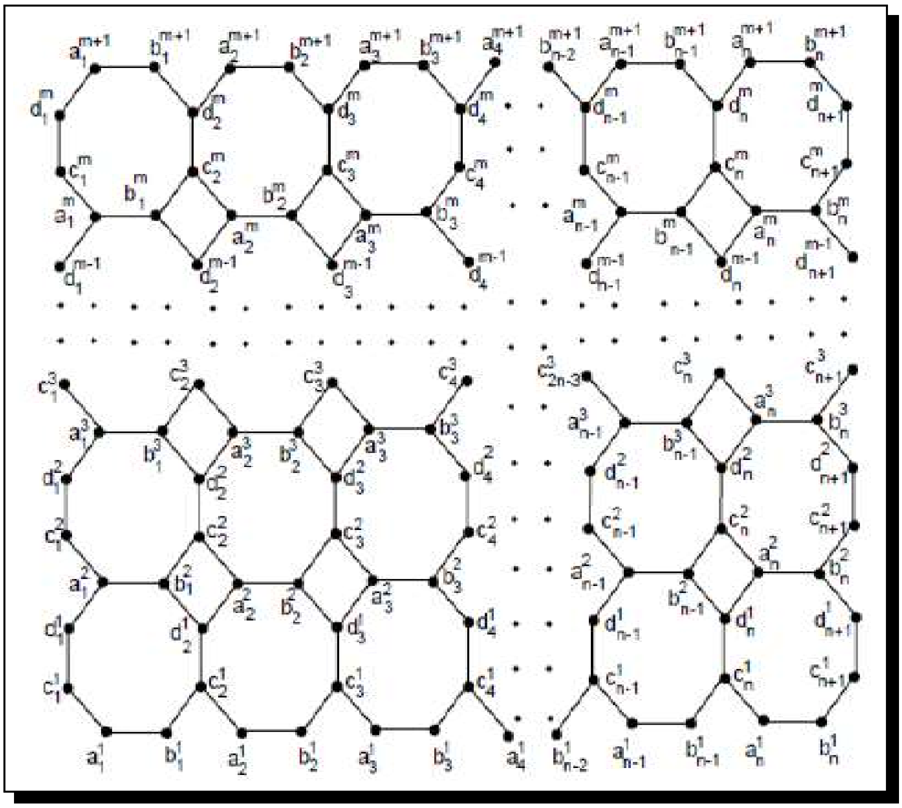

Figure 1. The Octagonal network $O_{\{n,m\}}$

2. Main Results

In this part we give our main computational results.

Theorem 2.1. *Let $O_{\{n,m\}}$ be the octagonal network. Then the M-polynomial of $O_{\{n,m\}}$ is*

$$M(DPZ_n,x,y) = (2n+2m+4)x^2y^2 + (4n+4m-8)x^2y^3 + (6mn-5n-5m+4)x^3y^3.$$

Proof. Let $O_{\{n,m\}}$ be the octagonal network. The edge set $E(O_{\{n,m\}})$ is divided into three edge partitions based on degrees of end vertices. The first edge partition $E_1(O_{\{n,m\}})$ contains $2n+2m+4$ edges uv, where $d_u = d_v = 2$. The second edge partition $E_2(O_{\{n,m\}})$ contains $4n+4m-8$ edges uv, where $d_u = 2$, $d_v = 3$. The third edge partition $E_3(O_{\{n,m\}})$ contains $6mn-5n-5m+4$ edges uv, where $d_u = d_v = 3$. From Definition 1.1 the M-polynomial of $O_{\{n,m\}}$ is given by

$$\begin{aligned} M(O_{\{n,m\}};x,y) &= \sum_{i\le j} m_{ij}x^iy^j \\ &= \sum_{2\le 2} m_{22}x^2y^2 + \sum_{2\le 3} m_{23}x^2y^3 + \sum_{3\le 3} m_{33}x^3y^3 \\ &= \sum_{uv\in E_1(O_{\{n,m\}})} m_{22}x^2y^2 + \sum_{uv\in E_2(O_{\{n,m\}})} m_{23}x^2y^3 + \sum_{uv\in E_3(O_{\{n,m\}})} m_{33}x^3y^3 \end{aligned}$$

$$= |E_1(O_{\{n,m\}})|x^2y^2 + |E_2(O_{\{n,m\}})|x^2y^3 + |E_3(O_{\{n,m\}})|x^3y^3$$
$$= (2n+2m+4)x^2y^2 + (4n+4m-8)x^2y^3 + (6mn-5n-5m+4)x^3y^3.$$

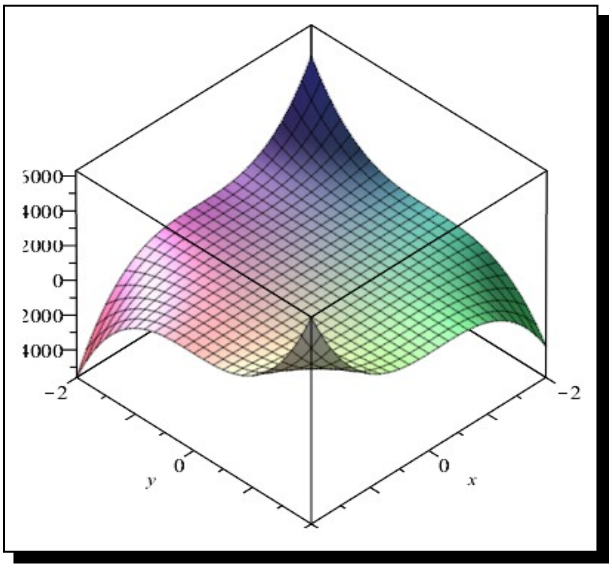

Figure 2. Plot of the M-polynomial of $O_{\{4,5\}}$

Now, we compute some degree-based topological indices from this M-polynomial.

Proposition 2.1. *Let* $O_{\{n,m\}}$ *be the octagonal network. Then*

$$M_1(O_{\{n,m\}}) = 36mn - 2m - 2n,$$
$$M_2(O_{\{n,m\}}) = 54mn - 13m - 13n + 4,$$
$$^mM_2(O_{\{n,m\}}) = \frac{11}{18}n + \frac{11}{18}m + \frac{1}{9} + \frac{2}{3}mn,$$
$$R_\alpha(O_{\{n,m\}}) = (2n+2m+4)4^\alpha + (4n+4m-8)6^\alpha + (6mn-5m-5n+4)9^\alpha,$$
$$R_\alpha(O_{\{n,m\}}) = \frac{(2n+2m+4)}{4^\alpha} + \frac{(4n+4m-8)}{6^\alpha} + \frac{(6mn-5m-5n+4)}{9^\alpha},$$
$$SSD(O_{\{n,m\}}) = \frac{8}{3}n + \frac{8}{3}m - \frac{4}{3} + 12mn,$$
$$H(O_{\{n,m\}}) = \frac{14}{15}n + \frac{14}{15}m + \frac{2}{15} + 2mn,$$
$$I(O_{\{n,m\}}) = -\frac{7}{10}n - \frac{7}{10}m + \frac{2}{5} + 9mn,$$
$$A(O_{\{n,m\}}) = -\frac{573}{64}n - \frac{573}{64}m + \frac{217}{16} + \frac{2187}{32}mn.$$

Proof. Let $M(O_{\{n,m\}};x,y) = f(x,y) = (2n+2m+4)x^2y^2+(4n+4m-8)x^2y^3+(6mn-5n-5m+4)x^3y^3$. Then

$$D_xf(x,y) = 2(2n+2m+4)x^2y^2 + 2(4n+4m-8)x^2y^3 + 3(6mn-5n-5m+4)x^3y^3,$$
$$D_yf(x,y) = 2(2n+2m+4)x^2y^2 + 3(4n+4m-8)x^2y^3 + 3(6mn-5n-5m+4)x^3y^3,$$

$$D_yD_xf(x,y) = 4(2n+2m+4)x^2y^2 + 6(4n+4m-8)x^2y^3 + 9(6mn-5n-5m+4)x^3y^3,$$

$$S_y(f(x,y)) = \frac{(2n+2m+4)}{2}x^2y^2 + \frac{(4n+4m-8)}{3}x^2y^3 + \frac{(6mn-5n-5m+4)}{3}x^3y^3,$$

$$S_xS_y(f(x,y)) = \frac{(2n+2m+4)}{4}x^2y^2 + \frac{(4n+4m-8)}{6}x^2y^3 + \frac{(6mn-5n-5m+4)}{9}x^3y^3,$$

$$D_y^\alpha(f(x,y)) = 2^\alpha(2n+2m+4)x^2y^2 + 3^\alpha(4n+4m-8)x^2y^3 + 3^\alpha(6mn-5n-5m+4)x^3y^3,$$

$$D_x^\alpha D_y^\alpha(f(x,y)) = 2^{2\alpha}(2n+2m+4)x^2y^2 + 2^\alpha 3^\alpha(4n+4m-8)x^2y^3 + 3^{2\alpha}(6mn-5n-5m+4)x^3y^3,$$

$$S_y^\alpha(f(x,y)) = \frac{(2n+2m+4)}{2^\alpha}x^2y^2 + \frac{(4n+4m-8)}{3^\alpha}x^2y^3 + \frac{(6mn-5n-5m+4)}{3^\alpha}x^3y^3,$$

$$S_x^\alpha S_y^\alpha(f(x,y)) = \frac{(2n+2m+4)}{2^{2\alpha}}x^2y^2 + \frac{(4n+4m-8)}{2^\alpha 3^\alpha}x^2y^3 + \frac{(6mn-5n-5m+4)}{3^{2\alpha}}x^3y^3,$$

$$S_yD_x(f(x,y)) = (2n+2m+4)x^2y^2 + \frac{2(4n+4m-8)}{3}x^2y^3 + (6mn-5n-5m+4)x^3y^3,$$

$$S_xD_y(f(x,y)) = (2n+2m+4)x^2y^2 + \frac{3(4n+4m-8)}{2}x^2y^3 + (6mn-5n-5m+4)x^3y^3,$$

$$Jf(x,y) = (2n+2m+4)x^4 + (4n+4m-8)x^5 + (6mn-5n-5m+4)x^6,$$

$$S_xJf(x,y) = \frac{(2n+2m+4)}{4}x^4 + \frac{(4n+4m-8)}{5}x^5 + \frac{(6mn-5n-5m+4)}{6}x^6,$$

$$JD_xD_yf(x,y) = 4(2n+2m+4)x^4 + 6(4n+4m-8)x^5 + 9(6mn-5n-5m+4)x^6,$$

$$S_xJD_xD_yf(x,y) = (2n+2m+4)x^4 + \frac{6(4n+4m-8)}{5}x^5 + \frac{9(6mn-5n-5m+4)}{6}x^6,$$

$$D_y^3f(x,y) = 2^3(2n+2m+4)x^2y^2 + 3^3(4n+4m-8)x^2y^3 + 3^3(6mn-5n-5m+4)x^3y^3,$$

$$D_x^3D_y^3f(x,y) = 2^6(2n+2m+4)x^2y^2 + 2^33^3(4n+4m-8)x^2y^3 + 3^6(6mn-5n-5m+4)x^3y^3,$$

$$JD_x^3D_y^3f(x,y) = 2^6(2n+2m+4)x^4 + 2^33^3(4n+4m-8)x^5 + 3^6(6mn-5n-5m+4)x^6,$$

$$Q_{-2}JD_x^3D_y^3f(x,y) = 2^6(2n+2m+4)x^2 + 2^33^3(4n+4m-8)x^3 + 3^6(6mn-5n-5m+4)x^4,$$

$$S_x^3JD_x^3D_y^3f(x,y) = 2^3(2n+2m+4)x^2 + 2^3(4n+4m-8)x^3 + \frac{3^6(6mn-5n-5m+4)}{4^3}x^4,$$

$$M_1(O_{\{n,m\}}) = (D_x+D_y)f(x,y)|_{x=y=1} = 36mn-2m-2n,$$

$$M_2(O_{\{n,m\}}) = D_yD_x(f(x,y))|_{x=y=1} = 54mn-13m-13n+4,$$

$${}^mM_2(O_{\{n,m\}}) = S_xS_y(f(x,y))|_{x=y=1} = \frac{11}{18}n + \frac{11}{18}m + \frac{1}{9} + \frac{2}{3}mn,$$

$$R_\alpha(O_{\{n,m\}}) = D_x^\alpha D_y^\alpha(f(x,y))|_{x=y=1} = (2n+2m+4)4^\alpha + (4n+4m-8)6^\alpha + (6mn-5m-5n+4)9^\alpha,$$

$$R_\alpha(O_{\{n,m\}}) = S_x^\alpha S_y^\alpha(f(x,y))|_{x=y=1} = \frac{(2n+2m+4)}{4^\alpha} + \frac{(4n+4m-8)}{6^\alpha} + \frac{(6mn-5m-5n+4)}{9^\alpha},$$

$$SSD(O_{\{n,m\}}) = (S_yD_x+S_xD_y)(f(x,y))|_{x=y=1} = \frac{8}{3}n + \frac{8}{3}m - \frac{4}{3} + 12mn,$$

$$H(O_{\{n,m\}}) = 2S_xJ(f(x,y))|_{x=1} = \frac{14}{15}n + \frac{14}{15}m + \frac{2}{15} + 2mn,$$

$$I(O_{\{n,m\}}) = S_xJD_xD_y(f(x,y))_{x=1} = -\frac{7}{10}n - \frac{7}{10}m + \frac{2}{5} + 9mn,$$

$$A(O_{\{n,m\}}) = S_x^3 Q_{-2} J D_x^3 D_y^3(f(x,y)) = -\frac{573}{64}n - \frac{573}{64}m + \frac{217}{16} + \frac{2187}{32}mn. \qquad \square$$

Theorem 2.2. *Let $O_{\{n,m\}}$ be the octagonal network. Then the first and second Zagreb polynomials of $O_{\{n,m\}}$ are*

(1) $M_1(O_{\{n,m\}},x) = 6mnx^6 - 5mx^6 - 5nx^6 + 4mx^5 + 4nx^5 + 4x^6 + 2mx^4 + 2nx^4 - 8x^5 + 4x^4$.

(2) $M_2(O_{\{n,m\}},x) = 6mnx^9 - 5mx^9 - 5nx^9 + 4x^9 + 4mx^6 + 4nx^6 - 8x^6 + 2mx^4 + 2nx^4 + 4x^4$.

Proof. Let $O_{\{n,m\}}$ be the octagonal network. Then

(1) by the definition the first Zagreb polynomial (Definition 1.5)

$$\begin{aligned} M_1(O_{\{n,m\}},x) &= \sum_{uv\in E(O_{\{n,m\}})} x^{[d_u+d_v]} \\ &= \sum_{uv\in E_1(O_{\{n,m\}})} x^{[d_u+d_v]} + \sum_{uv\in E_2(O_{\{n,m\}})} x^{[d_u+d_v]} + \sum_{uv\in E_3(O_{\{n,m\}})} x^{[d_u+d_v]} \\ &= |E_1(O_{\{n,m\}})|x^4 + |E_2(O_{\{n,m\}})|x^5 + |E_3(O_{\{n,m\}})|x^6 \\ &= 6mnx^6 - 5mx^6 - 5nx^6 + 4mx^5 + 4nx^5 + 4x^6 + 2mx^4 + 2nx^4 - 8x^5 + 4x^4. \end{aligned}$$

(2) Now by definition the second Zagreb polynomial (Definition 1.6)

$$\begin{aligned} M_2(O_{\{n,m\}},x) &= \sum_{uv\in E(O_{\{n,m\}})} x^{[d_u\times d_v]} \\ &= \sum_{uv\in E_1(O_{\{n,m\}})} x^{[d_u\times d_v]} + \sum_{uv\in E_2(O_{\{n,m\}})} x^{[d_u\times d_v]} + \sum_{uv\in E_3(O_{\{n,m\}})} x^{[d_u\times d_v]} \\ &= |E_1(O_{\{n,m\}})|x^4 + |E_2(O_{\{n,m\}})|x^6 + |E_3(O_{\{n,m\}})|x^9 \\ &= 6mnx^9 - 5mx^9 - 5nx^9 + 4x^9 + 4mx^6 + 4nx^6 - 8x^6 + 2mx^4 + 2nx^4 + 4x^4. \square \end{aligned}$$

Proposition 2.2. *Let $O_{\{n,m\}}$ be the octagonal network. Then the hyper-Zagreb index, first multiple Zagreb index and the second multiple Zagreb index of $O_{\{n,m\}}$ are*

(1) $HM(O_{\{n,m\}}) = 216mn - 48m - 48n + 8$.

(2) $PM_1(O_{\{n,m\}}) = \frac{331776}{390625} \times 46656^{mn} \times 6^{-5n-5m} \times 10^{4n+4m}$.

(3) $PM_2(O_{\{n,m\}}) = 531441^{mn} \times 3^{-10n-10m} \times 12^{4n+4m}$.

Proof. (1) By definition of hyper-Zagreb index (Definition 1.2)

$$\begin{aligned} HM(O_{\{n,m\}}) &= \sum_{uv\in E(O_{\{n,m\}})} [d_u+d_v]^2 \\ &= \sum_{uv\in E_1(O_{\{n,m\}})} [d_u+d_v]^2 + \sum_{uv\in E_2(O_{\{n,m\}})} [d_u+d_v]^2 + \sum_{uv\in E_3(O_{\{n,m\}})} [d_u+d_v]^2 \\ &= 16|E_1(O_{\{n,m\}})| + 25|E_2(O_{\{n,m\}})| + 36|E_3(O_{\{n,m\}})| \\ &= 16(2n+2m+4) + 25(4n+4m-8) + 36(6mn-5n-5m+4) \\ &= 216mn - 48m - 48n + 8. \end{aligned}$$

(2) By the definition of first multiple Zagreb index (Definition 1.3)

$$PM_1(O_{\{n,m\}}) = \prod_{uv\in E(O_{\{n,m\}})} [d_u+d_v]$$

$$= \prod_{uv\in E_1(O_{\{n,m\}})}[d_u+d_v]\times\prod_{uv\in E_2(O_{\{n,m\}})}[d_u+d_v]\times\prod_{uv\in E_3(O_{\{n,m\}})}[d_u+d_v]$$
$$= 4^{|E_1(O_{\{n,m\}})|}\times 5^{|E_2(O_{\{n,m\}})|}\times 6^{|E_3(O_{\{n,m\}})|}$$
$$= 4^{2n+2m+4}\times 5^{4n+4m-8}\times 6^{6mn-5n-5m+4}$$
$$= \frac{331776}{390625}\times 46656^{mn}\times 6^{-5n-5m}\times 10^{4n+4m}.$$

(3) By the definition of second multiple Zagreb index (Definition 1.4)

$$PM_2(O_{\{n,m\}}) = \prod_{uv\in E(O_{\{n,m\}})}[d_u\times d_v]$$
$$= \prod_{uv\in E_1(O_{\{n,m\}})}[d_u\times d_v]\times\prod_{uv\in E_2(O_{\{n,m\}})}[d_u\times d_v]\times\prod_{uv\in E_3(O_{\{n,m\}})}[d_u\times d_v]$$
$$= 4^{|E_1(O_{\{n,m\}})|}\times 6^{|E_2(O_{\{n,m\}})|}\times 9^{|E_3(O_{\{n,m\}})|}$$
$$= 4^{2n+2m+4}\times 6^{4n+4m-8}\times 9^{6mn-5n-5m+4}$$
$$= 531441^{mn}\times 3^{-10n-10m}\times 12^{4n+4m}.$$ □

3. Conclusions

In this article we computed many topological indices for the octagonal network. At first, we gave the general closed form of M-polynomial of this family and recover many degree-based topological indices out of it. We also computed multiple Zagreb indices and Zagreb polynomials of the octagonal network. These results can play a vital role in determining properties of this network and its uses in industry, electronics, and pharmacy [23, 25].

Acknowledgments

This research is supported by Gyeongsang National University, Jinju 52828, Korea.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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