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On Contra $\mathscr{I}_g$-continuity in Ideal Topological Spaces

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Abstract. In this paper, $\mathscr{I}_g$-closed sets and $\mathscr{I}_g$-open sets are used to define and investigate a new class of functions called contra $\mathscr{I}_g$-continuous functions in ideal topological spaces. We discuss the relationships with some other related functions.

1. Introduction

An *ideal* $\mathscr{I}$ on a topological space (X,τ) is a nonempty collection of subsets of X which satisfies

(i) $A \in \mathscr{I}$ and $B \subset A$ implies $B \in \mathscr{I}$ and
(ii) $A \in \mathscr{I}$ and $B \in \mathscr{I}$ implies $A \cup B \in \mathscr{I}$.

Given a topological space (X,τ) with an ideal $\mathscr{I}$ on X and if $\mathscr{P}(X)$ is the set of all subsets of X, a set operator $(\cdot)^* : \mathscr{P}(X) \to \mathscr{P}(X)$, called a *local function* [16] of A with respect to τ and $\mathscr{I}$ is defined as follows: for $A \subset X$, $A^*(\tau,\mathscr{I}) = \{x \in X / U \cap A \notin \mathscr{I}\}$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau / x \in U\}$. A Kuratowski closure operator $\mathrm{cl}^*(\cdot)$ for a topology $\tau^*(\tau,\mathscr{I})$ called the $*$-*topology*, finer than τ is defined by $\mathrm{cl}^*(A) = A \cup A^*(\tau,\mathscr{I})$ [9]. When there is no chance of confusion, we will simply write A^* for $A^*(\tau,\mathscr{I})$ and τ^* for $\tau^*(\tau,\mathscr{I})$. If $\mathscr{I}$ is an ideal on X then $(X,\tau,\mathscr{I})$ is called an ideal topological space. A subset A of an ideal space $(X,\tau,\mathscr{I})$ is $*$-*closed*(τ^*-*closed*) [8] if $A^* \subset A$. By a space, we always mean a topological space (X,τ) with no separation properties assumed. If $A \subset X$, $\mathrm{cl}(A)$ and $\mathrm{int}(A)$ will denote the closure and interior of A in (X,τ) respectively. A subset A of (X,τ) is said to be *regular open* [15] (resp. *regular closed* [15]) if $A = \mathrm{int}(\mathrm{cl}(A))$ (resp. $A = \mathrm{cl}(\mathrm{int}(A))$). In this paper, we introduce the notion of contra $\mathscr{I}_g$-continuity in ideal topological spaces and discuss their properties and give various characterizations.

2. Preliminaries

A subset A of an ideal space $(X,\tau,\mathscr{I})$ is $\mathscr{I}_g$-*closed* [10] if $A^* \subset U$ whenever $A \subset U$ and U is open. The complement of an $\mathscr{I}_g$-closed set is $\mathscr{I}_g$-open. The family of all

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$\mathscr{I}_g$-open sets is denoted by IGO(X). A subset of an ideal space $(X,\tau,\mathscr{I})$ is said to $\mathscr{I}_{rg}$-*closed* [12] if $A^* \subset U$ whenever $A \subset U$ and U is regular open. A subset A is called $\mathscr{I}_{rg}$-open if $X - A$ is $\mathscr{I}_{rg}$-closed and every rg-closed set is an $\mathscr{I}_{rg}$-closed set. An ideal topological space $(X,\tau,\mathscr{I})$ is said to be $\mathscr{I}_g$-*normal* [11] if each pair of nonempty disjoint closed sets can be seperated by disjoint $\mathscr{I}_g$-open sets. A function $f : (X,\tau,\mathscr{I}) \to (Y,\sigma)$ is called $\mathscr{I}_g$-continuous [7] if the inverse image of every closed set in Y is $\mathscr{I}_g$-closed in X. In a topological space (X,τ), a function $f : (X,\tau) \to (Y,\sigma)$ is said to be *contra continuous* [4] if for each open set V in $Y, f^{-1}(V)$ is closed in X and $f : (X,\tau) \to (Y,\sigma)$ is said to be *contra g-continuous* [3] if for each open set V in Y, $f^{-1}(V)$ is g-closed in X. A function $f : (X,\tau) \to (Y,\sigma)$ is called *preclosed* [6] if the image of every closed subset of X is preclosed in Y. A space (X,τ) is called *locally indiscrete* [13] if every open set is closed. A space (X,τ) is said to be *g-space* [3](resp. *gS-space* [3]) if every g-open set of X is open(resp. semiopen) in X. A space (X,τ) is said to be g-T_2 [2] if for each pair of distinct points x and y in X there exist two g-open sets U containing x and V containing y such that $U \cap V = \phi$. A space (X,τ) is said to be *g-normal* [3] if each pair of nonempty disjoint closed sets can be separated by disjoint g-open sets. A space (X,τ) is said to be an *Ultra Hausdorff space* [14] if for each pair of distinct points x and y in X there exist two clopen sets U containing x and V containing y such that $U \cap V = \phi$. A space (X,τ) is said to be *GO-connected* [1] if X cannot be expressed as two disjoint nonempty g-open sets of X.

3. Contra $\mathscr{I}_g$-continuity

Definition 3.1. *A* function $f : (X,\tau,\mathscr{I}) \to (Y,\sigma)$ is said to be *contra* $\mathscr{I}_g$-*continuous* if $f^{-1}(V)$ is $\mathscr{I}_g$-closed in $(X,\tau,\mathscr{I})$ for each open set V in (Y,σ).

Definition 3.2. *A* function $f : (X,\tau,\mathscr{I}) \to (Y,\sigma)$ is said to be *contra* $\mathscr{I}_{rg}$-*continuous* if $f^{-1}(V)$ is $\mathscr{I}_{rg}$-closed in $(X,\tau,\mathscr{I})$ for each open set V in (Y,σ).

Proposition 3.3. *Every contra g-continuous function is contra* $\mathscr{I}_g$-*continuous.*

Proof. Let $f : (X,\tau,\mathscr{I}) \to (Y,\sigma)$ be a contra g-continuous function and let V be any open set in Y. Then, $f^{-1}(V)$ is g-closed in X. Since every g-closed set is $\mathscr{I}_g$-closed, $f^{-1}(V)$ is $\mathscr{I}_g$-closed in X. Therefore f is contra $\mathscr{I}_g$-continuous. □

However, converse need not true as seen from the following example.

Example 3.4. Let $X = \{a,b,c,d\}$, $\tau = \{\phi,\{b\},\{b,c,d\},X\}$, $\sigma = \{\phi,\{a\},\{c\},\{a,c\},X\}$ and $\mathscr{I} = \{\phi,\{c\}\}$. Then the identity function $f : (X,\tau,\mathscr{I}) \to (Y,\sigma)$ is contra $\mathscr{I}_g$-continuous but not contra g-continuous.

Remark 3.5. The following example shows that $\mathscr{I}_g$-continuity and contra $\mathscr{I}_g$-continuity are independent.

Example 3.6. Let $X = \{a,b,c\}$, $\tau = \{\phi,\{a\},X\}$, $\sigma = \{\phi,\{b\},\{b,c\},X\}$ and $\mathscr{I} = \{\phi,\{c\}\}$. Then the identity function $f : (X,\tau,\mathscr{I}) \to (X,\sigma)$ is contra $\mathscr{I}_g$-continuous but not $\mathscr{I}_g$-continuous. The function $f : (X,\tau,\mathscr{I}) \to (X,\sigma)$ defined by $f(a) = c, f(b) = a$ and $f(c) = b$ is $\mathscr{I}_g$-continuous but not contra $\mathscr{I}_g$-continuous.

Proposition 3.7. *Every contra $\mathscr{I}_g$-continuous function is contra $\mathscr{I}_{rg}$-continuous.*

Proof. The proof follows from the fact that every $\mathscr{I}_g$-closed set is $\mathscr{I}_{rg}$-closed in X. □

Example 3.8. Let $X = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{c\},\{a,c\},X\}$, $\sigma = \{\phi,\{b\},\{a,c\},X\}$ and $\mathscr{I} = \{\phi\ \{c\}\}$. Then the identity function $f : (X,\tau,\mathscr{I}) \to (X,\sigma)$ is contra $\mathscr{I}_{rg}$-continuous but not contra $\mathscr{I}_g$-continuous.

Definition 3.9. A map $f : (X,\tau,\mathscr{I}) \to (Y,\sigma)$ is called *contra $*$-continuous* if the inverse image of every open set in (Y,σ) is $*$-closed in $(X,\tau,\mathscr{I})$.

Proposition 3.10. *Every contra $*$-continuous function is contra $\mathscr{I}_g$-continuous.*

Proof. Let $f : (X,\tau,\mathscr{I}) \to (Y,\sigma)$ be a contra $*$-continuous function and let V be any open set in Y. Then, $f^{-1}(V)$ is $*$-closed in X. Since every $*$-closed set is $\mathscr{I}_g$-closed, $f^{-1}(V)$ is $\mathscr{I}_g$-closed in X. □

However, converse need not true as seen from the following example.

Example 3.11. Let $X = \{a,b,c\}$, $\tau = \{\phi,\{a\},X\}$, $\sigma = \{\phi,\{b\},\{b,c\},X\}$ and $\mathscr{I} = \{\phi,\{c\}\}$. Then the identity function $f : (X,\tau,\mathscr{I}) \to (X,\sigma)$ is contra $\mathscr{I}_g$-continuous but not contra $*$-continuous.

Theorem 3.12. *Let $f : (X,\tau,\mathscr{I}) \to (Y,\sigma)$ be a function. Then the following are equivalent:*

(i) *f is contra $\mathscr{I}_g$-continuous.*
(ii) *The inverse image of each closed set in Y is $\mathscr{I}_g$-open in X.*
(iii) *For each point x in X and each closed set V in Y with $f(x) \in V$, there is an $\mathscr{I}_g$-open set U in X containing x such that $f(U) \subset V$.*

Proof. (i)⇒(ii). Let F be closed in Y. Then $Y - F$ is open in Y. By definition of contra $\mathscr{I}_g$-continuous, $f^{-1}(Y-F)$ is $\mathscr{I}_g$-closed in X. But $f^{-1}(Y-F) = X - f^{-1}(F)$. This implies $f^{-1}(F)$ is $\mathscr{I}_g$-open in X.

(ii)⇒(iii). Let $x \in X$ and V be any closed set in Y with $f(x) \in V$. By (ii), $f^{-1}(V)$ is $\mathscr{I}_g$-open in X. Set $U = f^{-1}(V)$. Then there is an $\mathscr{I}_g$-open set U in X containing x such that $f(U) \subset V$.

(iii)⇒(i). Let $x \in X$ and V be any closed set in Y with $f(x) \in V$. Then $Y - V$ is open in Y with $f(x) \in V$. By (iii), there is an $\mathscr{I}_g$-open set U in X containing x such that $f(U) \subset V$. This implies $U = f^{-1}(V)$. Therefore, $X - U = X - f^{-1}(V) = f^{-1}(Y - V)$ which is $\mathscr{I}_g$-closed in X. □

Theorem 3.13. *Let $f : (X,\tau,\mathscr{I}) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\mu)$. Then the following properties hold:*

(i) *If f is contra $\mathscr{I}_g$-continuous and g is continuous then $g \circ f$ is contra $\mathscr{I}_g$-continuous.*

(ii) *If f is contra $\mathscr{I}_g$-continuous and g is contra continuous then $g \circ f$ is $\mathscr{I}_g$-continuous.*

(iii) *If f is $\mathscr{I}_g$-continuous and g is contra continuous then $g \circ f$ is contra $\mathscr{I}_g$-continuous.*

Proof. (i) Let V be a closed set in Z. Since g is continuous, $g^{-1}(V)$ is closed in Y. Since f is contra $\mathscr{I}_g$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\mathscr{I}_g$-open in X. Therefore $g \circ f$ is contra $\mathscr{I}_g$-continuous.

(ii) Let V be any closed set in Z. Since g is contra continuous, $g^{-1}(V)$ is open in Y. Since f is contra $\mathscr{I}_g$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\mathscr{I}_g$-closed in X. Therefore $g \circ f$ is $\mathscr{I}_g$-continuous.

(iii) Let V be any closed set in Z. Since g is contra continuous, $g^{-1}(V)$ is open in Y. Since f is $\mathscr{I}_g$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\mathscr{I}_g$-open in X. Therefore $g \circ f$ is contra $\mathscr{I}_g$-continuous. □

Theorem 3.14. *If a function $f : (X,\tau,\mathscr{I}) \to (Y,\sigma)$ is contra $\mathscr{I}_g$-continuous and Y is regular, then f is $\mathscr{I}_g$-continuous.*

Proof. Let x be an arbitrary point of X and V be an open set of Y containing $f(x)$. Since Y is regular, there exists an open set W in Y containing $f(x)$ such that $\mathrm{cl}(W) \subset V$. Since f is contra $\mathscr{I}_g$-continuous, by Theorem 3.12, there exists an $\mathscr{I}_g$-open set U containing x such that $f(U) \subset \mathrm{cl}(W)$. Thus $f(U) \subset \mathrm{cl}(W) \subset V$. Hence f is $\mathscr{I}_g$-continuous. □

Definition 3.15. A space $(X,\tau,\mathscr{I})$ is said to be an $\mathscr{I}_g$*-space* if every $\mathscr{I}_g$-open set is $*$-open in $(X,\tau,\mathscr{I})$.

Theorem 3.16. *A function $f : (X,\tau,\mathscr{I}) \to (Y,\sigma)$ is contra $\mathscr{I}_g$-continuous and X is an $\mathscr{I}_g$-space then f is contra $*$-continuous.*

Proof. Let V be a closed set in Y. Since f is contra $\mathscr{I}_g$-continuous, $f^{-1}(V)$ is $\mathscr{I}_g$-open in X. Since X is an $\mathscr{I}_g$-space, $f^{-1}(V)$ is $*$-open in X. Therefore f is contra $*$-continuous. □

Definition 3.17. An ideal topological space $(X,\tau,\mathscr{I})$ is said to be $\mathscr{I}_g$-T_2 *space* if for each pair of distinct points x and y in $(X,\tau,\mathscr{I})$, there exists an $\mathscr{I}_g$-open set U containing x and an $\mathscr{I}_g$-open set V containing y such that $U \cap V = \phi$.

Theorem 3.18. *If $(X,\tau,\mathscr{I})$ is an ideal topological space and for each pair of distinct points x_1,x_2 in X, there exists a function f from $(X,\tau,\mathscr{I})$ into a Urysohn space Y such that $f(x_1) \neq f(x_2)$ and f is contra $\mathscr{I}_g$-continuous at x_1 and x_2, then X is $\mathscr{I}_g$-T_2.*

Proof. Let x_1 and x_2 be any two distinct points in X. Then by hypothesis, there is a function $f:(X,\tau,\mathscr{I})\to(Y,\sigma)$, such that $f(x_1)\neq f(x_2)$. Let $y_i=f(x_i)$ for $i=1,2$. Then $y_1\neq y_2$. Since Y is Urysohn, there exists open neighbourhoods V_{y_1} and V_{y_2} of y_1 and y_2 respectively in Y such that $\mathrm{cl}(V_{y_1})\cap\mathrm{cl}(V_{y_2})=\phi$. Since f is contra $\mathscr{I}_g$-continuous, there exists an $\mathscr{I}_g$-open set U_{x_i} of x_i in X such that $f(U_{x_i})\subset\mathrm{cl}(V_{y_i})$ for $i=1,2$. Hence we get $U_{x_1}\cap U_{x_2}=\phi$ because $\mathrm{cl}(V_{y_1})\cap\mathrm{cl}(V_{y_2})=\phi$. Thus X is $\mathscr{I}_g$-T_2. □

Corollary 3.19. *If f is a contra $\mathscr{I}_g$-continuous injection of an ideal topological space $(X,\tau,\mathscr{I})$ into a Urysohn space (Y,σ), then $(X,\tau,\mathscr{I})$ is $\mathscr{I}_g$-T_2.*

Proof. Let x_1 and x_2 be any pair of distinct points in X. Since f is contra $\mathscr{I}_g$-continuous and injective, we have $f(x_1)\neq f(x_2)$. Therefore by Theorem 3.18, X is $\mathscr{I}_g$-T_2. □

Corollary 3.20. *If f is a contra $\mathscr{I}_g$-continuous injection of an ideal topological space $(X,\tau,\mathscr{I})$ into a Ultra Hausdorff space (Y,σ), then $(X,\tau,\mathscr{I})$ is $\mathscr{I}_g$-T_2.*

Proof. Let x_1 and x_2 be any two distinct points in X. Then since f is injective and Y is Ultra Hausdorff, $f(x_1)\neq f(x_2)$ and there exists two clopen sets V_1 and V_2 in Y such that $f(x_1)\in V_1$, $f(x_2)\in V_2$ and $V_1\cap V_2=\phi$. Then $x_i\in f^{-1}(V_i)\in\mathrm{IGO}(X)$ for $i=1,2$ and $f^{-1}(V_1)\cap f^{-1}(V_2)=\phi$. Thus X is $\mathscr{I}_g$-T_2. □

Theorem 3.21. *If $f:(X,\tau,\mathscr{I})\to(Y,\sigma)$ is a contra $\mathscr{I}_g$-continuous, closed injection and Y is Ultra normal, then $(X,\tau,\mathscr{I})$ is $\mathscr{I}_g$-normal.*

Proof. Let F_1 and F_2 be disjoint closed subsets of X. Since f is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y. Since Y is Ultra normal, $f(F_1)$ and $f(F_2)$ are separated by disjoint clopen sets V_1 and V_2 respectively. Hence $F_i\subset f^{-1}(V_i)$, $f^{-1}(V_i)\in\mathrm{IGO}(X)$ for $i=1,2$ and $f^{-1}(V_1)\cap f^{-1}(V_2)=\phi$. Thus X is $\mathscr{I}_g$-normal. □

Definition 3.22. A graph $G(f)$ of a function $f:(X,\tau,\mathscr{I})\to(Y,\sigma)$ is said to be contra $\mathscr{I}_g$-closed if for each $(x,y)\in(X\times Y)\backslash G(f)$, there exist an $U\in\mathrm{IGO}(X)$ containing x and a closed set V of (Y,σ) containing y such that $f(U)\cap V=\phi$.

Theorem 3.23. *If $f:(X,\tau,\mathscr{I})\to(Y,\sigma)$ is contra $\mathscr{I}_g$-continuous and (Y,σ) is Urysohn, then $G(f)$ is contra $\mathscr{I}_g$-closed in $X\times Y$.*

Proof. Let $(x,y)\in(X\times Y)\backslash G(f)$, then $f(x)\neq y$ and there exists open sets V, W such that $f(x)\in V$, $y\in W$ and $\mathrm{cl}(V)\cap\mathrm{cl}(W)=\phi$. Since f is contra $\mathscr{I}_g$-continuous there exists $U\in\mathrm{IGO}(X)$ containing x such that $f(U)\subset\mathrm{cl}(V)$. Since $\mathrm{cl}(V)\cap\mathrm{cl}(W)=\phi$, we have $f(U)\cap\mathrm{cl}(W)=\phi$. This shows that $G(f)$ is contra $\mathscr{I}_g$-closed in $X\times Y$. □

Remark 3.24. The following example shows that the condition Urysohn on the space (Y,σ) in Theorem 3.23 cannot be dropped.

Example 3.25. Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$, $\sigma = \{\phi, \{b\}, \{b, c\}, X\}$ and $\mathscr{I} = \{\phi, \{c\}\}$. Clearly X is not a Urysohn space. Also the identity function $f : (X, \tau, \mathscr{I}) \to (X, \sigma)$ is contra $\mathscr{I}_g$ -continuous but not contra $\mathscr{I}_g$ -closed.

Corollary 3.26 ([3], Theorem 2.26). *If $f : (X, \tau) \to (X, \sigma)$ is contra g-continuous function and (Y, σ) is a Urysohn space, then $G(f)$ is contra-g-closed in $X \times Y$.*

Proof : The proof follows from the Theorem 3.23 if $\mathscr{I} = \{\phi\}$.

Definition 3.27. An ideal topological space $(X, \tau, \mathscr{I})$ is said to be *$\mathscr{I}_g$-connected* if $(X, \tau, \mathscr{I})$ cannot be expressed as the union of two disjoint nonempty $\mathscr{I}_g$-open subsets of $(X, \tau, \mathscr{I})$.

Theorem 3.28. *A contra $\mathscr{I}_g$-continuous image of a $\mathscr{I}_g$-connected space is connected.*

Proof. Let $f : (X, \tau, \mathscr{I}) \to (Y, \sigma)$ be a contra $\mathscr{I}_g$-continuous function of an $\mathscr{I}_g$-connected space $(X, \tau, \mathscr{I})$ onto a topological space (Y, σ). If possible, let Y be disconnected. Let A and B form a disconnection of Y. Then A and B are clopen and $Y = A \cup B$ where $A \cap B = \phi$. Since f is contra $\mathscr{I}_g$-continuous, $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are nonempty $\mathscr{I}_g$-open sets in X. Also $f^{-1}(A) \cap f^{-1}(B) = \phi$. Hence X is not $\mathscr{I}_g$-connected. This is a contradiction. Therefore Y is connected. □

Corollary 3.29 ([3], Theorem 2.27). *A contra g-continuous image of a g-connected space is connected.*

Proof. The proof follows from the theorem 3.28 if $\mathscr{I} = \{\phi\}$. □

Lemma 3.30. *For an ideal topological space $(X, \tau, \mathscr{I})$, the following are equivalent.*

(i) *X is $\mathscr{I}_g$-connected.*

(ii) *The only subset of X which are both $\mathscr{I}_g$-open and $\mathscr{I}_g$-closed are the empty set ϕ and X.*

Proof. (i)⇒(ii). Let F be an $\mathscr{I}_g$-open and $\mathscr{I}_g$ -closed subset of X. Then $X - F$ is both $\mathscr{I}_g$-open and $\mathscr{I}_g$-closed. Since X is $\mathscr{I}_g$-connected, X can be expressed as union of two disjoint nonempty $\mathscr{I}_g$-open sets X and $X - F$, which implies $X - F$ is empty.

(ii)⇒(i). Suppose $X = U \cup V$ where U and V are disjoint nonempty $\mathscr{I}_g$-open subsets of X. Then U is both $\mathscr{I}_g$-open and $\mathscr{I}_g$-closed. By assumption either $U = \phi$ or X which contradicts the assumption U and V are disjoint nonempty $\mathscr{I}_g$-open subsets of X. Therefore X is $\mathscr{I}_g$-connected. □

Theorem 3.31. *Let $f : (X, \tau, \mathscr{I}) \to (Y, \sigma)$ be a surjective preclosed contra $\mathscr{I}_g$-continuous function. If X is an $\mathscr{I}_g$-space, then Y is locally indiscrete.*

Proof. Suppose that V is open in Y. By hypothesis f is contra $\mathscr{I}_g$-continuous and therefore $f^{-1}(V) = U$ is $\mathscr{I}_g$-closed in X. Since X is an $\mathscr{I}_g$-space, U is $closed$ in X. Since f is preclosed, then V is also preclosed in Y. Now we have $\mathrm{cl}(V) = \mathrm{cl}(\mathrm{int}(V)) \subset V$. This means that V is closed and hence Y is locally indiscrete. □

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