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A Two Unit Standby System with Preventive Maintenance and Inverse Gaussian Repair Time Distribution

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Abstract. The present paper deals with the profit function analysis of a two identical unit standby system model with the concept of preventive maintenance of the operative unit after some significant time that is not fixed. Time to preventive maintenance and preventive maintenance time are considered correlated and their joint distribution is taken as bivariate exponential. The failure time distribution of an operative unit is taken as exponential whereas the repair time distribution is inverse gaussian. The system model has been analyzed by using regenerative point technique. The graphical behaviors of MTSF and profit function have also been studied.

1. Introduction

The incorporation of redundancy is one of the important techniques to improve the reliability of the system. The system models with active and passive (standby) redundancies have been frequently analysed by various authors including [5, 6, 11] with different assumptions due to their vital existence in modern industries and business.

In practice, a sort of repair called preventive maintenance is given to the system to improve its working capability and to delay its actual failure so that we can enhance the benefits made by the system. A few attempts have been made to study the models with the concept of preventive maintenance [1, 2, 7, 8, 9]. The concept of correlation between failure and repair times has been frequently used by various authors including [3, 4, 6, 10]. The present paper is introducing the concept of correlation between time to preventive maintenance and time taken in preventive maintenance, which is of much importance in real existing system models.

2. System Description and Assumptions

The assumptions about the model under study are given as under:

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(i) The system comprises of two identical units. Initially, one unit is operative and other is kept as cold standby.

(ii) After some significant time the operative unit goes for preventive maintenance (p.m.).

(iii) A single repairman is always available with the system to repair a failed unit and for preventive maintenance of an operating unit. The repair or preventive maintenance is done on FCFS basis.

(iv) The switching device, used to detect the failed unit and to switch the standby unit into operation, is perfect and instantaneous.

(v) The time to preventive maintenance and time taken in preventive maintenance are considered to be correlated and to follow the bivariate exponential distribution having the density as follows:

$$f(x,y)=\alpha\beta(1-r)e^{-\alpha x-\beta y}I_0(2\sqrt{\alpha\beta rxy}),\quad x,y,\alpha,\beta>0,\ 0\le r<1,$$

where

$$I_0=\sum_{k=0}^{\infty}\frac{(z/2)^{2k}}{(k!)^2}\,.$$

(vi) The failure time distribution of a unit is taken as negative exponential while the repair time distribution is Inverse Gaussian.

3. Notation and States of the System

We define the following notations:

E : Set of regenerative states $\equiv \{S_0,S_1,S_2\}$

$\bar{E}$: Set of non-regenerative states $\equiv \{S_3,S_4,S_5,S_6\}$

λ : Constant failure rate of an operative unit.

X : Time to preventive maintenance of an operative unit.

Y : Time taken in preventive maintenance of a unit.

$h(\cdot), H(\cdot)$: p.d.f. and c.d.f. of time to repair the failed unit, s.t.

$$h(t)=\frac{1}{\sqrt{2\pi}}t^{-3/2}\exp\left\{\frac{-(t-\mu)^2}{2\mu^2 t}\right\};\ t>0,\ \mu>0.$$

$g(x)$: Marginal p.d.f. of $X=\alpha(1-r)e^{-\alpha(1-r)x}$.

$K(y|x)$: Conditional c.d.f. of y given that the unit is sent for p.m. after an operation of time x.

Symbols of the states of the system

N_O, N_S : Unit in N (Normal) mode and operative/standby state.

N_{pm}, N_{wpm} : Unit in N (Normal) mode and under preventive maintenance/ waiting for preventive maintenance.

F_r, F_{wr} : Unit in F (Failure) mode and under repair/waiting for repair.

Considering these symbols in view of assumptions stated earlier, we have the following states of the system:

Up States:

$$S_0 \equiv (N_o, N_s), \quad S_1 \equiv (N_{pm}, N_o), \quad S_2 \equiv (N_o, F_r)$$

Failed States:

$$S_3 \equiv (F_{wr}, F_r), \quad S_4 \equiv (N_{pm}, N_{wpm}), \quad S_5 \equiv (N_{pm}, F_{wr}), \quad S_6 \equiv (N_{wpm}, F_r)$$

The transition diagram of the system model along with failure rates/repair time c.d.f.s is shown in Figure 1. In the figure we observe that the epoch of the transition entrance into the state S_4 from S_1, S_5 from S_1, S_3 from S_2 and S_6 from S_2 are non-regenerative. All other entrance epochs are regenerative.

4. **Transition Probabilities and Sojourn Times**

By simple probabilistic arguments, the unconditional and conditional transition probabilities are as follows:

$$(4.1) \qquad p_{01} = \frac{\alpha(1-r)}{\lambda+\alpha(1-r)}, \qquad p_{02} = \frac{\lambda}{\lambda+\alpha(1-r)},$$

$$p_{10/x} = \beta' e^{-\alpha r(1-\beta')x}, \qquad p_{11/x}^{(4)} = \frac{\alpha(1-r)}{\lambda+\alpha(1-r)}[1-\beta' e^{-\alpha r(1-\beta')x}],$$

$$p_{12/x}^{(5)} = \frac{\lambda}{\lambda+\alpha(1-r)}[1-\beta' e^{-\alpha r(1-\beta')x}], \qquad \beta' = \frac{\beta}{\beta+\lambda+\alpha(1-r)};$$

$$p_{20} = \exp\left\{\frac{1-\sqrt{1+2\mu^2(\lambda+\alpha(1-r))}}{\mu}\right\},$$

$$p_{21}^{(6)} = \frac{\alpha(1-r)}{\lambda+\alpha(1-r)}\left[1-\exp\left\{\frac{1-\sqrt{1+2\mu^2(\lambda+\alpha(1-r))}}{\mu}\right\}\right],$$

$$p_{22}^{(3)} = \frac{\lambda}{\lambda+\alpha(1-r)}\left[1-\exp\left\{\frac{1-\sqrt{1+2\mu^2(\lambda+\alpha(1-r))}}{\mu}\right\}\right].$$

It can be easily verified that

$$(4.2) \qquad p_{01}+p_{02}=1, \quad p_{10/x}+p_{11/x}^{(4)}+p_{12/x}^{(5)}=1, \quad p_{20}+p_{21}^{(6)}+p_{22}^{(3)}=1.$$

Unconditional transitional probabilities are

$$(4.3) \qquad p_{10} = \frac{\beta'(1-r)}{(1-r\beta')}, \quad p_{11}^{(4)} = \frac{\alpha(1-r)(1-\beta')}{(1-r\beta')\{\lambda+\alpha(1-r)\}},$$

$$p_{12}^{(5)} = \frac{\lambda(1-\beta')}{(1-r\beta')\{\lambda+\alpha(1-r)\}}.$$

We observe the following relations:

(4.4) $p_{01}+p_{02}=1, \quad p_{10}+p_{11}^{(4)}+p_{12}^{(5)}=1, \quad p_{20}+p_{21}^{(6)}+p_{22}^{(3)}=1\,.$

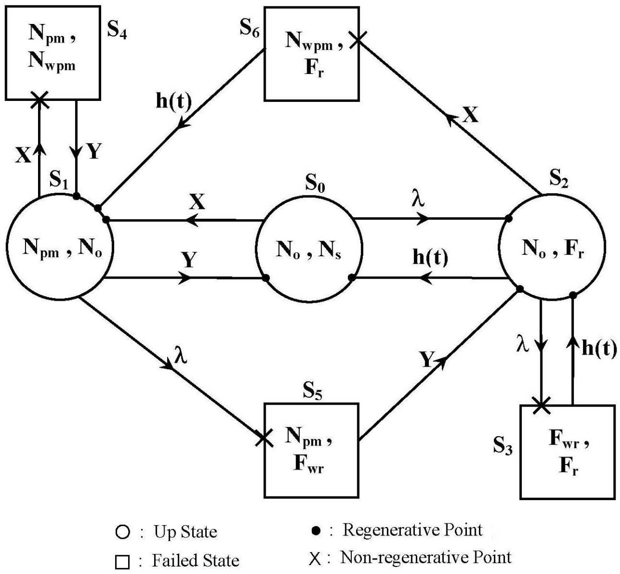

Figure 1

Let X_i denotes the sojourn time in state S_i, then the mean sojourn time in state S_i is given by

$$\psi_i=\int P(X_i>t)dt\,.$$

The mean sojourn times in various states are as follows:

(4.5) $$\psi_0=\frac{1}{\lambda+\alpha(1-r)}, \qquad \psi_{1/x}=\frac{[1-\beta' e^{-\alpha r(1-\beta')x}]}{\lambda+\alpha(1-r)},$$

$$\psi_1=\frac{(1-\beta')}{(1-r\beta')\{\lambda+\alpha(1-r)\}},$$

$$\psi_2=\frac{1}{\lambda+\alpha(1-r)}\left[1-\exp\left\{\frac{1-\sqrt{1+2\mu^2(\lambda+\alpha(1-r))}}{\mu}\right\}\right],$$

$\psi_3=\mu=\psi_6,$ $\qquad \mu$: mean of the inverse Gaussian distribution ;

$$\psi_{4/x}=\frac{(1+\alpha r x)}{\beta}, \qquad \psi_4=\frac{1}{\beta(1-r)},$$

$$\psi_{5/x}=\frac{1+\alpha r x}{\beta}, \qquad \psi_5=\frac{1}{\beta(1-r)}\,.$$

5. **Analysis of Results**

5.1. *Reliability and MTSF*

Let the r.v. T_i, be the time to system failure when the system starts its operation from state $S_i \in E$, then the reliability of the system is given by

$$R_i(t) = P(T_i > t).$$

To determine $R_i(t)$, we assume that the failed states (S_3 to S_6) of the system as absorbing. Using the simple probabilistic arguments, one can easily develop the recurrence relations among $R_i(t)$; $i = 0, 1, 2$. Taking the Laplace Transforms of the relations and simplifying the resulting set of algebraic equations for $R_0^*(s)$, we get after omitting the arguments 's' for brevity:

$$(5.11) \quad R_0^*(s) = \frac{Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^*}{1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^*},$$

where Z_0^*, Z_1^* and Z_2^* are the L.T. of

$$(5.12) \quad Z_0(t) = e^{-\{\lambda + \alpha(1-r)t\}},$$

$$Z_1(t) = e^{-\{\lambda + \alpha(1-r)t\}} \int \overline{K}(t|x) g(x) dx,$$

$$Z_2(t) = e^{-\{\lambda + \alpha(1-r)t\}} \int_t^\infty \frac{1}{\sqrt{2\pi}} u^{-3/2} \exp\left\{\frac{(u-\mu)^2}{2\mu^2 u}\right\} du.$$

Taking inverse L.T. of (5.11), we get the reliability of the system. The MTSF is given by

$$(5.13) \quad E(T_0) = \lim_{s \to 0} R_0^*(s) = \frac{\psi_0 + p_{01}\psi_1 + p_{02}\psi_2}{1 - p_{01}p_{10} - p_{02}p_{20}}.$$

5.2. *Availability*

Let us define $A_i(t)$ as the probability that the system is up at epoch t when it initially starts from state $S_i \in E$. Using the definition of $A_i(t)$ and probabilistic concepts, the recurrence relations among $A_i(t)$, $i = 0, 1, 2$ can easily be developed. Using the technique of L.T., the value $A_0(t)$ in terms of its L.T. is as follows:

$$(5.21) \quad A_0^*(s) = \frac{N_2(s)}{D_2(s)},$$

where

$$N_2(s) = \{(1 - q_{11}^{(4)*})(1 - q_{22}^{(3)*}) - q_{12}^{(5)*} q_{21}^{(6)*}\} Z_0^* + q_{01}^* \{(1 - q_{22}^{(3)*}) Z_1^* + q_{12}^{(5)*} Z_2^*\}$$
$$+ q_{02}^* \{q_{21}^{(6)*} Z_1^* + (1 - q_{11}^{(4)*}) Z_2^*\}$$

and

(5.22) $$D_2(s) = (1-q_{11}^{(4)*})(1-q_{22}^{(3)*}) - q_{12}^{(5)*}q_{21}^{(6)*} - q_{01}^*\{q_{10}^*(1-q_{22}^{(3)*}) + q_{12}^{(5)*}q_{20}^*\} - q_{02}^*\{q_{21}^{(6)*}q_{10}^* + (1-q_{11}^{(4)*})q_{20}^*\}$$

The steady-state availability of the system is given by

(5.23) $$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) = N_2/D_2\,,$$

where

$$N_2 = \{(1-p_{11}^{(4)})(1-p_{22}^{(3)}) - p_{12}^{(5)}p_{21}^{(6)}\}\psi_0 + p_{01}\{(1-p_{22}^{(3)})\psi_1 + p_{12}^{(5)}\psi_2\} + p_{02}\{p_{21}^{(6)}\psi_1 + (1-p_{11}^{(4)})\psi_2\}$$

and

(5.24) $$D_2 = \{p_{10}p_{21}^{(6)} + p_{20}(1-p_{11}^{(4)})\}\psi_0 + \{p_{21}^{(6)} + p_{20}p_{10}\}\psi_1 + \{p_{12}^{(5)} + p_{10}p_{02}\}\psi_2\,.$$

The expected up time of the system during $(0,t)$ is given by

$$\mu_{up}(t) = \int_0^t A_0(u)du$$

so that

(5.25) $$\mu_{up}^*(s) = A_0^*(s)/s\,.$$

5.3. *Busy Period Analysis*

(i) *For preventive maintenance*

We define $B_i^p(t)$ as the probability that the repairman is busy in performing the preventive maintenance of a unit when the system initially starts from state $S_i \in E$. Using simple probabilistic arguments, the value of $B_0^p(t)$ in terms of its L.T. is as follows:

(5.31) $$B_0^{p*}(s) = N_3(s)/D_3(s)\,,$$

where

$$N_3(s) = \{q_{01}^*(1-q_{22}^{(3)*}) + q_{02}^*q_{21}^{(6)*}\}(Z_1^* + q_{14}^*Z_4^* + q_{15}^*Z_5^*)$$

and $D_3(s)$ is same as $D_2(s)$.

In the long run the probability that the repairman is busy in preventive maintenance is given by

(5.32) $$B_0^p = N_3/D_3\,.$$

The expected busy period of the repairman for preventive maintenance during $(0,t)$ is given by

$$\mu_b^p(t) = \int B_0^p(u)du$$

so that

(5.33) $\mu_b^{p*}(s) = B_0^{p*}(s)/s\,.$

(ii) *For repair*

We define $B_i^r(t)$ as the probability that the repairman is busy in repairing a failed unit when the system initially starts from state $S_i \in E$. Using simple probabilistic arguments, the value of $B_0^r(t)$ in terms of its L.T. can easily be obtained as follows:

(5.34) $B_0^{r*}(s) = N_4(s)/D_4(s)\,,$

where

$$N_4(s) = \{q_{01}^* q_{12}^{(5)*} + q_{02}^*(1 - q_{11}^{(4)*})\}(Z_2^* + q_{23}^* Z_3^* + q_{26}^* Z_6^*)$$

and $D_4(s)$ is same as $D_2(s)$.

In the long run the probability that the repairman is busy in repairing the failed unit is given by

(5.35) $B_0^r = N_4/D_4\,.$

The expected busy period of the repairman in repairing the failed unit during $(0,t)$ is given by

$$\mu_b^r(t) = \int B_0^r(u)du$$

so that

(5.36) $\mu_b^{r*}(s) = B_0^{r*}(s)/s\,.$

6. **Profit Function Analysis**

Profit function $P(t)$ can easily be obtained for the system model under study with the help of the characteristic obtained earlier.

The net expected profit incurred during $(0,t)$ is

(6.1) $P(t) = K_1\mu_{up}(t) - K_2\mu_b^p(t) - K_3\mu_b^r(t)\,,$

where

K_1 = revenue per unit time of the system.
K_2 = preventive maintenance cost per unit of time.

and

K_3 = repair cost per unit of time.

The expected profit per unit time in steady state is

(6.2) $P = K_1 A_0 - K_2 B_0^p - K_3 B_0^r\,.$

7. Graphical Study of the System Behaviour

For a more concrete study of system behavior, we plot the curves for MTSF and profit with respest to λ for three different values of mean of Inverse Gaussian distribution μ $(5, 7$ and $9)$ and two different values of correlation coefficient r $(0.25$ and $0.50)$.

The curves for MTSF are shown in Figure 2. From these cures, we observe that MTSF decreases uniformly as the values of λ increase. Also with the increase in the values of μ, expected life of the system decreases. Moreover, with the increase in the value of r, MTSF increases as it should be.

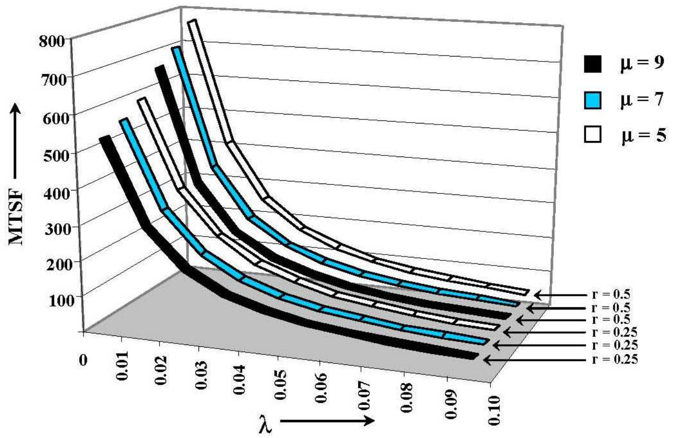

Figure 2

The curves for Profit function are shown in Figure 3. From these curves it is obvious that the profit decrease almost with linear trend as the values of λ increase. Here also with the change in the values of μ and r, the same trends as that of MTSF are observed.

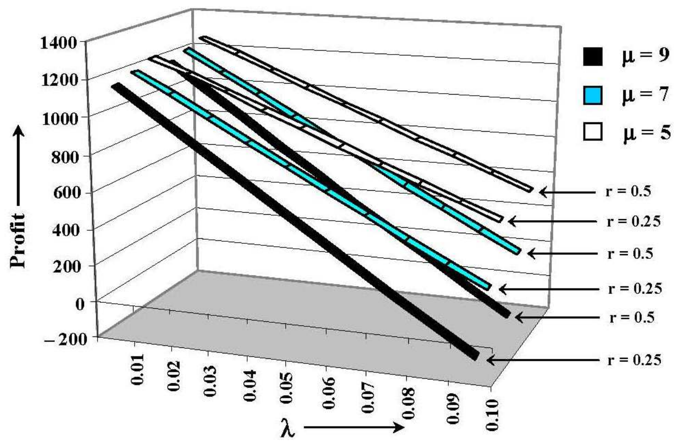

Figure 3

Thus, we conclude that the higher correlation between time to preventive maintenance and preventive maintenance provides the better system performances.

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