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GEM and the $\Upsilon(1 S)$
D. White


#### Abstract

The Gluon Emission Model (GEM), first proposed by F. Close in 1979, has been shown to serve very nicely as a basis for calculations of not only the widths of the $\rho$ meson, the $\phi$ meson, the $K^{*}$ (892), and the $J$ meson, but also for the determination of the strong coupling parameter, $\alpha_{s}$, over essentially the entire range of experimentally reachable energy, leading to an evaluation of $\alpha_{s}$ at the $Z$ boson energy of $0.121 \pm 0.003$. The GEM has built into its framework two precepts of prime importance for the carrying out of the above types of calculations: (1) the specification of a quark spin-flip matrix element as the central determinant of a vector meson resonance and (2) the virtual photon and the gluon as two aspects of the same entity, viz., the four-momentum propagator. The prime significance of (1) is that the square of the quark spin-flip matrix elements in vector meson width calculations are proportional to $q_{i}^{4}$, where $q_{i}$ represents the magnitude of the charge of quark type " $i$ ". The significance of (2) is that the virtual photon and the gluon essentially obtain their identities from what the vertices of origin and termination are in the relevant Feynman Diagram. Close, as a point of fact, represents the virtual photon as transmuting into a gluon ... and vice versa $\ldots$ where necessary, all transmutation couplings being of magnitude, 1. The ramifications of (1) are that, as $(2 / 3)^{4}$ is 16 times $(1 / 3)^{4}$, it is quite easy to determine that the $c c^{*}$ (charm - anti-charm) structure of the $J(3097)$ must transmute to an $s s^{*}$ (strange - anti-strange) in point-like manner, such that it is the $s s^{*}$ structure that undergoes the spin-flip at the $J(3097)$ resonance. Likewise, the $\Upsilon(1 S)$ must transmute in point-like manner from its original $b b^{*}$ (bottom -anti-bottom) structure to a $c c^{*}$ structure before decaying. The ramification of (2) is that the leptonic width to hadronic width ratio associated with the same basic decaying structure must be in the ratio of $\alpha$ to $\alpha_{s}$, where $\alpha$ represents the fine structure constant $=(1 / 137.036)$.

At the present juncture in the literature is found that the GEM predicts the hadronic width of the $\Upsilon(1 S)$ to be $\sim 41 \mathrm{Kev}$, whereas the figure for same as stated in the 2008 Meson Table from the Particle Data Group (PDG) is $\sim 50 \mathrm{Kev}$. The discrepancy noted above ( $23 \%$ ) is extremely important, because, if we were to assume that the GEM was in error by such amount, it turns out that all other GEM calculations, currently essentially exactly on the mark as to the $\rho$, the $\phi$, the $K^{*}(892)$, the $J$, and $\alpha_{s}$ at the $Z$ mass, would have to be rendered as $23 \%$ too large by bringing the GEM's determination of the $\Upsilon(1 S)$ in line with the PDG's determination of same through adjustment of the GEM's determination of $\alpha_{s}$. Hence, in order to make the GEM as currently constructed fit the PDG as to the hadronic width of the $\Upsilon(1 S)$, all other GEM calculations would be discrepant by the same amount, i.e., $23 \%$, at each diverse point of the energy spectrum where


[^0]
#### Abstract

the GEM has been successfully applied. Clearly, then, what must be addressed in the present work are the details in the GEM's determination of the width of the $\Upsilon(1 S)$, with an eye towards any reasonable modifications that might remove the above-mentioned disparity. Unlike the theoretical structures prevalent in the literature that one encounters as to determining the width of the $\Upsilon(1 S)$, the GEM theory is about as simple as it gets: One fundamental process is posited for the formation and decay of any spin one meson, i.e., a quark spin-flip; the gluon absorption cross-section for said process is then integrated over energy, and from there, the Feynman Diagram resulting in hadron or lepton pairs is then calculated. We review the development of the GEM and its applications, from its beginnings in 1979 through 2009 ... including the $23 \%$ disparity noted above. We then postulate the existence of an additional process involved in the decay of the $\Upsilon(1 S) \ldots$ one not assumed to be extant in the other, less massive vector mesons to which the GEM has been successfully applied. We find that the "additional route of decay" removes completely the noted disparity without affecting the GEM in its other applications. Finally, the GEM ansatz as presented herein is applied to the $\Upsilon(2 S)$ with noteable success.


## 1. Introduction

As stated in the Abstract, the Gluon Emission Model (GEM), proposed by F. Close in 1979 (see F. Close (1979)), has been quite successful in determining on a purely theoretical basis the widths of the $K^{*}$ (892) and the $J$ (3097), once three constants are determined in the general expression, assumed universal, for the width of any vector meson. At the present juncture it would be useful to put forth the basic framework of the GEM as published in White (2008-R), "The Gluon Emission Model for Hadron Production Revisited". Immediately below is found an excerpt from the Introduction of said article which lays the foundation for the GEM:

In all quantum systems in which natural decay occurs between an excited level and the ground state, the integrated absorption cross-section goes as

$$
\begin{equation*}
\sigma(\omega)=K \alpha|V|^{2}(1 / m)^{2}(1 / \omega) L(\omega) \tag{1}
\end{equation*}
$$

where $K$ is a constant, $\omega$ represents photon frequency, $|V|^{2}$ represents the square of the matrix element descriptive of the photon emission process, the system has mass $m, L(\omega)$ is a Lorentz Amplitude with a peak at $\omega=\omega_{0}$ and with a width $\Gamma$, and $\alpha=(1 / 137.036)$ represents the fine structure constant.

Assuming "asymptotic freedom", i.e., that we may ignore the masses of the decay products (light hadron pairs) in relation to the total energy involved in the system under investigation, we may employ Eq. (1) to predict the width of vector mesons by making the following substitutions to take us from a general quantum electrodynamics (QED) to a specific quantum chromodynamics (QCD) process:

We substitute for the photon frequency $\omega$ the gluon energy $Q_{0}$.
We evaluate the right hand side of Eq. (1) at a specific vector meson mass, $m_{v}$, i.e., $Q_{0}=m=m_{v}$. (Hence, the associated Lorentz Amplitude equals unity.)

We require $|V|^{2}$ to be proportional to $\Sigma_{i}\left(q_{i}\right)^{4}$, where $q_{i}=$ quark charge (in units of electron charge magnitude) associated with the quarks comprising the relevant vector meson. (The above criterion is consistent with spin-spin interaction [see also R. Dalitz (1977), p. 604] proportional to $q_{i}^{2}$, where $i$ denotes quark flavor, giving rise to spin-flip transitions, and the sum is required only in the case of the $\rho$, as it comprises both the up quark ( $u$ ) of charge $q_{u}=2 / 3$ and the down quark (d) of charge $q_{d}=-1 / 3$.)

We postulate $|V|^{2}$ to be proportional to only $\Sigma_{i}\left(q_{i}\right)^{4}$, i.e., the precise form of the interaction is universal to all vector mesons in their ground states, except for quark charge differences.

We replace $\alpha$ by $\alpha_{s}$, the strong coupling parameter, which has the well-known form from QCD gauge invariance theories (see [2, S. Gasiorowicz and J.L. Rosher, American Journal of Physics 49(1981), 954 and ff]) of:

$$
\begin{equation*}
\alpha_{s}=B\left[\ln \left(Q_{0} / \Lambda\right)\right]^{-1} \tag{2}
\end{equation*}
$$

where $B$ is a constant and $\Lambda$ is a parameter to be determined. Again, we emphasize that commensurate with the above replacements is that we must assume that the initial energy involved in the formation of a given vector meson is extremely high, i.e., in the "asymptotically free" region of energy space, where the masses of emerging hadron pairs as decay products can be neglected. Accordingly, then, we find in terms of the above ansatz (normalizing to the $\rho$ )

$$
\begin{equation*}
\Gamma_{v}=A\left(m_{p} / m_{v}\right)^{3}\left(\Sigma_{i}\left(q_{i}\right)^{4}\right)\left[\ln \left(m_{v} / \Lambda\right)\right]^{-1} \tag{3}
\end{equation*}
$$

where $\Gamma_{v}$ represents the width of a given vector meson, $v$, and $A$ is a constant to be determined.

The constants, $A$ and $\Lambda$, may be determined by simultaneously fitting the width of the $\rho$ and the width of the kaon branch of the $\phi$ to the form of Eq. (3) in the above quotation, and $B$ may be determined by evaluating $\alpha_{s}$ at the $\Upsilon(1 S)$ energy through the utilization of the experimentally determined partial width associated with the $\Upsilon(1 S) \rightarrow e^{+} e^{-}$decay in conjunction with the GEM-theoretical hadronic width of the $\Upsilon(1 S)$ (see White (2008-R)). In conventional terms (see White (2010)) the hadronic width of any vector meson may be expressed as the following:

$$
\begin{equation*}
\Gamma_{v} \approx\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)\left(m_{p} / m_{v}\right)^{3}\left(\Sigma_{i}\left(q_{i}\right)^{4}\right) \tag{1}
\end{equation*}
$$

where $m_{e}$ represents the electron mass of 0.511 Mev , so that $2 m_{e}=1.022 \mathrm{Mev}$, $\alpha_{s}$ represents the strong coupling parameter, given by $\alpha_{s}=1.2\left[\ln \left(m_{v} / 50 \mathrm{Mev}\right)\right]^{-1}$, $m_{p}$ represents the mass of the $\rho$ meson, $m_{v}$ represents the mass of the vector meson with designate " $v$ ", and $q_{i}$ represents the charge of the relevant quark type(s) " $i$ " to undergo the spin flip to form the vector meson under consideration. The $q_{i}$
involved in $\rho$ formation, for example, are the $q_{u}=2 / 3$ and $q_{d}=-1 / 3$, where " $u$ " designates an "up quark" and "d" designates a "down quark". Only $q_{s}=1 / 3$, where " $s$ " designates a "strange quark", is involved in the formation of the kaon branch of the $\phi$, whereas $q_{u}, q_{d}$, and $q_{s}$ are all involved in the formation of the $K^{*}(892)$ (see White (2008-R, 2008-K)). In addition, as we will see below, the $q_{i}$ associated with the $J(3097)$ is actually $q_{s}$, and that associated with the $\Upsilon(1 S)$ is actually $q_{c}=-2 / 3$, where " $c$ " is the designate for the "charm quark" (see also White (2008-R)).

Now, since the GEM treats the virtual photon and the gluon as, essentially, two aspects of the same entity, which we will call "the four-momentum propagator" and designate as " $\zeta$ ", by definition, the ratio of the partial width associated with a given decaying pair of quarks comprising a given vector meson associated with electron-positron decay to the hadronic width of same is simply $\left(\alpha / \alpha_{s}\right)$, where " $\alpha$ " represents the fine structure constant $=(1 / 137.036)$. Hence, the general form for the partial width of a vector meson undergoing $e^{+} e^{-}$decay would be given by

$$
\begin{equation*}
\Gamma_{v-e e} \approx(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)\left(m_{\rho} / m_{v}\right)^{3}\left(\Sigma_{i}\left(q_{i}\right)^{4}\right) \tag{2}
\end{equation*}
$$

A relevant Feynman Diagram will make the various aspects of the GEM easier to picture, so let us look now to Figure 1 below, which represents the Feynman Diagram (FD) associated the formation and decay of vector meson " $X$ " in its simplest possible form.


Figure 1. Basic Feynman Diagram for Conventional Vector Meson Formation/Decay via the GEM.

In Figure $1 \zeta_{1}$ represents, in part, a virtual photon created at the $e^{+} e^{-}$ annihilation vertex, coupling at said vertex represented as $\alpha$; then, in Close's terms, the virtual photon couples to a gluon with coupling strength " 1 ", which then couples to the $x x^{*} \ldots$ a given quark - anti-quark pair, also with coupling strength " 1 ". In our notation $\zeta_{1}$ simply represents a four-momentum propagator, created at the $e^{+} e^{-}$vertex and absorbed (as a gluon) at the $x x^{*}$ node. The details of the absorption of $\zeta_{1}$ are contained in the integrated absorption cross-section as exhibited in the quotation from White (2008), and $|V|^{2}$, proportional to $q_{x}^{4}$, describes the formation of the spin one resonance. From there $\zeta_{2}$ (a gluon) is emitted, resulting in coupling to hadrons $\left(h ; h^{*}\right)$, the coupling at the latter vertex of magnitude $\alpha_{s}$. The calculation of the width of the $x x^{*}$ state is, given the stated
mechanism of a spin flip of one of the " $x$ quarks" due to a spin - spin interaction proportional to $q_{x}^{2}$, proceeds straight along the dictates of standard QED, except for the replacement of $\alpha$ by $\alpha_{s}$ at the $h h^{*}$ vertex.

For comparison, immediately below we present the FD associated with the same $X$ meson, assumed to exist in the realm of asymptotic freedom, decaying into an electron-positron pair.


Figure 2. Basic Feynman Diagram for Conventional Vector Meson Formation and Decay into an Electron/Positron Pair via the GEM.

The only fundamental difference between Figure 1 and Figure 2 is that in Figure $2 \zeta_{2}$ starts out as a gluon and ends up as a virtual photon at the right hand vertex, at which point the coupling, of course, is now $\alpha$. Hence, all in the width calculation associated with Figure 1 is the same in Figure 2, except that $\alpha_{s}$ in Eq. (1) is replaced by $\alpha$. Of note, too, and we shall return to the point made here, Figure 2 represents rigorously a straight-forward calculation in QED, again, given the stated mechanism for the formation of the resonance state. However, it is also important to note that Figure 2 applies only to vector mesons existing in the realm of "asymptotic freedom", i.e., to the $J(3097)$, the $\Upsilon(1 S)$, and "Toponium", or the " $T$ " meson.

In Section 2, which follows, we will view the detailed FDs required by the GEM to describe the widths of the $\rho$, the $\phi$, the $K^{*}(892) \ldots$ a very interesting case, as the $K^{*}(892)$ is not conventionally thought of as a vector meson per se, though it is of the spin one variety $\ldots$ the $J(3097)$, the $\Upsilon(1 S)$, and the " $T$ ". In addition we will review briefly the successful match with experiment that the GEM-derived strong coupling parameter function currently demonstrates over a very wide range of energy.

## 2. Applications of the GEM (1979-2009)

## 2.A. The $\rho$ Meson

Although the width of the $\rho$ (and the $\phi$ ) as determined by the GEM is guaranteed to be a match to experiment by construction, the $\rho$ is a good place to start with the elucidation of the application of the GEM to the various spin one mesons because of the simplicity involved. Let us begin by viewing Figure 3 below $\ldots$ the FD associated with the formation and decay of the $\rho$ meson.


Figure 3. Basic Feynman Diagram for Formation and Decay of the $\rho$ meson via the GEM.

In Figure $3 \zeta_{1}$ represents a virtual photon created at the $e^{+} e^{-}$vertex which transmutes to a gluon, which, in turn, is absorbed by the $\left[q_{u} q_{u}^{*}+q_{d} q_{d}^{*}\right]$ combination; $\zeta_{2}$ represents the emitted gluon, which converts to pion pairs. The application of Eq. (1) results in the following for the hadronic width of the $\rho$ :

$$
\begin{align*}
& \Gamma_{\rho}\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)\left(\Sigma_{i}\left(q_{i}\right)^{4}\right) \\
& \quad \approx\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)(17 / 81) \tag{3a}
\end{align*}
$$

where

$$
\begin{equation*}
\alpha_{s}=1.2[\ln (776 / 50)]^{-1}=0.4376 \tag{3b}
\end{equation*}
$$

Hence,

$$
\begin{equation*}
\Gamma_{\rho} \approx 150 \mathrm{Mev} \tag{3c}
\end{equation*}
$$

Though adaptation of Figure 2 and Eq. (2) do not formally apply, as asymptotic freedom does not apply to the $\rho$, we note that in the event that it were to apply, we would obtain for the electron/positron partial width, $\Gamma_{\rho-e e}$, the following:
$\Gamma_{\rho-e e} \approx(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)(17 / 81) \approx 2.50 \mathrm{Mev}$ a figure about 355 times too high (see PDG (2009-M)), indicating that the transmutation coupling of the $\zeta_{2}$ gluon to its virtual photon identity is only 0.0028 , as opposed to 1 in the asymptotically free energy regime.

## 2.B. The $\phi$ Meson

Application of the GEM to the kaon branch of the $\phi$ meson $\left(\phi_{K}\right)$ follows similar lines as to the $\rho$. The FD associated with the formation and decay of the kaon branch of the $\phi$ may be seen below.

For the hadronic width of the kaon branch of the $\phi$ we obtain:

$$
\begin{align*}
\Gamma_{\phi-K} & \approx\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)\left(m_{p} / m_{v}\right)^{3}\left(\Sigma_{i}\left(q_{i}\right)^{4}\right) \\
& \approx\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)(776 / 1019)^{3}(1 / 81) \tag{4a}
\end{align*}
$$

where

$$
\begin{equation*}
\alpha_{s}=1.2[\ln (1019 / 50)]^{-1}=0.3981 \tag{4b}
\end{equation*}
$$



Figure 4. Basic Feynman Diagram for Formation and Decay of the Kaon Branch of the $\phi$ Meson via the GEM.

Hence,

$$
\begin{equation*}
\Gamma_{\phi-K} \approx 3.55 \mathrm{Mev} \tag{4c}
\end{equation*}
$$

Again applying Eq. (2) to the kaon branch of the $\phi$, we obtain for its $e^{+} e^{-}$partial width the following:

$$
\Gamma_{\phi-K-e e} \approx(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)(776 / 1019)^{3}(1 / 81) \approx 0.0650 \mathrm{Mev}
$$

a figure still way too high as compared to experiment (see PDG (2009-M)), but here about 52 times so, indicating that the $\zeta_{2}$ gluon to virtual photon transmutation coupling has risen to 0.0194 .

## 2.C. The $K^{*}(892)$

The situation regarding the $K^{*}(892)$ is highly interesting. Close had developed the GEM in the 1970s to describe two distinct processes: (1) the production of pion pairs associated with the $\rho$ resonance and (2) the production of kaon pairs associated with the $\phi$ resonance. In a sense, then, the GEM was first envisioned to be "route specific", i.e., the spin flip process involving up and down quarks, which resonates at the $\rho$ mass, was thought of as "the pion route" in thinking of the decay of quark - anti-quark structures, while the spin flip process involving the strange quark, which resonates at the $\phi$ mass, was thought of as the corresponding "kaon route". At that time no one had thought of applying the GEM to the $K^{*}(892)$, because, although energetically possible, the $K^{*}(892)$ did not exhibit "a pion route" in its decay; rather, the $K^{*}(892)$ decays almost exclusively into various $\{\pi, K\}$ combinations, with equal probability of occurrence among the various allowed decay products. Such circumstance led to the invention of the "isospin" quantum number, a half integer value for which signifying a forbidden decay route that is energetically possible. However, since the spin associated with the $K^{*}(892)$ is one, it is quite feasible that the GEM, appropriately mitigated to fit the situation pertaining to the $K^{*}(892)$ 's isospin, may be applied to the $K^{*}(892)$ resonance. In fact, the GEM has been applied to the $K^{*}$ (892) quite successfully (see White (2008-R and 2008-K)). The reasoning leading to the proper mitigation is as follows:

Since pions and kaons are the decay products of the $K^{*}$ (892), with the various types of pions combining with correspondingly allowed various types of kaons and all types showing up with equal probability, it is reasonable to assume that the $K^{*}(892)$...for purposes of discussion here considered as a composite entity of mass, 894 Mev , i.e., no distinction as to charged mode versus neutral mode being made, comprises a linear combination of $\left\{u u^{*}, d d^{*}\right.$, and $\left.s s^{*}\right\}$ in equal measure. Symbolically, we may represent the $K^{*}(892)$, therefore, as

$$
\begin{equation*}
K^{*}(892)=(1 / \sqrt{3})\left[u u^{*}+d d^{*}+s s^{*}\right] . \tag{5}
\end{equation*}
$$

Now, the associated value of $\left(\Sigma_{i}\left(q_{i}\right)^{4}\right)$ would be (18/81), but the "pion route" does not occur, though it is energetically possible. So, segmenting the decay in terms of "routes", the $\{\pi, K\}$ route, whose $\left(\Sigma_{i}\left(q_{i}\right)^{4}\right)=(18 / 81)$ does occur, whereas the "pion route", whose $\left(\Sigma_{i}\left(q_{i}\right)^{4}\right)=(17 / 81)$ does not occur. The allowed route is thus favored over the forbidden route by the factor (18/17), therefore. Hence, we postulate that the isospin quantum number $=(1 / 2)$ assigned to the $K^{*}(892)$ signifies that of the energetically possible routes available to the $K^{*}(892)$ resonance, $(18 / 35)$ of them manifests in the decay process (the $\{\pi, K\}$ route), whereas $(17 / 35)$ of them fails to materialize (the pion route). We thus multiply the right hand side of Eq. (1) by (18/35) to obtain the width of the $K^{*}(892)$. First, let us view the associated FD:


Figure 5. Basic Feynman Diagram for Formation and Decay of the $K^{*}(892)$ via the GEM.

The GEM yields for the width of the $K^{*}(892)$ the following:

$$
\begin{align*}
\Gamma_{K^{*}} & \approx(18 / 35)\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)\left(m_{k^{*}} / m_{v}\right)^{3}\left(\Sigma_{i}\left(q_{i}\right)^{4}\right) \\
& \approx(18 / 35)\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)(776 / 894)^{3}(18 / 81) \tag{6a}
\end{align*}
$$

where

$$
\begin{equation*}
\alpha_{s}=1.2[\ln (894 / 50)]^{-1}=0.4161 \tag{6b}
\end{equation*}
$$

Hence,

$$
\begin{equation*}
\Gamma_{K^{*}} \approx 50.80 \mathrm{Mev} \tag{6c}
\end{equation*}
$$

From White (2008-K) the average of the widths associated with the charged and neutral modes of the $K^{*}(892)$ (from PDG (2004-M)) is stated as $\Gamma_{K^{*}}($ PDG $)=$
50.75 Mev . Hence, the GEM as applied to the $K^{*}(892)$ provides for fabulous agreement with experiment. Moreover, the GEM demonstrates quite clearly that the $K^{*}(892)$ is not a strange meson in the usual sense, i.e., it is seen not as a $u s^{*}$, $s u^{*}, d s^{*}$, or $s d^{*}$ structure at all; rather it is seen, similar to the theoretical structures of the $\rho$ and the $\phi$, as comprising a linear combination of more than one type of quark - anti-quark pair, its specific nature expressed via Eq. (5).

## 2.D. The $J(3097)$

Application of the GEM in accord with Figure 1, with $x=c$, seems reasonably straight-forward, but it turns out to be problematic. However, when one sees that the hadronic width of the $J(3097)$, designated as simply the " $J$ " henceforth, given by the application of Eq. (1) in accord with Figure 1 with $x=c$, is roughly sixteen times too large, as compared to experimental results, coupled with the fact that the hadronic width of the $\Upsilon(1 S)$ given by the application of Eq. (1) in accord with Figure 1 with $x=b$ is roughly sixteen times too small, as compared with experimental results, it becomes obvious as to what physically must transpire as regards both the $J$ and the $\Upsilon(1 S)$. Restricting the discussion to the $J$ for the time being, in what we call "the zeroth order approximation", the basic $c c^{*}$ structure of the $J$ must make a point-like transition to an $s s^{*}$ structure of equal mass, whereupon one of the $s$ quarks undergoes a spin flip to form the associated resonance (see White (2009-J)). The point-like transition from $c c^{*}$ to $s s^{*}$ is instantaneous, thus having no influence on the $J$ 's width. Indeed, the resonance does not even form until an $s$ (or $s^{*}$ ) quark undergoes a spin flip. That the $c c^{*}$ to $s s^{*}$ transition is necessary is quite understandable: The $J$ is not massive enough for it to be able to decay into hadrons via emission of two $c$ quarks; hence, it must transition to a quark pair of lesser bare mass each. The simplest possible assumption is that the $c c^{*}$ transitions to the quark pair type characterized by the next smallest mass, viz., the $s$ type. Nothing prevents the $c c^{*}$ structure from decaying into leptons ( $e^{+} e^{-}$and $\mu^{+} \mu^{-}$), however. It is found in White (2009-J), in fact, that in order for both the hadronic width of the $J$ and the leptonic width of the $J$ as determined via the GEM to match the results of experiment, $(8 / 9)^{\text {ths }}$ of the $c c^{*}$ structure must undergo a slightly "un-point-like" transition to $s s^{*}$, described by a form factor, $f<1$, which, in turn, decays into both hadrons and leptons as per Eq. (1) and Eq. (2), respectively, while (1/9)th of the original $c c^{*}$ structure remains to decay into leptons exclusively. We may picture the complete details of the J formation and decay via the following two arrays of FDs, the first such array descriptive of what we may now call "the first order approximation" to the width of the $J$, the second such array descriptive of what we call "the second order approximation", which follows along the lines of White (2009-J).

In Figure 6 " $l$ " represents a leptonic decay product, $\xi_{2 a}$ represents the gluon involved in a point-like transition from $c c^{*}$ to $s s^{*}$, and all other " $\zeta$ " designates


Figure 6a. Feynman Diagram Array Characterizing the Formation and Decay of the $J(3097)$ in First Order Approximation via the GEM.
should be understood from previous discussion. Transforming the schematic representation of Figure 6 into the calculation of the full (hadronic plus leptonic) width of the $J$ in first order approximation, denoted as $\Gamma_{J-f u l l-1}$, proceeds as follows (the factors of " 2 " in Eq. (7a), immediately in front of the factors " $\alpha / 2 \pi$ )" take into account muon pair production in accord with " $e-\mu$ universality"):

$$
\begin{align*}
\Gamma_{J-f u l l-1} \approx & (8 / 9)\left\{\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)\left(m_{\rho} / m_{J}\right)^{3}\left(q_{s}\right)^{4}\right. \\
& \left.+2(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)\left(m_{\rho} / m_{J}\right)^{3}\left(q_{s}\right)^{4}\right\} \\
& +(1 / 9)\left\{2(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)\left(m_{p} / m_{j}\right)^{3}\left(q_{c}\right)^{4}\right\} . \tag{7a}
\end{align*}
$$

Thus,

$$
\begin{aligned}
\Gamma_{J-\text { full-1 }} \approx & (8 / 9)\left\{\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)(776 / 3097)^{3}(1 / 81)\right. \\
& \left.+2(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)(776 / 3097)^{3}(1 / 81)\right\} \\
& +(1 / 9)\left\{2(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)(776 / 3097)^{3}(16 / 81)\right\} .
\end{aligned}
$$

The value of the strong coupling parameter at the $J$ mass is given by

$$
\begin{equation*}
\alpha_{s}=1.2[\ln (3097 / 50)]^{-1}=0.2908 \tag{7b}
\end{equation*}
$$

Therefore,

$$
\begin{align*}
\Gamma_{J-f u l l-1} & \approx(8 / 9)\{92.2491 \mathrm{Kev}+4.6298 \mathrm{Kev}\}+(1 / 9)\{74.0769 \mathrm{Kev}\} \\
& \approx 94.35 \mathrm{Kev} . \tag{7c}
\end{align*}
$$

The value for $\Gamma_{J-f u l l-l}$ obtained via the first approximation of the GEM is a match to experiment, as according to PDG ( $2009-\mathrm{M}$ ), the full width of the $J$ via experiment is $(93.2 \pm 2.1) \mathrm{Kev}$. As well, the hadronic width alone via the first approximation of the GEM is a match to experiment ( 82.00 Kev via the GEM vs. ( $81.7 \pm 0.5$ ) Kev via experiment (PDG (2009-M)); the leptonic width via the first approximation of the GEM is 12.35 Kev , which is about $11 \%$ more than that reported by the PDG currently ( $11.10 \pm 0.16$ ) Kev (PDG (2009-M)).

The first approximation assumes that $(8 / 9)^{\text {ths }}$ of the original $c c^{*}$ state undergo a point-like transition to an excited $s s^{*}$ state, leaving $(1 / 9)^{\text {th }}$ of the original $c c^{*}$ state to decay into leptons. A point-like transition is instantaneous, so it has no effect on the width of the original construction (i.e., the $J$ ). In terms of a form factor, $f$, a point-like transition is consistent with $f=1$. As it is difficult to see how any fraction of the original $c c^{*}$ state could "know" to make an instantaneous transition, leaving a remnant to do other things, we believe a second order approximation, again, along the lines of White (2009-J) is in order. Our reasoning is simply that, logically, we feel that there simply must be some type of communication between the $c c^{*}$ and $s s^{*}$ states before the $c c^{*}$ to $s s^{*}$ transition takes place in order for the proper remnant to consistently remain to decay into leptons. Hence, we reason that $f<1$ describes the $c c^{*}$ to $s s^{*}$ transition. Statistically, $f=\left(1-q_{s}^{2}\right)=(8 / 9)$ is necessary to describe the hadronic width of the $J$. Since $f$ is not appreciably different than 1 , the leptonic width of the $J$, relative to the first order approximation, will be mitigated slightly. The second order FD for the $J$ follows:

In Figure $6 \mathrm{~b} f=(8 / 9)$ multiplies the entire array. Denoting the full width of the $J$ in second order approximation by $\Gamma_{j \text {-full- }}$, we find in accord with Figure 6 b :

$$
\begin{align*}
\Gamma_{J-f u l l-2} \approx & (8 / 9)\left[\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)^{3}\left(q_{s}\right)^{4}\right. \\
& +2(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)\left(m_{\rho} / m_{J}\right)^{3}\left(q_{s}\right)^{4} \\
& \left.+(1 / 9)\left\{2(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)\left(m_{\rho} / m_{J}\right)^{3}\left(q_{c}\right)^{4}\right\}\right] \tag{7d}
\end{align*}
$$

Thus,

$$
\begin{aligned}
\Gamma_{J-\text { full-2 }} \approx & (8 / 9)\left[\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)(776 / 3097) 3(1 / 81)\right. \\
& +2(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)(776 / 3097)^{3}(1 / 81) \\
& \left.+(1 / 9)\left\{2(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)(776 / 3097)^{3}(16 / 81)\right\}\right] .
\end{aligned}
$$

Again, the value of the strong coupling parameter at the $J$ mass is given by Eq. (7b), viz.,

$$
\alpha_{s}=1.2[\ln (3097 / 50)]^{-1}=0.2908
$$



Figure 6b. Feynman Diagram Array Characterizing the Formation and Decay of the $J(3097)$ in Second Order Approximation via the GEM.

Therefore,

$$
\begin{aligned}
\Upsilon_{J-f u l l-2} & \approx(8 / 9)[92.2491][\mathrm{Kev}+4.6298 \mathrm{Kev}+(1 / 9)\{74.0769 \mathrm{Kev}\}] \\
& \approx 93.43 \mathrm{Kev} .
\end{aligned}
$$

The full width of the $J$ under second order approximation is thus nearly an exact match to experiment ( 93.4 Kev via the GEM vs. 93.2 Kev from PDG (2009-M)). The hadronic width of the $J$ is unchanged from first to second approximation; so, it remains a match with experiment (82.0 Kev via the GEM vs. 81.7 Kev from PDG (2009-M)). As well, the leptonic width of the J via the GEM ( 11.4 Kev ) is now only $2.7 \%$ higher than that reported by the PDG ( $(11.1 \pm 0.2) \mathrm{Kev})$.

## 2.E. The $\Upsilon(1 S)$

Analogous to the $J$, the $\Upsilon(1 S)$, originally a $b b^{*}$ construction, must transition to a $c c^{*}$ excited state of the same mass as that of the $b b^{*}$ state in order to decay into hadrons. Unlike the $J$, however, there is no reason to suspect that leptons emerge from the $b b^{*}$ state. Hence, we assume that all types of $\Upsilon(1 S)$ decays ensue from the $c c^{*}$ excited state. Corroborative evidence abounds in support of such assumption, as we shall see, so let us proceed with the viewing of the two FDs which depict the hadronic decay of the $\Upsilon(1 S)$ and the leptonic decay of the $\Upsilon(1 S)$, respectively:


Figure 7a. Basic Feynman Diagram for $\Upsilon(1 S)$ Formation and Decay into Hadrons via the GEM.


Figure 7b. Basic Feynman Diagram for $\Upsilon(1 S)$ Formation and Decay into Leptons via the GEM.

From Eq. (1) the hadronic width of the $\Gamma(1 S)$, denoted by $\Gamma_{\Upsilon-H}$, via the GEM theoretical structure is given by:

$$
\begin{align*}
\Gamma_{Y-H} & \approx\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)\left(m_{\rho} / m_{Y}\right)^{3}\left(q_{c}\right)^{4} \\
& \approx\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)(776 / 9460)^{3}(16 / 81) \tag{8a}
\end{align*}
$$

where

$$
\begin{equation*}
\alpha_{s}=1.2[\ln (9460 / 50)]^{-1}=0.2289 \tag{8b}
\end{equation*}
$$

Hence,

$$
\begin{equation*}
\Gamma_{\Upsilon-H} \approx 40.76 \mathrm{Kev} \tag{8c}
\end{equation*}
$$

The PDG in the 2008 Meson Table (PDG (2008-M), p.119) reports the corresponding figure as

$$
\begin{equation*}
\Gamma_{\Upsilon-H}(P D G)=49.99 \mathrm{Kev}, \tag{8d}
\end{equation*}
$$

a figure $23 \%$ higher than the GEM-theoretical result.
However, if we look at the leptonic width of the $\Upsilon(1 S)$, denoted by $\Gamma_{Y-L}$, as derived via the GEM, we find from Eq. (2) (the right hand side of same multiplied by " 3 " to take into account muon and tauon pairs in accord with "e- $\mu-\tau$ universality") that

$$
\begin{align*}
\Gamma_{Y-L} & \approx 3(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)\left(m_{\rho} / m_{\Upsilon}\right)^{3}\left(q_{c}\right)^{4} \\
& \approx 3(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)(776 / 9460)^{3}(16 / 81) \tag{9a}
\end{align*}
$$

Hence,

$$
\begin{equation*}
\Gamma_{Y-L} \approx 3.90 \mathrm{Kev}, \tag{9b}
\end{equation*}
$$

which represents a match to the PDG's report from the same 2008 Meson Table of

$$
\begin{equation*}
\Gamma_{Y-L}(\mathrm{PDG})=(4.03 \pm 0.14) \mathrm{Kev} . \tag{9c}
\end{equation*}
$$

Specific to the $e^{+} e^{-}$partial width $\left(\Gamma_{Y-e e}\right)$, the GEM obviously determines $\Gamma_{Y-e e}$ $\approx 1.30 \mathrm{Kev}$, while the PDG in the above-mentioned source (p. 119) states $\Gamma_{Y-e e}\left(\mathrm{PDG}_{1}\right) \approx 1.34 \mathrm{Kev}$ directly, but indirectly, in terms of its stated fractional branching ratio on p.119, a different value is inferred, viz., $\Gamma_{Y \text {-ee }}\left(\mathrm{PDG}_{2}\right) \approx$ 1.29 Kev. From the latter we infer that according to the PDG ( $2008-\mathrm{M}$ ), the experimentally determined value for the $e^{+} e^{-}$partial width of the $\Upsilon(1 S)$ is given by

$$
\begin{equation*}
\Gamma_{Y-e e}(\mathrm{PDG})=(1.31 \pm 0.03) \mathrm{Kev}, \tag{9d}
\end{equation*}
$$

a match to that of the GEM, i.e.,

$$
\begin{equation*}
\Gamma_{Y-e e} \approx 1.30 \mathrm{Kev} . \tag{9e}
\end{equation*}
$$

Herein (i.e., the match between Eq. (9d) and Eq. (9e)) lies the source of a paradox that the hadronic width as given by the GEM (i.e., $\sim 41 \mathrm{Kev}$ ) is correct, though it is so seriously discrepant with that reported by the PDG (i.e., 50 Kev ). The paradox unfolds as follows: In order to obtain the constant " $B$ " in the general expression for $\alpha_{s}$, once $\Lambda$ was determined (see White (2008-R), Section 3; note there, too, that in Eq. (6) on p. 547 the factor " $\alpha$ " is missing as the multiplier of "( $41 / 1.31)$ "), the assumption was made that, since the $\Upsilon(1 S)$ exists well into the realm of asymptotic freedom,
$\alpha / \alpha_{s}=\left(e^{+} e^{-}\right.$partial width)/(hadronic partial width) as associated with the $\Upsilon(1 S)$.
In White (2008-R) we inserted $\Gamma_{Y-e e}(\mathrm{PDG})=1.31 \mathrm{Kev}$ for the $e^{+} e^{-}$partial width, and for the hadronic partial width, we inserted the GEM-theoretical width, i.e., $\Gamma_{Y-H} \approx 41 \mathrm{Kev}$. We then obtained the general relation,

$$
\begin{equation*}
\alpha_{s}=B\left[\ln (9460 / 50]^{-1}=\alpha(41 / 1.31),\right. \tag{10}
\end{equation*}
$$

from which we solved for " $B$ " to obtain, $B=1.2$.
In turn, as " $B$ " is a multiplier on the right hand sides of all width calculations via the GEM theory, and as all width calculations, as seen above, represent nearly exact matches with experiment in all cases except as to the hadronic width of the $\Upsilon(1 S)$, it has been difficult to fathom the source of the disparity between $\Gamma_{Y-H}=41 \mathrm{Kev}$ and $\Gamma_{Y-H}(\mathrm{PDG})=50 \mathrm{Kev}$.

## 2.F. The T Meson

To address the $T$ meson, thought to be a $t t^{*}$ (where " $t$ " represents the top quark) state but never "discovered" to date, is quite speculative on our part,
but we think it important to do so because the GEM provides a perfectly logical reason as to why the $T$ has yet to be "found", i.e., unequivocally shown to exist by experiment. Said reason is just the opposite of the prevailing view as to the "invisibility" of the $T$, which is: "the $T$ doesn't last long enough for it to be found". In a sense such is true; after all, the $b b^{*}$ of the $\Upsilon(1 S)$ transitions instantaneously to a $c c^{*}$ state according to the GEM, but the mass of the original $b b^{*}$ state is preserved in the resulting $c c^{*}$ state, thus allowing for the "finding" of a resonance at the $\Upsilon(1 S)$ mass. Assuming the $T$ to act in like manner to the $\Upsilon(1 S)$, the following FD would apply as regards hadron production:


Figure 8. Basic Feynman Diagram for $T$ Formation and Decay into Hadrons via the GEM.

The hadronic width of the $T$, from Eq. (1) would be:

$$
\begin{align*}
\Gamma_{T} & \approx\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)\left(m_{\rho} / m_{T}\right)^{3}\left(q_{b}\right)^{4} \\
& \approx\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)(776 / 340000)^{3}(1 / 81) \tag{11a}
\end{align*}
$$

where

$$
\begin{equation*}
\alpha_{s}=0.90[\ln (340000 / 50)]^{-1}=0.1020 \tag{11b}
\end{equation*}
$$

(In Eq. (11b) the constant " 1.2 " in the expression for $\alpha_{s}$ becomes " 0.90 " beyond 100000 Mev (see White (2010) and in Eq. (11a) $q_{b}=-1 / 3$.)

Hence,

$$
\begin{equation*}
\Gamma_{T} \approx 0.024 \mathrm{Mev} \tag{11c}
\end{equation*}
$$

Thus, we see that, contrary to the "convenient explanation" as to why the $T$ has not so far been observed, the $T$ lives for a very long time (about 0.03 ps )! It's just that its width to mass ratio makes it impossible right now for the experimental apparatus to pick up such a narrow signal amongst the "noise" inherent in the energy background needed to produce the $T$.

## 2.G. The GEM-Derived $\alpha_{s}$

In a very detailed and comprehensive work (White (2010)), recently published in the Journal of Interdisciplinary Mathematics, the GEM-derived strong coupling parameter, $\alpha_{s}$ (GEM), is shown to be valid over the entire range of energy heretofore reachable by experiment, i.e., from essentially the hadron threshold ( $\sim 270 \mathrm{Mev}$ ) to about twice the mass of the $Z$ boson ( $\sim 2$ [ $91,000 \mathrm{Mev}]$ ).

Furthermore, it is shown in White (2010) that "if $\alpha_{s}$ (GEM) agrees with any experiment designed to determine $\alpha_{s}$ (the experiment performed at any given energy) to within $10 \%$, it is roughly $90 \%$ certain that said experiment will predict the accepted value of $\alpha_{s}\left(\mathrm{EXP} ; m_{z}\right)$ to within $2.5 \%$ ". In the above quote $\alpha_{s}\left(\mathrm{EXP} ; m_{z}\right)$ represents the evolutionary value of $\alpha_{s}$ extrapolated to the $Z$ boson mass $\left(m_{z}\right)$ based on the experimental result for $\alpha_{s}$. The converse is also true, i.e., "in order for a given experiment to be able to accurately determine the accepted value of $\alpha_{s}\left(\mathrm{EXP} ; m_{z}\right)$, it must first agree with $\alpha_{s}$ (GEM), at least at the $90 \%$ level at the energy scale at which the experiment is conducted". The GEM-derived value for $\alpha_{s}$ associated with the energy scale $=m_{z}\left(\alpha_{s}\left(\mathrm{GEM} ; m_{z}\right)\right)$ is found in White (2010) to be given by:

$$
\begin{equation*}
\alpha_{s}\left(\mathrm{GEM} ; m_{z}\right)=0.121 \pm 0.003 \tag{12a}
\end{equation*}
$$

a figure inclusive of the currently accepted result (see (PDG (2004-C), p. 18), given by:

$$
\begin{equation*}
\alpha_{s}\left(\text { PDG } ; m_{z}\right)=0.119 \pm 0.002 \tag{12b}
\end{equation*}
$$

Perhaps the most striking feature of $\alpha_{s}(G E M)$ is that it retains its form, i.e.,

$$
\begin{equation*}
\alpha_{s}(\mathrm{GEM})=1.2[\ln (E / 50 \mathrm{Mev})]^{-1} \tag{13}
\end{equation*}
$$

for any energy, $E$, from $\sim 270 \mathrm{Mev}$ all the way to the $\Upsilon(1 S)$ mass of 9460 Mev , while functioning strikingly well, as we have seen, as a major constituent in the width calculations of the $\rho$, the $\phi$, the $K^{*}(892)$, and the $J(3097)$. Beyond 9460 Mev , the scale factor in $\alpha_{s}$ (GEM) retains its value of 50 Mev as the multiplicative factor of " 1.2 " is allowed to go to its required high energy asymptotic value of " $6 \pi / 21$ " $=" 0.90$ " at $E=m_{z}$ (White (2010)). Thus, $\alpha_{s}$ (GEM) is overall a very simple function, yet very accurate essentially everywhere. As another example of such, we cite Aguilar et al. (2004)), wherein on page 254, the authors put forth a value for $\alpha_{s}$ evaluated at the tauon mass ( 1777 Mev ), such value claimed to be scheme independent, it serving as a standard, therefore, to be employed in order to test the reliability of four schemes therein introduced for obtaining $\alpha_{s}$. Their result,
 match to the GEM's determination of the same entity of 0.336 .

Now, as we have seen from Section 2.G, the constant " $B$ " $=$ " 1.2 " is derived in part from basing the ratio of the experimental partial width, $\Gamma_{Y-e e}($ PDG $)$, to the GEM-theoretical hadronic width, $\Gamma_{Y-H}$, as equal to ( $\alpha / \alpha_{s}$ ) evaluated at the $\Upsilon(1 S)$ mass. The $\Upsilon(1 S)$, existing as it does well out into the realm of asymptotic freedom must have $\left(\Gamma_{\Upsilon-e e} / \Gamma_{Y-H}\right)=\left(\alpha / \alpha_{s}\right)$; the FDs as constructed according to Figures 7a and 7 b demand such. Acceding to such demand leads, however, to the paradoxical situation discussed in Section 2.E. Hence, of all the applications of the GEM, $\Gamma_{Y-H}$ as equal to 40.76 Mev is essentially the only GEM-derived quantity associated with
anything from the $\rho$ mass to beyond the $Z$ mass that, apparently, "does not work". To "make it work" within the context of the GEM as described so far, "B" would have to become " 1.2 )(50/41)" $=$ " 1.46 ", thus destroying utterly the plethora of agreements with experiment outlined above. We address the problem as it stands immediately below in Section 3.

## 3. A Postulated Additional Route in $\Upsilon(1 S)$ Decay (2010)

There is, actually, a very simple, and at the same time a very plausible solution to the paradox mentioned above in Section 2.E and Section 2.G, viz., we postulate an additional route for $\Upsilon(1 S)$ decay into hadrons, a route assumed not to have a high probability of occurrence for the $J$ or the other vector mesons of mass less than that of the $J$. As the basis for the existence of the additional route available to the $\Upsilon(1 S)$, we point to the fact that there is roughly three times the energy spectrum available to the $\Upsilon(1 S)$ in its decay ( 9460 Mev worth) as compared to the next lightest vector meson, i.e., the $J$ ( 3097 Mev worth). With three times the energy spectrum (as compared to the $J$ ) available to the $\Upsilon(1 S)$, we think it plausible that decays resulting in hadrons as products may be allowed to take place through the bifurcation of the gluon emitted from the resonance state (or more simply stated: via emission of two gluons), rather than what has heretofore been assumed in accord with Figure 7a, in which a single gluon, $\zeta_{3 a}$, converts to hadrons to mark the final stage of the decay process. Specifically, we propose that, in addition to the route as described immediately above, that a route exists in which $\zeta_{3 a}$ bifurcates into two gluons, each of which then converts to hadrons. The FD associated with the proposed additional route is seen immediately below.


Figure 9. Basic Feynman Diagram for Postulated Additional Route for $\Upsilon(1 S)$ Formation and Decay into Hadrons ( $h, h^{\prime}, h^{\prime \prime}$, and $h^{\prime \prime \prime}$ ) via the GEM.

The additional route, which we denote as the "bifurcated gluon route for hadron decay" (BGRHD), effectively adds $\alpha_{s}$ times $\Upsilon_{Y-H}$, or ( 0.2289 ) ( 40.76 Kev ) $=$ 9.33 Kev to the GEM-theoretical width of the $\Upsilon(1 S)$. The reformulated situation regarding the $\Upsilon(1 S)$ may be summarized, therefore, as follows:

Denoting the partial width due to the BGRHD as $\Gamma_{Y-B G H}$, we have

$$
\begin{equation*}
\Gamma_{Y-B G H}=9.33 \mathrm{Kev} . \tag{14}
\end{equation*}
$$

From Section 2.E we have

$$
\Gamma_{Y-H}=40.76 \mathrm{Kev}
$$

Also from Section 2.E we have

$$
\Gamma_{Y-L}=3.90 \mathrm{Kev}
$$

The net hadronic width of the $\Upsilon(1 S)$ as per the GEM would now be given by given by

$$
\begin{equation*}
\Gamma_{Y-(H+B G H)}(\text { GEM2010 })=50.09 \mathrm{Kev}, \tag{15}
\end{equation*}
$$

which now represents a nearly perfect match to $\Gamma_{Y-H(\mathrm{PDG})}=49.99 \mathrm{Kev}$.
In addition the full width of the $\Upsilon(1 S)$ as per the GEM would now be given by

$$
\begin{equation*}
\Gamma_{Y-f u l l}(\text { GEM2010 })=53.99 \mathrm{Kev}, \tag{16}
\end{equation*}
$$

which also represents a nearly perfect match to $\Gamma_{Y \text {-full }}(P D G)=(54.02 \pm 1.25) \mathrm{Kev}$.
With the addition of the BGRHD the calculation of " $B$ " in the expression for $\alpha_{s}$ is uncompromised, while at the same time the major discrepancy between the hadronic width of the $\Upsilon(1 S)$ as determined via the GEM versus via the methods engaged by the PDG is completely removed. For that reason we believe the postulate as to the addition of the BGRHD is a viable one. In fact, if we postulate that in addition to the BGRHD there is a companion route for leptons, i.e., a bifurcated gluon route for lepton decay (BGRLD), whose FD is identical to that of Figure 9, except that on the far right hand side of the diagram, each " $\alpha_{s}$ " is replaced by " $\alpha$ " and " $h, h^{\prime}, h^{\prime \prime}$, and $h^{\prime \prime \prime}$ " are replaced by " $l_{i}^{+}, l_{i}^{-}, l_{j}^{+}$, and $l_{j}^{-}$", respectively, where " $i$ " and " $j$ " denote lepton types and $i=j$ is allowed, $(3.90 \mathrm{Kev} / 137.036)=0.03 \mathrm{Kev}$ would be added to $\Gamma_{\Upsilon \text {-full }}(\mathrm{GEM} 2010)$ above, thus bringing

$$
\begin{equation*}
\Gamma_{\Upsilon \text {-full }}(\text { GEM2010 }) \rightarrow 54.02 \mathrm{Kev}, \tag{17}
\end{equation*}
$$

i.e., the realization of an exact match to experiment.

## 4. Application of the GEM to the $\Upsilon(2 S)$ (2010)

As a test of the basic theoretical structure thus far presented, we now seek to apply the GEM to the calculation of the width of the $\Upsilon(2 S)$. Guided by White (2009- $\psi$ ), in which the companion to the $J$, i.e., the $\psi(2 S)$, was explored via the GEM, we first note that according to PDG (2008-M) that the $e^{+} e^{-}$partial width of the $\Upsilon(2 S)$, denoted by $\Gamma_{20-e e}$ (PDG), is given by

$$
\begin{equation*}
\Gamma_{20-e e}(P D G)=(0.61 \pm 0.01) \mathrm{Kev} . \tag{18}
\end{equation*}
$$

Application of Eq. (2), assuming that $100 \%$ of the original $b b^{*}$ state associated with the $\Upsilon(2 S)$ transitions in point-like manner to a $c c^{*}$ state (an unphysical situation, as some $b b^{*}$ must remain in order to convert to the $\Upsilon(1 S)$ ) yields the corresponding partial width as:

$$
\begin{equation*}
\Gamma_{20-e e}(\text { GEM }) \approx(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)\left(m_{\rho} / m_{\Upsilon(2 S)}\right)^{3}\left(q_{c}\right)^{4} \tag{19a}
\end{equation*}
$$

or

$$
\begin{align*}
\Gamma_{20-e e}(\mathrm{GEM}) & \approx(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)(776 / 10,023)^{3}(16 / 81) \\
& =1.09 \mathrm{Kev} . \tag{19b}
\end{align*}
$$

From here, it is not difficult to show that if $\sim(4 / 9)^{\text {ths }}$ of the $b b^{*}$ remain, some to decay into lepton pairs, the others to make a transition to the $\Upsilon(1 S)$, with $(5 / 9)^{\text {ths }}$ of the original $b b^{*}$ state transitioning to a $c c^{*}$ state, the experimental result expressed by Eq. (18) can be met via the GEM. The situation is, in fact, in exact analogy to the situation regarding the $J$. In "second approximation" regarding the $J$ recall that the form factor, $f=\left(1-q_{s}^{2}\right)=(8 / 9)$, was introduced as multiplying the entire FD array associated with the second order approximation associated with $J$ decay (see Figure 6b). Here, we have a form factor, $f^{\prime}=\left(1-q_{c}^{2}\right)=(5 / 9)$, which we can, by analogy, set up to multiply the entire FD array characterizing the $\Upsilon(2 S)$ direct decay route to dissolution, as seen immediately below:


Figure 10. Feynman Diagram Array Characterizing the Formation and Direct Decay to Dissolution of the $\Upsilon(2 S)$ in Second Order Approximation via the GEM.

We may thus, in complete analogy to Eq. (7b), arrive at the full partial width of the decay of the $\Upsilon(2 S)$ resulting in complete dissolution, denoted by $\Gamma_{20 \text {-full }}(\mathrm{GEM})$, as follows:

$$
\begin{aligned}
\Gamma_{20-\text {-fll }}(\mathrm{GEM}) \approx & (5 / 9)\left[\left(\alpha_{s} / 2 \pi\right)(10,042)\left(2 m_{e}\right)\left(m_{\rho} / m_{y}(2 S)\right)^{3}\left(q_{c}\right)^{4}\right. \\
& +3(\alpha / 2 \pi)(10,042)(2 m e)\left(m_{\rho} / m_{y(2 S)}\right)^{3}\left(q_{c}\right)^{4} \\
& \left.+(4 / 9)\left\{3(\alpha / 2 \pi)(10,042)\left(2 m_{e}\right)\left(m_{\rho} / m_{y(2 S)}\right)^{3}\left(q_{b}\right)^{4}\right\}\right] .
\end{aligned}
$$

The factor " 3 " in front of " $(\alpha / 2 \pi)$ " in two spots in the above equation takes into account muon and tauon production in accord with " $e-\mu-\tau$ universality".

At the $\Upsilon(2 S)$ mass we have

$$
\begin{equation*}
\alpha_{s}=1.2[\ln (10,023 / 50)]^{-1}=0.2264 \tag{20}
\end{equation*}
$$

Hence,

$$
\begin{equation*}
\Gamma_{20-\text {-ful }}(\mathrm{GEM})=(5 / 9)[33.90 \mathrm{Kev}+3.28 \mathrm{Kev}+0.09 \mathrm{Kev}], \tag{21a}
\end{equation*}
$$

which can be simplified to:

$$
\begin{equation*}
\Gamma_{20-f u l l}(\mathrm{GEM})=18.83 \mathrm{Kev}+1.87 \mathrm{Kev}=20.70 \mathrm{Kev} . \tag{21b}
\end{equation*}
$$

We note that the hadronic part of $\Gamma_{20 \text {-full }}(G E M)$ is 18.83 Kev and that the leptonic part of $\Gamma_{20 \text {-full }}(G E M)$ is 1.87 Kev , a figure representing a near match to the PDG's figure of $(1.83 \pm 0.03) \mathrm{Kev}$. Thus, similar to the case of the $J$, the leptonic width of the $\Upsilon(2 S)$ as determined via the GEM is about $2.2 \%$ higher than that reported by the PDG.

In addition $\Gamma_{20 \text {-full }}(\mathrm{GEM})=20.70 \mathrm{Kev}$ represents $64.7 \%$ of the full width of the $\Upsilon(2 S)$ as reported in PDG (2008-M) ... a figure within $\sim 10 \%$ of the experimentally determined fraction for same of $72.2 \%$ reported in the same source. Because information is still somewhat sketchy as to the details of the $\Upsilon(2 S)$ decay (listed individual decay routes in PDG (2008-M) comprise only $33 \%$ of the full width of the $\Upsilon(2 S)$ ), that $\left\{\Gamma_{20 \text {-full }}(\mathrm{GEM}) / \Gamma_{\Upsilon(2 S) \text {-full }}\right\}$ is only $\sim 10 \%$ discrepant with experiment at the present time is an encouraging result.

## 5. Concluding Remarks

We know of no other complete, comprehensive, QED-based theory of vector mesons, besides the GEM theory of same. Nor do we know of a single complete, comprehensive theory of the strong coupling parameter $\left(\alpha_{s}\right)$, besides that based on the GEM. With the additions made to the GEM as seen in Section 3, the GEM is now fully descriptive of all aspects of all known vector mesons (plus the $\left.K^{*}(892)\right)$ to a very high degree of accuracy, and, as has been shown in White (2010), in order for any determination of $\alpha_{s}$ to be employed successfully (i.e., within $2.5 \%$ ) as the basis for the determination of $\alpha_{s}$ associated with the $Z$ boson energy, such determination must agree to within $10 \%$ with the GEM-theoretical
result at the energy where $\alpha_{s}$ is determined. Perhaps we should not be surprised at such findings: After all, the GEM assumes that a known mechanism is universally involved in the formation and decay of any vector meson and then proceeds along the road of the calculation of associated Feynman Diagrams ... straight along the guidelines as laid out in QED, with the exception of, in the case of hadron production, the substitution of $\alpha_{s}$ for $\alpha$ in order to represent gluon absorption at the hadron production vertex. Lepton production, of course, follows strictly the guidelines laid out in QED in the limits of asymptotic freedom, generally accepted to set in at $E \sim 3000 \mathrm{Mev}$. History has shown time and time again that if the mechanism involved in a quantum mechanical process is well understood, the calculation of the associated FDs will yield results which are very representative of what is seen via reliable experiment. Our analysis of the GEM above indeed yields nothing short of spectacular agreement with experiment on the assumption that vector mesons arise by means of an electromagnetic, spin - spin interaction, causing a spin flip of one of the quarks of a di-quark pair which has absorbed a four-momentum propagator (as a gluon).

By contrast, in the literature having to do with theoretical calculation of vector meson widths is apparent no comprehensive view of vector mesons at all. The $\rho$ is handled differently than the $\phi$, which is handled differently than the $J$, which, in turn, is handled differently than the $\Upsilon(1 S)$. Indeed, as mentioned in Section 2.F, the $T$ is considered as "invisible" by the "physics establishment". There is prevalently not even the hint of a mechanism which would lead to a spin one state in QCD-based work on vector mesons. Rather, the existence of the resonance state is taken as the starting point for width calculations. Such stance throws completely out the window, so to speak, any chance to include the formation stage of vector mesons in the relevant calculations ... and therefore the influence that such stage has on them as to the width is completely lost. Close's brilliant insight to treat the gluon and the virtual photon as two aspects of the same entity (the fourmomentum propagator) in complementary fashion is also not widely employed in the QCD literature. Such leads to such FDs as seen below as the prototype for describing lepton production from $\Upsilon(1 S)$ decay (from Hart et al. (2005)):


Figure 11. Basic Feynman Diagram for $\Upsilon(1 S)$ Meson Decay into an electron/positron pair proposed by Hart et al.

One will note that the FD of Figure 11 bears very little resemblance to the corresponding figure (Figure 7b) describing $e^{+} e^{-}$production via the GEM. In the
context of the GEM a $c c^{*}$ state of mass 9460 Mev is what decays; in Figure 11 it is a $b b^{*}$ state. Within the context of the GEM the four-momentum propagator, $\zeta_{3 a}$, couples to the resonance (on the left) with strength " 1 "; in Figure 11 the corresponding coupling is $\alpha q_{b}^{2}=(1 / 9) \alpha$, because the four-momentum propagator of Figure 11 is considered as a virtual photon $\left(\gamma_{v}\right)$ all the way to the pair vertex In the words of the authors of Figure 11, "there is no exact mathematical result for $M$ ", whereas the corresponding term (V) in the GEM is well known. In addition, in their expression for the $e^{+} e^{-}$partial width, the overall mass dependence is $\left[m_{Y}\right]^{-2}$ (indicative of the $e^{+} e^{-}$collision cross-section as associated with the $\Upsilon(1 S)$ ), whereas the QED-based GEM demands a mass dependence of $\left[m_{Y}\right]^{-3}$ as associated with any quantum system which spontaneously decays to dissolution (see for example, Merzbacher (1970), p.486). Noteably missing is the term " $q_{c}^{4 "}$, which describes the basic, known interaction, derived from QED as essential for the representation of the formation of the $\Upsilon(1 S)$. Is it any wonder that, whereas armed with a hand-held scientific calculator, via the GEM one may determine the electron/positron partial width of the $\Upsilon(1 S)$ in less than a minute, it takes the authors of Figure 11 seventy-two full days using a single super computer processor (or as they report, "18 hours of running ... using 96 processors of the Sunfire supercomputer") to accomplish the same thing?

The GEM is "color blind", i.e., there is no reference to quark color in the GEM theory, whereas the literature is replete with such objects as "color octets" contained in "Fock states", though their effects "must be estimated" (PDG (2004-C), p. 10). The GEM contains no radiative corrections as part of the width calculations associated with any vector mesons. The PDG, on the other hand, utilizes radiative corrections from QED in order to determine from experiment the widths of the $J$ and the $\Upsilon(1 S)$, as they cannot be measured directly (PDG (2006$\mathrm{W})$ ). Our point is simply one of interest here: The GEM theoretical structure is supremely simple, yet it produces the same results with far less "effort" than other, purely formal methodology contained in QCD. Ockham's Razor comes immediately to mind: If one can determine the same quantity ... the electron/positron partial width of the $\Upsilon(1 S)$, for example, utilizing far less input than and in less than $(1 / 103680)^{\text {th }}$ the time in calculation (if only one super computer processor is available) compared to another method, which method is the more viable? There are complexities along either avenue, but the basic question to be answered in pursuing the GEM is, "what Feynman Diagrams will fit the basic picture set forth by QED?", whereas it seems that to pursue the road devoid of the basic electromagnetic process responsible for the formation of the spin one resonance, the basic question is, "how many Feynman Diagrams and how many theoretical constructs from QCD can we mash together ... along with how much computer time must we use ... to force out a reasonable answer?" One avenue has a firm base upon which to stand; the other ... baseless, really ... opts for machination,
both of the mental and of the material types. The latter is wherefrom the moniker "industrial physics" originates. The GEM certainly does not fall in such category. If anything, the GEM is representative of what we perceive as a "lost art" . . pure QED at its core ... proposed by names unknown and, alike, by names linked to some degree of fame (Close and Dalitz, respectively, representative of such names) 30 years ago, when QCD was just getting its start. In closing we sincerely hope that our review of the 30-year history of the GEM has shown it to be not only a theory of a lost art, but a theory of great and beautiful art, as well.

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D. White, Department of Biological, Chemical, and Physical Sciences, Roosevelt University, 430 S. Michigan Ave., Chicago, Illinois 60605, USA
E-mail: dwhite@roosevelt.edu

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