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Research Article

Stochastic Integrals in Discounting Proactive Operations

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Abstract. Proactivity constitutes a structural factor of decision making in many practical disciplines. It is generally adopted that stochastic discounting models substantially contribute to the development and implementation of proactive operations. The paper concentrates on the formulation of a stochastic discounting model by incorporating a stochastic integral and a positive random variable. Moreover, the paper provides interpretation of the stochastic discounting model in proactive environments.

Keywords. Proactivity; Stochastic integral; Infinite divisibility; Discounting model

Mathematics Subject Classification (2020). 47N90; 47A48; 60H05; 90B50

1. Introduction

It is quite easily understood that the concept of infinite divisibility constitutes a valuable factor for the implementation of research activities in various areas of probability distributions. More precisely, it is very well known the particular significance of results related to the family of infinitely divisible distributions [15]. It is of some particular theoretical and practical importance to mention that the number of new research activities on infinitely divisible distributions have been significantly increased during the last six decades [16]. In particular, research activities concentrating on the preservation of infinitely divisibility under certain transformations have substantially contributes to the establishment of significant theoretical results with very useful practical interpretations [14]. Such research activities are generally implemented by making use of the canonical representation of the corresponding characteristic functions [13]. Research activities in the area of transformations of characteristic functions incorporate the following three steps. The first step is the formulation of a transformation for characteristic functions. The second step is the investigation of the formulated characteristic function. More precisely, the second step concentrates on the establishment of theoretical properties for the formulated characteristic function. The third step is the interpretation in practice of the formulated characteristic function. In other words, the third step is readily recognized as a principal component of stochastic models formulation.

The contribution of the present paper consists of the implementation of two purposes. The first purpose is the establishment of a characterization of a transformed infinitely divisible characteristic function. The second purpose is the interpretation of the established characterization in the area of continuous stochastic discounting. More precisely, the established characterization incorporates the product of two infinitely divisible characteristic functions. One of these characteristic functions belongs to a stochastic integral. In consequence, the presence of a stochastic integral contributes to the implementation of the second purpose of the paper.

2. Interconnecting Infinitely Divisible Characteristic Functions

The present section is devoted to the establishment interconnections between four infinitely divisible characteristic functions. The stochastic derivations and the practical interpretations of interconnections between infinitely divisible characteristic functions are generally recognized as particularly useful in formulating and investigating stochastic models [4,6,10].

We suppose that L is an infinitely divisible random variable with characteristic function $\varphi_L(u)$ then the function

$$\varphi_V(u) = \exp\left\{a\int_0^u \frac{\log\varphi_L(w)}{w}dw\right\} \tag{2.1}$$

is the characteristic function of an infinitely divisible random variable V, where $a > 0$ [7].

We also suppose that S is an infinitely divisible random variable with characteristic function $\varphi_S(u)$ then the function

$$\varphi_C(u) = \exp\left\{\frac{a}{u^a}\int_0^u \log\varphi_S(w)w^{a-1}dw\right\} \tag{2.2}$$

is the characteristic function of an infinitely divisible random variable where, $a > 0$ [7,9,16].

The infinitely divisible characteristic function $\varphi_V(u)$ and the infinitely divisible characteristic function $\varphi_C(u)$ have been established as very strong tools in the area of stochastic integration [2,16]. The present section is devoted to the consideration of structural interconnections between the infinitely divisible characteristic functions

$$\varphi_L(u), \varphi_V(u), \varphi_C(u), \varphi_S(u).$$

Such interconnections significant facilitate the formulation, investigation, and interpretation of stochastic models [1,17].

More precisely, the results of this section are useful to revealing the important role for undertaking research activities in stochastic modeling of the random variable L, the random variable V, the random variable C, and the random variable S by making use of the corresponding characteristic functions [8, 18].

Theorem. *We suppose that the infinitely divisible random variable L is independent of the infinitely divisible random variable C and the characteristic function $\varphi_L(u)$ is differentiable. Moreover, we consider the infinitely divisible random variable S with differentiable characteristic function $\varphi_S(u)$ then*

$$\varphi_L(u)\exp\left\{\frac{a}{u^a}\int_0^u \log\varphi_S(w)w^{a-1}dw\right\} = \varphi_S(u) \tag{2.3}$$

if, and only if,

$$L + V \overset{d}{=} S\ , \tag{2.4}$$

where $\overset{d}{=}$ denotes equality in distribution.

Proof. Only the sufficiency condition will be proved since the necessity condition can be proved by reversing the argument. If we use characteristic functions in (2.4) we get that

$$\varphi_L(u)varphi_V(u) = \varphi_S(u). \tag{2.5}$$

From (2.4) and (2.5) we get the integral equation

$$\varphi_L(u)\exp\left\{a\int_0^u \frac{\log\varphi_L(w)}{w}dw\right\} = \varphi_S(u)$$

or equivalently the integral equation

$$\log\varphi_L(u) + a\int_0^u \frac{\log\varphi_L(u)}{w}dw = \log\varphi_S(u). \tag{2.6}$$

Since the characteristics functions $\varphi_L(u)$ and $\varphi_S(u)$ are infinitely divisible then $\varphi_L(u)$ and $\varphi_S(u)$ have not real roots. Moreover, $\varphi_L(u)$ and $\varphi_S(u)$ are differentiable. Hence, the integral equation (2.6) implies the differential equation

$$\frac{d}{du}\log\varphi_L(u) + a\frac{\log\varphi_L(u)}{u} = \frac{d}{du}\log\varphi_S(u) \tag{2.7}$$

with $u \neq 0$. Multiplying both sides of the differential equation (2.7) by u^a we get the differential equation

$$u^a\frac{d}{du}\log\varphi_L(u) + au^{a-1}\log\varphi_L(u) = u^a\frac{d}{du}\log\varphi_S(u). \tag{2.8}$$

From (2.8) we get that

$$\int_0^u w^a d\log\varphi_L(w) + a\int_0^u w^{a-1}\log\varphi_L(w)dw = \int_0^u w^a d\log\varphi_S(w). \tag{2.9}$$

Integrating in the integral equation (2.9) we get the integral equation

$$u^a\log\varphi_L(u) - a\int_0^u w^{a-1}\log\varphi_L(w)dw + a\int_0^u w^{a-1}\log\varphi_L(w)dw$$
$$= u^a\log\varphi_S(u) - a\int_0^u w^{a-1}\log\varphi_S(w)dw$$

which can be written in the form

$$\log\varphi_L(u)=\log\varphi_S(u)-\frac{a}{u^a}\int_0^u \log\varphi_S(w)w^{a-1}dw \tag{2.10}$$

for $u\neq 0$. From (2.10) it follows that

$$\varphi_L(u)\exp\left\{\frac{a}{u^a}\int_0^u \log\varphi_S(w)w^{a-1}dw\right\}=\varphi_S(u). \tag{2.11}$$

In the following section, the interpretation of (2.4) in the implementation of proactive operations is established.

3. Stochastic Integrals in Continuous Discounting Models

We consider an asset with indefinite lifetime and we suppose that $\{X(t),t=0\}$ is a stochastic process with the random variable $X(t)$ denoting the income produced by the asset in the time interval $[0,t]$. If $1/a$ is the force of interest then the existence, the investigation and the interpretation of the stochastic integral

$$V=\int_0^\infty e^{-t/a}dX(t)$$

are known [7]. More precisely, the stochastic integral V denotes the present value, as viewed from the time point 0, of the income produced by the asset during life time [7]. We also suppose that the random variable J denotes a cash flow arising at the time point 0, and J is distributed as the random variable L. Hence the stochastic model

$$S=J+V$$

or equivalently the statistic models

$$S=J+\int_0^\infty e^{-t/a}dX(t),$$

denotes the sum of two cash flows arising at time point 0. It is quite obvious that the above stochastic model facilitates the operations of strategic thinking and decision making related to the future evolution of the asset. More precisely, the facilitation of these extremely useful operations is achieved by providing modelers and practitioners with the ability to act proactively [11,12]. It is readily understood that the incorporation of an activity treating practical situations usually makes use of stochastic models [5]. The present section formulates a stochastic discounting model, incorporating a stochastic integral as the main structural factor, in order to take advantage of the extremely important theoretical and practical properties of the valuable concept of proactivity for the fundamental discipline of decision making under conditions of uncertainty [3].

4. Conclusions

It constitutes a general adoption that proactivity is a structural factor of the decision making activities under conditions of uncertainty. The presence of proactivity in practical situations requires the formulations, and interpretations of stochastic models. It is obvious that stochastic discounting models form a family of stochastic models quite suitable for introducing proactivity in practical situations. The present paper makes use of a stochastic integral and a positive random variable in formulating a stochastic discounting model for introducing the concept of proactivity in decision making under conditions of uncertainty. In addition, the presence of the property of infinite divisibility in the incorporated stochastic integral strongly facilitates the investigation and interpretation of the presence of proactivity in the formulation and interpretation of the corresponding stochastic model.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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