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Research Article

Soft D_μ-Compactness in Soft Generalized Topological Spaces

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Abstract. The aim of this paper is to introduce a further generalization of compactness in soft generalized topological spaces. We define and study the concept of soft D_μ-compact spaces in soft generalized topological spaces. Basic properties and characterizations of soft D_μ-compact spaces are established. Soft D_μ-compactness in subspaces of soft generalized topological spaces are also investigated.

Keywords. Soft generalized topological spaces; Soft D_μ-set; Soft D_μ-compactness; Soft μ-D_2 space

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1. Introduction

The concept of *generalized topological space* (GTS) was introduced by Csaszar [2] is one of the most important developments of general topology in recent years. The concept of γ-compactness in Generalized Topological Spaces have been introduced by Csaszar [3]. Sunil Jacob John introduced the concept of Soft μ-Compact [8] in Soft Generalized Topological Spaces. The purpose of the present paper is to show that the concept of a soft compact space can be generalized by replacing soft μ-open sets by soft D_μ-sets. We establish some of the basic properties and characterizations. We also examine the basic theorems about soft D_μ-compactness in subspaces, soft μ-D_2 spaces in soft generalized topological spaces.

2. Preliminaries

We recall some basic definitions and notations of most essential concepts needed in the following. Let X be a non-empty set and denote $\exp(X)$ the power set of X. According to [2], a collection $\mu \subseteq \exp(X)$ of subsets of X is called a *generalized topology* (GT) on X and (X,μ) is called a *generalized topological space* (GTS) if μ has the following properties:

(i) $\varphi \in \mu$

(ii) Any union of elements of μ belongs to μ. Let μ be a GT on a set $X \neq \varphi$.

Note that $X \in \mu$ must not hold; if $X \in \mu$ then we say that the GT μ is strong [2]. Let $8M_\mu$ denote the union of all elements of μ; of course, $M_\mu \in \mu$, and $M_\mu = X$ if and only if μ is a strong GT. The space (X,μ) or simply X will always mean a strong generalized topological space with the strong generalized topology μ. A subset U of X is called μ-open if $U \in \mu$. A subset V of X is called μ-closed if $X - V \in \mu$. A subset U of X is called μ-clopen if U is both μ-open and μ-closed.

Jyothis and Sunil [4] introduced the concept of *Soft Generalized Topological Space* (SGTS) and studied Soft μ-compactness in SGTSs. The generalized topology is different from general topology by its axioms. According to Csaszar, a collection of subsets of X is a generalized topology on X if and only if it contains the empty set and arbitrary union of its elements. But soft generalized topology is based on soft set theory. Jyothis and Sunil [5] discussed some separation axioms in soft generalized topological space.

Throughout, this paper U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a non-empty subset of E.

Definition 2.1 ([6])**.** Let a soft set F_A over the universe U is defined by the set of ordered pairs $F_A = \{(e, f_A(e))/e \in E, f_A(e) \in P(U)\}$, where f_A is a mapping given by $f_A : A \to P(U)$ such that $f_A(e) = \phi$ if $e \notin A$. Here f_A is called an approximate function of soft set F_A. The set of all soft sets over U is denoted by $S(U)$.

Definition 2.2 ([6])**.** Let $F_A \in S(U)$. A *Soft Generalized Topology* (SGT) on F_A, denoted by μ or μ_{F_A} is a collection of soft subsets of F_A having the following properties: (i) $F_\emptyset \in \mu$ and (ii) The soft union of any number of soft sets in μ belong to μ. The pair (F_A,μ) is called a *Soft Generalized Topological Space* (SGTS). The *Soft Generalized Topological Space* (SGTS) is said to be strong if $A = E$.

Throughout, this paper we consider *Strong Soft Generalized Topological Spaces* (SSGTS).

Definition 2.3. A subset F_B of a space (F_E,μ) is called a soft D_μ-set if there are two sets $F_U,F_V \in \mu$ such that $F_U \neq F_E$ and $F_B = F_U - F_V$.

Definition 2.4. A collection $\Re$ of subsets of soft generlized topological space (F_E,μ) is said to be a cover of F_E if the union of the elements of $\Re$ is equal to F_E.

It is called a soft D_μ-cover of F_E if its elements are soft D_μ-subsets of F_E.

The SGTS (F_E,μ) is called soft D_μ-compact if every soft D_μ-cover of F_E has finite subcover.

Definition 2.5. A space (F_E,μ) is called soft μ-D_2 if for any pair of distinct points α_1, α_2 of F_E, there exist disjoint soft D_μ-sets F_G and F_H of F_E containing α_1 and α_2, respectively.

Soft D_μ-compact Space in SSGTS

Theorem 1. *If (F_E,μ) is finite soft generalized topological space. Then F_E is soft D_μ-compact.*

Proof. Let $F_E = \{F_{a_1},F_{a_2},\dots,F_{a_n}\}$. Let $\Re$ be a soft D_μ-covering of F_E. Then each element in F_E belongs to one of the members of $\Re$ say $a_1 \in F_{G_1}$, $a_1 \in F_{G_2},\dots,a_n \in F_{G_n}$, where $\in F_{G_i} \in \Re$ and $F_{G_i} = F_{u_i} - F_{v_i}, F_{u_i}, F_{v_i}$ are soft μ-open. $F_{u_i} \neq F_E$, $i = 1,2,3,\dots,n$. Since each F_{G_i} is soft D_μ-set, the collection $\{F_{G_1},F_{G_2},\dots,F_{G_n}\}$ is a finite subcollection of soft D_μ-sets which covers F_E. Hence F_E is soft D_μ-compact. □

Theorem 2. *Let F_B be soft D_μ-compact subsets of soft μ-D_2 space (F_E,μ) and $\alpha \in F_E$ is not in F_B, then there is a soft μ-open set F_G such that $F_B \subset F_G$.*

Proof. Suppose F_B is soft D_μ-compact subsets of soft μ-D_2 space (F_E,μ) and $\alpha \in F_E$ is not in F_B. Since (F_E,μ) is soft μ-D_2, for each $\beta \in F_B$, there exists soft D_μ-sets F_{U_α} and F_{V_β} such that $\alpha \in F_{U_\alpha}$, $\beta \in F_{V_\beta}$, $F_{U_\alpha} \cap F_{V_\beta} = F_\varphi$, where $F_{U_\alpha} = F_{C_\alpha} - F_{D_\alpha}$, $F_{V_\beta} = F_{F_\beta} - F_{G_\beta}$, F_{C_α}, F_{D_α}, F_{F_β}, F_{G_β} are soft μ-open sets. Now, the collection $\{F_{V_\beta} : \beta \in F_B\}$ is a soft D_μ-covering of F_B. Since F_B is soft D_μ-compact, there exist a finite subcollection, say $\{F_{V_{\beta_1}},F_{V_{\beta_2}},\dots,F_{V_{\beta_n}}\}$ of soft D_μ-sets covering F_B. Thus, $F_B \subset \bigcup_{i=1}^{n} F_{V_{\beta_i}} = \bigcup_{i=1}^{n} F_{F_{\beta_i}} - F_{G_{\beta_i}} \subset \bigcup_{i=1}^{n} F_{F_{\beta_i}}$. Since F_{F_β} is soft μ-open, $\bigcup_{i=1}^{n} F_{F_{\beta_i}}$ is soft μ-open. Hence the proof. □

Theorem 3. *Let (F_E,μ) be strong soft generalized topological spaces. Then finite union of soft D_μ-compact sets is soft D_μ-compact.*

Proof. Assume that $F_G \subseteq F_E$ and $F_F \subseteq F_E$ are any soft D_μ-compact subsets of F_E. Let $\Re$ be a soft D_μ-cover of $F_G \cup F_F$. Then $\Re$ will also soft D_μ-cover of both F_G and F_F. So, by hypothesis, there exist a finite subcollection of $\Re$ of soft D_μ-sets say $\{F_{G_1},F_{G_2},\dots,F_{G_n}\}$ and $\{F_{F_1},F_{F_2},\dots,F_{F_m}\}$ covering F_G and F_F respectively, where $F_{G_i} = F_{A_i} - F_{B_i}$, $F_{A_i} \neq F_E$ and F_{A_i}, F_{B_i} are soft μ-open, $i = 1,2,\dots,n$, $F_{F_j} = F_{C_j} - F_{D_j}$, $F_{C_j} \neq F_E$ and F_{C_j}, F_{D_j} are soft μ-open. Clearly, the collection $\{F_{G_1},F_{G_2},\dots,F_{G_n},F_{F_1},F_{F_2},\dots,F_{F_m}\}$ is a finite subcollection of $\Re$ of soft soft D_μ-sets covering $F_G \bigcup F_F$. By induction, every finite union of soft D_μ-compact sets is soft D_μ-compact. □

Theorem 4. *Let (F_E,μ) be strong SGTS. If μ is a collection of all soft μ-clopen sets then non-empty subsets of a soft D_μ-compact space is soft D_μ-compact.*

Proof. Let the SGTS (F_E,μ) be soft D_μ-compact space and F_G be non-empty soft subset of F_E. By hypothesis, there exists two soft μ-open $F_P,F_Q,F_P \neq F_E$ such that $F_G = F_P - F_Q$.
$F_E - F_G = F_E - (F_P - F_Q)$ which implies $F_E - F_G$ is soft D_μ-sets. Consider the collection $\Re = \{F_{A_\alpha}\}_{\alpha\in J}$ where $F_{A_\alpha} = F_{B_\alpha} - F_{C_\alpha}$, $F_{B_\alpha} \neq F_E$, F_{B_α}, F_{C_α} are soft μ-open sets, be a soft D_μ-cover of F_G. Then the collection $\{\{F_{A_\alpha}\}_\alpha \in J, \{F_E - F_G\}\}$ is a soft D_μ-covering of F_E. It is given that F_E is soft D_μ-compact, then there exist a collection $\Re$ of soft D_μ-sets covering F_E which can be either

(i) $\{F_{A_{\alpha_1}},F_{A_{\alpha_2}},\dots,F_{A_{\alpha_n}}\}$ or

(ii) $\{F_{A_{\alpha_1}},F_{A_{\alpha_2}},\dots,F_{A_{\alpha_n}},F_E - F_G\}$.

Consider (i) Since $\bigcup_{i=1}^n F_{A_{\alpha_i}} = F_E$ and $F_G \subseteq F_E$, $F_G = \bigcup_{i=1}^n F_{A_{\alpha_i}}$. Then the collection $F_{A_{\alpha_i}\, i=1,2,\dots,n}$ of soft D_μ-sets is a finite subcollection of $\Re$ covering F_G. Hence F_G soft D_μ-compact.
Consider (ii) Since $(\bigcup_{i=1}^n F_{A_{\alpha_i}})\bigcup(F_E - F_G) = F_E$, then $F_G \subseteq \bigcup_{i=1}^n F_{A_{\alpha_i}}$ because if $\alpha \in F_G$ implies $\alpha \in F_E$ implies $\alpha \in (\bigcup_{i=1}^n F_{A_{\alpha_i}})\bigcup(F_E - F_G)$. Then $\alpha \in (\bigcup_{i=1}^n F_{A_{\alpha_i}})$ or $\alpha \in (F_E - F_G)$. So, $\alpha \in \bigcup_{i=1}^n F_{A_{\alpha_i}}$, since $\alpha \in F_G \alpha \notin (F_E - F_G)$. Hence $F_G \subseteq (\bigcup_{i=1}^n F_{A_{\alpha_i}})$. Now the collection $F_{A_{\alpha_i}\, i=1,2,\dots,n}$ of soft D_μ-sets is a finite subcollection of $\Re$ covering F_G. Hence F_G is soft D_μ-compact. □

Theorem 5. *Let (F_E,μ) be a strong SGTS. Then the following statements are equivalent.*

(i) *F_E is soft D_μ-compact.*

(ii) *For every collection $\Re$ of complement of soft D_μ-subsets of F_E, the intersection of all the elements of $\Re$ is empty then the collection $\Re$ contains a finite subcollection with empty intersection.*

Proof. (i)$\Longrightarrow$(ii): Suppose F_E is soft D_μ-compact space. Let $\mathfrak{C} = \{F_A - F_B : F_A,F_B \in$ soft μ-open, $F_A \in F_E\}$ be the collection of all soft D_μ-subsets of F_E and let $\Re = \{F_E - (F_A - F_B) : (F_A - F_B) \in \mathfrak{C}\}$ be the collection of all complements of soft D_μ-subsets of F_E. Suppose the intersection of all the elements of $\Re$ is empty. (i.e.) $\bigcap_i[F_E - (F_A - F_B)] = F_\varphi$. Then $F_E - \bigcap_i[F_E - (F_A - F_B)] = F_E - F_\varphi$. i.e. $[\bigcap_i[F_E - (F_A - F_B)]]^c = F_E$. Therefore, by De-Morgans Law, $\bigcup_i(F_A - F_B) = F_E$. Then the collection $\{(F_{A_i} - F_{B_i})\}_i$ of soft D_μ-subsets is a covering of F_E. Since (F_E,μ) is soft D_μ-compact, there is finite subcollection say $\{F_{A_1} - F_{B_1}, F_{A_2} - F_{B_2}, F_{A_n} - F_{B_n}\}$ of $\{(F_{A_i} - F_{B_i})\}_i$ covering F_E that is $\bigcup_{i=1}^n (F_{A_i} - F_{B_i}) = F_E$. Then $F_E - \bigcup_{i=1}^n (F_{A_i} - F_{B_i}) = F_\varphi$ which implies $\bigcap_{i=1}^n [(F_{A_i} - F_{B_i})]^c = F_\varphi$. Hence $\bigcap_{i=1}^n [F_E - (F_{A_i} - F_{B_i})] = F_\varphi$.

(ii)$\Longrightarrow$(i) Assume that for every collection $\Re = \{F_E - (F_A - F_B) : F_A,F_B$ are soft μ-open sets, $F_A \in F_E\}$ of complements of soft D_μ-subsets of F_E, the intersection of all the elements of $\Re$ is empty implies the collection $\Re$ contains a finite subcollection with empty intersection. Let $\wp = \{F_{A_i} - F_{B_i} : F_{A_i} - F_{B_i}$ where $F_{A_i} - F_{B_i}$ is soft D_μ-sets, $F_{A_i} \neq F_E$ for all $i\}$ be a soft D_μ-cover of F_E that is $\bigcup_i(F_{A_i} - F_{B_i}) = F_E = [\bigcup_i(F_{A_i} - F_{B_i})]^c = F_\varphi$. By De-Morgans Law,

$[\bigcap_i[(F_{A_i}-F_{B_i})]^c]^c = F_E$. By hypothesis, $[\bigcap_{i=1}^n[(F_{A_i}-F_{B_i})]^c]^c = F_E$. Again by De-Morgans Law, $\bigcup_{i=1}^n(F_{A_i}-F_{B_i}) = F_E$ that is the collection $\{F_{A_1}-F_{B_1}, F_{A_2}-F_{B_2}, F_{A_n}-F_{B_n}\}$ of soft D_μ subsets is a finite subcollection of $\wp$ covering F_E. Hence (F_E,μ) is soft D_μ-compact. □

Theorem 6. *Let F_B be a soft subset of a SGTS $(F_{A,}\mu)$. Then the following are equivalent:*

(i) *F_B is soft D_μ-compact with respect to μ.*

(ii) *F_B is soft $D_\mu/_{F_B}$ compact with respect to the subspace SGT $\mu/_{F_B}$ on F_B.*

Proof. (i)⇒(ii) Suppose F_B is soft D_μ-compact. Let $\Re = \{F_{G_\alpha}\}_{\alpha\in I}$ be a soft $D_\mu/_{F_B}$ covering of F_B for each α. Then there exist F_{B_α}, $F_{C_\alpha} \in D_\mu/_{F_B}$ such that $F_{G_\alpha} = F_{B_\alpha} - F_{C_\alpha}$. Since F_{B_α}, $F_{C_\alpha} \in D\mu/_{F_B}$, there exist soft μ-open sets F_{E_α}, F_{F_α} such that $F_{B_\alpha} = F_{E_\alpha} \bigcap F_B$ and $F_{C_\alpha} = F_{F_\alpha} \bigcap F_B$. Hence $F_{G_\alpha} = (F_{E_\alpha}\bigcap F_B) - (F_{F_\alpha}\bigcap F_B) = (F_{E_\alpha} - F_{F_\alpha})\bigcap F_B = F_{H_\alpha}\bigcap F_B$ for each α, where $F_{H_\alpha} = F_{E_\alpha} - F_{F_\alpha}$ is soft D_μ-set. Therefore, the collection $(F_{H_\alpha})_{\alpha\in I}$ of soft D_μ-sets is a D_μ-covering of F_B. By hypothesis, there is a finite subcollection of soft D_μ-sets $\{F_{H_{\alpha_1}}, F_{H_{\alpha_2}}, \dots, F_{H_{\alpha_n}}\}$ covering F_B. Then the collection $\{F_{H_{\alpha_1}}\bigcap F_B F_{H_{\alpha_2}}\bigcap F_B, \dots, F_{H_{\alpha_n}}\bigcap F_B\} = \{F_{G_{\alpha_1}}, F_{G_{\alpha_2}}, \dots, F_{G_{\alpha_n}}\}$ of soft $D_\mu/_{F_B}$-sets is a finite subcollection of $\Re$ covering F_B. Hence F_B is soft $D_\mu/_{F_B}$ compact with respect to the μ_{F_B}.

(ii)⇒(i) Suppose F_B is soft $D\mu/_{F_B}$ compact with respect to the μ_{F_B}. Let $\mathfrak{U} = (F_{H_\alpha})_{\alpha\in I}$ be a soft D_μ-covering of F_B where F_{H_α} is soft D_μ-set for all α. Since F_{H_α} is soft D_μ-set, there exist soft μ-open sets F_{E_α}, F_{F_α} such that $F_{H_\alpha} = F_{E_\alpha} - F_{F_\alpha}$. Set $F_{G_\alpha} = F_{H_\alpha}\bigcap F_B$. Then $F_{G_\alpha} = (F_{E_\alpha} - F_{F_\alpha})\bigcap F_B = (F_{E_\alpha}\bigcap F_B) - (F_{F_\alpha}\bigcap F_B)$ implies F_{G_α} is soft $D_\mu/_{F_B}$. But the $\{F_{G_\alpha}\}_{\alpha\in I}$ of soft $D_{\beta * g\alpha\mu/_{F_B}}$ is a covering of F_B with respect to $\mu/_{F_B}$. By hypothesis there is finite subcollection $\{F_{G_{\alpha_1}}, F_{G_{\alpha_2}}, \dots, F_{G_{\alpha_n}}\}$ of soft $D_\mu/_{F_B}$ sets covering F_B. That is $\{F_{H_{\alpha_1}}\bigcap F_B F_{H_{\alpha_2}}\bigcap F_B, \dots, F_{H_{\alpha_n}}\bigcap F_B\}$ is a finite subcollection of soft $D_{\beta * g\alpha\mu/_{F_B}}$-sets covering F_B. Then the collection $\{F_{H_{\alpha_1}}, F_{H_{\alpha_2}}, \dots, F_{H_{\alpha_n}}\}$ of soft D_μ-sets is a finite subcollection of $\mathfrak{U}$ covering F_B. Hence F_B is soft D_μ-compact. □

3. Conclusion

Hence, we introduce soft D_μ-sets and Soft D_μ-compact spaces in terms of soft D_μ-sets in Soft Generalized Topological Spaces. Also, some properties and characterizations are investigated.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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