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Breathing Modes in Rotating Bose-Condensed Gas: An Exact Diagonalization Study

Research Article

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Abstract. We present an exact diagonalization study of the breathing mode collective excitations for a rotating Bose-Einstein condensate of $N = 10$ spinless bosons interacting via repulsive finite-range Gaussian potential and harmonically confined in quasi-two-dimension. The yrast state and the low-lying excited states are variationally obtained in given subspaces of the quantized total angular momentum L employing the beyond lowest Landau level approximation in slowly rotating regime with $0 \leq L < 2N$. For a given L, the low-energy eigenspectra (bands) are obtained in weakly to moderately interacting regime. Further, for a given interaction, the split in low-lying eigenenergies with increasing L is the precursor to spontaneous symmetry breaking of the axisymmetry associated with the entry of the first vortex. With increase in repulsive interaction, the value of the first breathing mode increases for stable total angular momentum states $L = 0$ and N, but decreases for intermediate $0 < L < N$ metastable states. The position of the observed first breathing modes in the eigenspectrum remains unchanged as the interaction is varied over several orders of magnitude.

Keywords. Bose-Einstein condensate; Exact diagonalization; Beyond lowest Landau level (LLL) approximation; Breathing mode; Finite-range Gaussian interaction potential

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1. Introduction

Ever since the experimental realization of Bose-Einstein condensation (BEC) in ultra-cold alkali atomic vapours in a harmonic trap [1–3], the study of collective excitations in such systems has been an important subject of research in quantum many-body physics [4, 5].

The decisive experimental control over the density, the effective dimensionality and the atom-atom interaction strength [6], makes these systems an outstanding one to study subtle quantum many-body effects, in particular, collective excitations. The breathing mode (monopole oscillation or uniform radial expansion and contraction) is one of the most important collective excitations [7,8] used as a diagnostic tool for many-body effects [9]. In recent years, a number of theoretical studies have demonstrated that the quantum breathing mode is ideally suited to estimate the atom-atom coupling strength, its kinetic and interaction energies and other such observables [10], in a trapped atomic vapour system. This leads to a novel kind of spectroscopy of trapped systems. For few-body systems realized in lower dimension [11,12], the behaviour of breathing modes can be studied to a very high degree of precision [13,14]. An understanding of the physics of few-body systems may then be extrapolated to larger systems to gain an insight into the beyond mean-field physics of macroscopic ensembles. In this note, we follow this approach and present an exact diagonalization study of the many-body effects of interaction and rotation (with quantized L) on the dynamics of breathing modes.

2. Theoretical Formalism

We consider a system of N interacting spinless bosons each of mass M, trapped in a harmonic potential $V(\mathbf{r}) = \frac{1}{2}M\left(\omega_\perp^2 r_\perp^2 + \omega_z^2 z^2\right)$. The trap is subjected to an external rotation about the z-axis with angular velocity $\widetilde{\Omega} \equiv \widetilde{\Omega}\hat{e}_z$. The trapping potential $V(\mathbf{r})$ is assumed to be highly anisotropic with $\lambda_z \equiv \omega_z/\omega_\perp \gg 1$, so that the many-body dynamics along z-axis is frozen. The system is thus effectively quasi-two-dimensional (quasi-2D) with x-y rotational symmetry. Choosing $\hbar\omega_\perp$ and $a_\perp = \sqrt{\hbar/M\omega_\perp}$ as units of energy and length respectively, the many-body Hamiltonian in the co-rotating frame is given as

$$H = \sum_{j=1}^{N}\left[-\frac{1}{2}\boldsymbol{\nabla}_j^2 + \frac{1}{2}\mathbf{r}_j^2 - \Omega L\left(\mathbf{r}_j\right)\right] + \frac{1}{2}\sum_{i\neq j}^{N} U\left(|\mathbf{r}_i - \mathbf{r}_j|\right) \tag{2.1}$$

where $\Omega = \widetilde{\Omega}/\omega_\perp$ (≤ 1) is the dimensionless angular velocity and L (scaled by $\hbar$) is the z projection of the total angular momentum operator. The inter-particle interaction U is described by the Gaussian potential

$$U\left(|\mathbf{r}_i - \mathbf{r}_j|\right) = \frac{\mathrm{g}_2}{2\pi\sigma_\perp^2}\exp\left[-\frac{\left(r_{\perp i} - r_{\perp j}\right)^2}{2\sigma_\perp^2}\right]\delta\left(z_i - z_j\right) \tag{2.2}$$

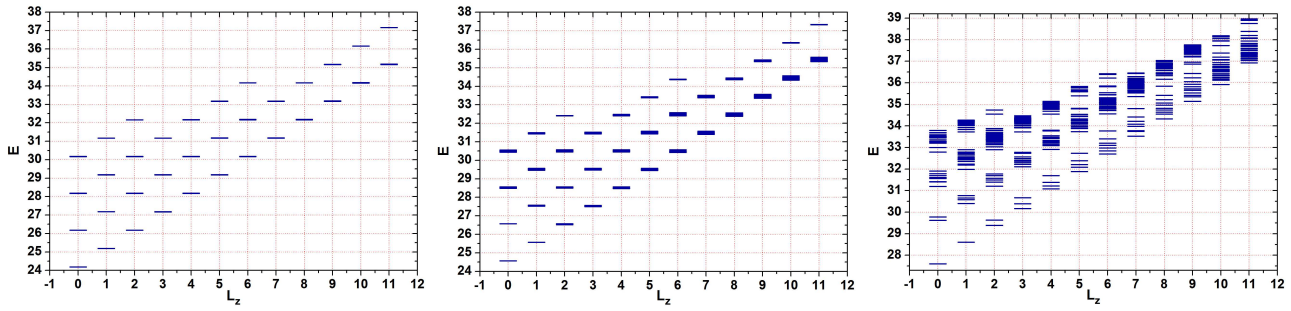
with width $\sigma_\perp$ (scaled by $a_\perp$) being the effective range of two-body interaction. The dimensionless parameter $\mathrm{g}_2 = 4\pi a_s/a_\perp$ measures the strength of the two-body interaction with a_s being the s-wave scattering length for particle-particle collision. We assume that the scattering length is positive ($a_s > 0$) so that the effective finite-range interaction is repulsive. The above finite-range Gaussian interaction potential is expandable within a finite number of single-particle basis and hence computationally feasible [15,16]. In the limit $\sigma_\perp \to 0$, the Gaussian potential in equation (2.2) reduces to the usual zero-range contact potential $\mathrm{g}_2\delta\left(\mathbf{r}_i - \mathbf{r}_j\right)$.

To obtain the eigenenergies and the corresponding eigenstates, we employ exact diagonalization of the Hamiltonian matrix in different subspaces of L using Davidson algorithm

[17] with inclusion of higher Landau levels in constructing the many-body basis states [18]. It is to be noted that for a many-body system under consideration here, the characteristic energy scale for the interaction is determined by the dimensionless parameter $Na_s/a_\perp$. Owing to the increasing dimensionality of the Hilbert space with N making the computation impractical, we vary a_s so as to achieve the value of $Na_s/a_\perp$ relevant to experimental situation [9].

3. Results and Discussions

We consider a system of $N = 10$ bosonic atoms of ^{87}Rb in a quasi-2D harmonic trap with confining frequency $\omega_\perp = 2\pi \times 220$ Hz and the z-asymmetry parameter $\lambda_z \equiv \omega_z/\omega_\perp = \sqrt{8}$. The condensate has extension $a_z = \sqrt{\hbar/M\omega_z}$ in the z-direction and its dynamics along this axis is taken to be completely frozen. Recent advancements in atomic physics has made it possible to tune the low-energy atom-atom scattering length in ultra-cold atomic vapours using Feshbach resonance [6]. Accordingly in the calculations presented here, the parameters of the two-body interaction potential in equation (2.2) has been chosen as $\sigma_\perp = 0.1$ and $a_s = 10a_0$, $100a_0$ and $1000a_0$ where, $a_0 = 0.05292\,nm$ is the Bohr radius. The corresponding dimensionless parameter $g_2 = 4\pi a_s/a_\perp$ defined above turns out to be 0.009151, 0.09151 and 0.9151, respectively.



(a) $a_s = 10a_0$ (b) $a_s = 100a_0$ (c) $a_s = 1000a_0$

Figure 1. For $N = 10$ condensed bosons in a quasi-2D harmonic trap, the low-lying energy eigenspectrum (in units of $\hbar\omega_\perp$) *versus* the total angular momentum L (scaled by $\hbar$) for three different values of repulsive interaction g_2 (parametrized by a_s) and $\sigma_\perp = 0.1$ in equation (2.2). In the present calculation, up to thirty low-lying energy eigenstates have been found.

Following [13], we examine here the breathing modes in a rotating system of $N = 10$ bosons within the quantized total angular momentum regime $0 \le L < 2N$. In Figure 1, we present the low-energy eigenspectra (bands) for different total angular momentum L states with three different values of interaction parameter g_2 (parametrized by the scattering length a_s). Eigenstates having the same total angular momenta constitute a L series (or bands). The ith eigenstate of the L series is denoted as L_i and the corresponding eigenenergy as $E(L_i)$. We observe that corresponding to each L, the quasi-degenerate low-lying eigenenergies split to form energy bands for weakly, Figures 1(a) and 1(b), to moderately interacting regime, Figure 1(c), and the respective energy gaps associated with the low-lying eigenstates are reduced. We also observe that for a given interaction strength, the low-lying eigenenergies further split with increase in L to fill the energy gaps, as seen in Figures 1(b) and 1(c). This split in low-lying

eigenenergies with increasing L leads to spontaneous symmetry breaking of the axisymmetry associated with the entrance of the first vortex [19] in the angular momentum regime $0 \le L \le N$.

Table 1. The eigenenergies of the L_i states for $N = 10$ bosons with interaction parameters $g_2 = 0.09151$ and $\sigma_\perp = 0.1$ in the total angular momentum regime $0 \le L \le 11$. The L_1 states are the yrast states (or ground modes) and the L_i states 0_2, 1_2, 2_3, 3_4, 4_5, 5_6, 6_8, 7_9, 8_{11}, 9_{13}, 10_{24}, 11_{27} are the observed first breathing modes.

i	$L=0$	$L=1$	$L=2$	$L=3$	$L=4$	$L=5$	$L=6$	$L=7$	$L=8$	$L=9$	$L=10$	$L=11$
1	24.5663	25.5663	26.5217	27.4976	28.4778	29.4524	30.4299	31.4062	32.3822	33.3583	34.3339	35.3340
2	26.5664	27.5217	26.5665	27.5216	28.4963	29.4774	30.4416	31.4309	32.4039	33.3830	34.3578	35.3459
3	26.5688	27.5447	28.4996	27.5670	28.5202	29.4952	30.4545	31.4361	32.4102	33.3864	34.3630	35.3563
4	28.4833	27.5668	28.5218	29.4781	28.5675	29.5184	30.4752	31.4545	32.4308	33.4009	34.3746	35.3582
5	28.5218	27.5687	28.5231	29.4977	30.4540	29.5679	30.4940	31.4716	32.4321	33.4109	34.3819	35.3648
6	28.5224	29.4607	28.5392	29.4990	30.4759	31.4315	30.5164	31.4926	32.4522	33.4261	34.3830	35.3721
7	28.5438	29.4832	28.5671	29.5051	30.4779	31.4440	30.5681	31.5143	32.4672	33.4313	34.3846	35.3769
8	28.5448	29.5007	28.5680	29.5204	30.4807	31.4523	32.4074	31.5678	32.4911	33.4482	34.3948	35.3819
9	28.5662	29.5018	30.4454	29.5228	30.4953	31.4557	32.4285	33.3834	32.5119	33.4627	34.4009	35.3826
10	28.5676	29.5221	30.4598	29.5349	30.4982	31.4619	32.4324	33.4047	32.5671	33.4894	34.4056	35.3848
11	28.5687	29.5228	30.4787	29.5661	30.5002	31.4734	32.4357	33.4060	34.3591	33.5091	34.4071	35.3921
12	30.4398	29.5233	30.4798	29.5683	30.5184	31.4759	32.4399	33.4106	34.3808	33.5658	34.4144	35.3986
13	30.4617	29.5365	30.4819	31.4223	30.5205	31.4779	32.4454	33.4138	34.3848	35.3347	34.4225	35.4007
14	30.4627	29.5416	30.4990	31.4448	30.5312	31.4903	32.4529	33.4257	34.3876	35.3601	34.4279	35.4064
15	30.4833	29.5444	30.4998	31.4557	30.5647	31.4960	32.4549	33.4306	34.3915	35.3635	34.4308	35.4127
16	30.4836	29.5659	30.5006	31.4578	30.5686	31.4977	32.4579	33.4322	34.4033	35.3718	34.4344	35.4203
17	30.4944	29.5676	30.5051	31.4591	32.4027	31.5161	32.4680	33.4331	34.4051	35.3814	34.4470	35.4268
18	30.4970	29.5684	30.5153	31.4771	32.4137	31.5171	32.4724	33.4371	34.4097	35.3831	34.4530	35.4304
19	30.5037	31.4247	30.5209	31.4779	32.4251	31.5278	32.4753	33.4407	34.4113	35.3867	34.4723	35.4317
20	30.5040	31.4407	30.5224	31.4791	32.4330	31.5629	32.4849	33.4480	34.4127	35.3883	34.4858	35.4450
21	30.5212	31.4456	30.5231	31.4803	32.4353	31.5687	32.4932	33.4530	34.4159	35.3914	34.4936	35.4517
22	30.5219	31.4601	30.5238	31.4815	32.4429	33.3831	32.4956	33.4552	34.4275	35.3954	34.5161	35.4699
23	30.5228	31.4614	30.5339	31.4916	32.4468	33.4038	32.5127	33.4624	34.4296	35.3993	34.5672	35.4854
24	30.5229	31.4625	30.5383	31.4957	32.4534	33.4078	32.5141	33.4677	34.4305	35.4033	36.3342	35.4926
25	30.5242	31.4676	30.5442	31.4980	32.4566	33.4090	32.5245	33.4709	34.4330	35.4069	36.3346	35.5146
26	30.5354	31.4769	30.5642	31.4986	32.4571	33.4129	32.5608	33.4794	34.4367	35.4092	36.3466	35.5669
27	30.5373	31.4798	30.5662	31.5006	32.4585	33.4253	32.5683	33.4899	34.4407	35.4098	36.3558	37.3287
28	30.5416	31.4822	30.5689	31.5040	32.4619	33.4262	34.3632	33.4937	34.4490	35.4123	36.3572	37.3336
29	30.5426	31.4834	32.4033	31.5088	32.4665	33.4297	34.3816	33.5085	34.4519	35.4203	36.3586	37.3344
30	30.5589	31.4912	32.4232	31.5188	32.4749	33.4340	34.3843	33.5114	34.4572	35.4246	36.3599	37.3352

To analyze the breathing modes, we present in Table 1 the low-lying eigenenergies $E(L_i)$ corresponding to angular momentum states $0 \le L \le 11$ with the interaction parameter value $g_2 = 0.09151$. The lowest energy state L_1 corresponding to an L is referred to as the yrast state (we would occasionally call it the ground mode). We observe that for the non-rotating $L = 0$ states, $E(0_2) - E(0_1) = 2.0001$ (in units of $\hbar\omega_\perp$). The states 0_1 and 0_2 are, thus, taken to be the ground mode (the yrast state) and the first breathing mode, respectively, for $L = 0$. It was pointed out by Pitaevskii and Rosch [7] that a purely 2D bosonic system with zero-range interaction exhibits breathing modes arising from $SO(2,1)$ symmetry. The energy difference between adjacent breathing mode levels is $2\hbar\omega_\perp$. Although in the present work, the finite-range Gaussian interaction potential has been used to replace the zero-range (δ-function) interaction, the feature of $2\hbar\omega_\perp$ spacing is seen to persist in the energy eigenspectrum. Further, the $2\hbar\omega_\perp$

spacing also appears for the rotating $L > 0$ states. For example, we find $E(1_2) - E(1_1) = 1.9554$ for $L = 1$, $E(5_6) - E(5_1) = 1.9791$ for $L = 5$ and $E(10_{24}) - E(10_1) = 2.0003$ for $L = 10$. Thus, the breathing modes existing in 2D systems with zero-range (δ-function) interaction potential may also exist in our quasi-2D system with Gaussian interaction potential.

For the non-rotating $L = 0$ case (column 2 in the Table 1), the position of the first breathing mode 0_2 is always immediately above the yrast state 0_1. However as L increases, the positions (labelled by the index i in column 1) of the first breathing modes are shifted to higher eigenstates, forming a stair-like pattern of eigenenergies corresponding to the breathing modes. We further observe from the Table 1 that the shifts in the positions of the first breathing modes are small for states with $L < N$; however, for the first vortex state with $L = N = 10$, we notice an appreciable shift in the position of the first breathing mode. It might be due to rearrangement of bosons on the nucleation of first central vortex. We have found that as the repulsive interaction is increased over three orders of magnitude, the positions of the first breathing modes in the energy eigenspectra for a given L, remain unchanged.

We also observe the second and third breathing modes with approximately $4\hbar\omega_\perp$ and $6\hbar\omega_\perp$ spacing. For example, the states 0_4, 1_6, 2_9, 3_{13}, 4_{17}, 5_{22} and 6_{28} are the second breathing modes. Similarly, the states 0_{12}, 1_{19} and 2_{29} are the third breathing modes. Moreover as is the case for the first breathing mode, for a given L, the positions of these higher breathing modes are also found to be independent of the inter-particle interaction strength.

Table 2. For $N = 10$ condensed bosons, the values of the breathing modes E_{BM} (in units of $\hbar\omega_\perp$) i.e. the difference between the eigenenergies corresponding to the breathing state (L_i) and the yrast state (L_1), for three different values of the dimensionless interaction parameter g_2 (parametrized by a_s) with Gaussian width $\sigma_\perp = 0.1$ in equation (2.2). With increase in repulsive interaction, the value of the first breathing mode increases for stable total angular momentum states $L = 0$ and N, but decreases for intermediate $0 < L < N$ metastable states.

L	E_{BM}	$E_{BM}(10a_0)$	$E_{BM}(100a_0)$	$E_{BM}(1000a_0)$
0	$E(0_2) - E(0_1)$	1.9999	2.0001	2.0111
1	$E(1_2) - E(1_1)$	1.9952	1.9554	1.7837
2	$E(2_3) - E(2_1)$	1.9978	1.9779	1.8062
3	$E(3_4) - E(3_1)$	1.9978	1.9805	1.9357
4	$E(4_5) - E(4_1)$	1.9976	1.9762	1.8287
5	$E(5_6) - E(5_1)$	1.9978	1.9791	1.8542
6	$E(6_8) - E(6_1)$	1.9976	1.9775	1.8620
7	$E(7_9) - E(7_1)$	1.9976	1.9772	1.8403
8	$E(8_{11}) - E(8_1)$	1.9976	1.9768	1.8331
9	$E(9_{13}) - E(9_1)$	1.9976	1.9764	1.7263
10	$E(10_{24}) - E(10_1)$	2.0000	2.0003	2.0189
11	$E(11_{27}) - E(11_1)$	1.9994	1.9947	1.9792

The results of the first breathing modes in subspaces of quantized L are presented in Table 2 for three different values of the interaction parameter g_2. We observe that for a given L, the values of the first breathing mode calculated with weak interactions corresponding to the choice $g_2 = 0.009151$ and 0.09151 are very close (column 3 and 4) but significantly different from the ones calculated for moderate interaction corresponding to the choice $g_2 = 0.9151$ (column 5). For the non-rotating $L = 0$ and the single-vortex $L = N = 10$ states, the values of the breathing

modes *increase* albeit very weakly with increase in repulsive interaction. On the contrary for the intermediate $0 < L < N$ metastable states, the values of the breathing modes are appreciably different from the non-rotating and single-vortex states and *decreases* significantly with increase in repulsive interaction.

4. Conclusions

In conclusion, we have examined the breathing mode collective excitations of rotating Bose-Einstein condensate with finite-range Gaussian interaction in a quasi-2D harmonic trap. By exact diagonalization of the many-body Hamiltonian matrix in beyond lowest-Landau-level approximation, the eigenenergies of the ground and the low-lying excited states are obtained with total angular momenta $0 \leq L < 2N$, corresponding to slowly rotating regime. We have presented the low-energy eigenspectra (bands) of the rotating system for three representative values of different interacting regimes in subspaces of quantized L. For a given L, the degeneracy of eigenenergies is lifted (forming energy bands) for weakly to moderately interacting regime and correspondingly the energy gaps associated with the low-lying eigenstates are reduced. We find that the repulsive interaction influences the values of the first breathing modes in two different ways. For stable $L = 0, N$ states the value of the first breathing mode increases whereas for intermediate $0 < L < N$ metastable states it decreases with increase in repulsive interaction. For metastable states with angular momentum $L < N$, we observe a small shift in the position of the first breathing mode in the eigenspectrum whereas for the vortex state ($L = N$), the shift becomes significant. The position of the observed first breathing mode in the eigenspectrum remains unchanged as the interaction is varied over several orders of magnitude.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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