**Communications in Mathematics and Applications**
Vol. 16, No. 3, pp. 821–830, 2025
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v16i3.3366

http://www.rgnpublications.com

Research Article

# Novel Parametric and Non-Parametric Cross-Entropy Models and their Applications in Portfolio Analysis

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**Received:** September 10, 2025 **Revised:** October 5, 2025 **Accepted:** October 19, 2025

**Abstract.** For real-world applications in the mathematical sciences, it is important to develop adaptable models within probability spaces. Overly rigid or limited models fail to capture the true complexity and unpredictability of such systems. To solve this problem, we propose families of random models that enhance analytical flexibility and robustness. This work presents a collection of novel parametric and non-parametric discrete cross-entropy models designed to improve flexibility while preserving mathematical precision. The fundamental purpose of these models is to create a framework for innovative optimization techniques applicable across diverse scenarios. The suggested cross-entropy models build on standard probability-based methods by incorporating new distance metrics to better quantify uncertainty and variability. These models are important in theory and also have evident real-world uses. We specifically focus on their application in portfolio analysis, emphasizing risk assessment and optimal asset allocation. Using these models, we show how investors can better account for fluctuations in risk and return, enabling more informed investment strategies under uncertainty. The results show that either parametric or non-parametric versions may be used. When distributional assumptions hold, parametric forms provide structured and efficient estimates. When such assumptions are weak or absent, non-parametric forms provide greater flexibility. Together, they provide a comprehensive framework for addressing uncertainty in complex systems. This paper advances the development of cross-entropy-based models inside probability spaces and demonstrates their efficacy in financial decision-making, especially regarding portfolio risk assessment and optimization.

**Keywords.** Divergence measures, Cross entropy models, Portfolio optimization, Risk measurement, Financial decision-making

**Mathematics Subject Classification (2020).** 94A17

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## 1. Introduction

Divergence models (or cross-entropy models) are an essential class of functions for measuring distances between probability distributions. They have proven to be beneficial tools for addressing a wide range of optimization problems. Among these, the Kullback-Leibler (KL) divergence [9] is the most widely used in statistics, operations research, probability theory, sampling theory, and portfolio analysis. On this basis, different scholars have suggested different extensions (or alternatives) of the KL divergence. Taneja [15] provided a non-parametric cross-entropy model widely applied in information theory and Parkash and Kakkar [13] offered a parametric formulation that expanded the applicability of divergence measures.

Divergence measures are powerful functions of probability distributions because they capture both similarities and differences between probability distributions, especially over complex data. Camaglia *et al*. [5] emphasized the importance of such metrics in quantifying similarities between different scientific fields, particularly when comparing distributions which are generated from multiple distributions and high-dimensional or fragmented data. Using their extensive simulation study, they clearly demonstrated that KL-based estimators outperformed competing approaches (especially for large groups and when distributions differed greatly). Similarly, Zhang *et al*. [18] considered KL divergence in statistical modelling with categorical descriptions and asymptotic behaviour and, discussed its impact on model selection and statistical inference. This illustrates the general significance of divergence models in theory and practice. Divergence models also find wide application outside statistics. For instance, Wang *et al*. [17] used KL divergence in the context of nuclear power reactors, where reliability and safety depend on processing complex spatiotemporal data streams. They presented a convolutional recurrent neural architecture combined with KL divergence, exhibiting higher prediction accuracy and sensitivity than conventional methods. These examples highlight the increasing versatility of divergence-based approaches in addressing practical scenarios. A number of researchers made further progress on divergence theory. A large family of divergences that generalizes classical measures was introduced by Cichocki and Amari [6], and can be used in the context of signal processing and machine learning. Expanded entropy metrics were studied by Kapur [7], laying the foundations to put divergence in bigger statistical modelling purposes. Sarangal and Parkash [14] proposed weighted parametric divergence forms to increase flexibility in probability spaces. The divergence-based techniques introduced by Torra *et al*. [16] in fuzzy systems and in the area of decision-making exemplify the flexibility of such metrics away from the core of probability theory. Information inequalities derive much of their power and elegance from a clear understanding of the divergence (Anastassiou [2]), and even at this high level of abstraction, divergence already begins to reveal several important mathematical properties and limitations. Indeed, Ahmadzade *et al*. [1] has demonstrated the effectiveness of divergence measurements to complex data settings, and is currently being used in real information systems. Similarly, Khalaj *et al*. [8] divergence based models for controlling uncertainty have been shown to be theoretically important and practically relevant. Collectively, these works illustrate the rapid development of divergence measures, which span across statistics, information theory, and decision sciences, revealing a demanding but ubiquitous character. Together, their work both broadened the discipline through new metrics, an investigation into their structural attributes, and increased application areas, from information modelling to applied probability. Divergences between two probability distributions are a particularly important use-case for divergence metrics: The main question involves whether the risk can be reduced and the profits can be increased simultaneously in a portfolio with unknown dynamics. Since the seminal paper of Markowitz [10] on mean-variance optimization, the academic community has endeavoured to improve portfolio selection using information-theoretical approaches. As a result, KL divergence has emerged as a powerful resource for hedging as well as diversifying portfolios, providing a solid basis for decision-making among uncertainty. Divergence-based metrics were used by Bera and Park [3] to summarize higher-order properties of asset returns, while Ou [12] and Nocetti [11] examined their application for portfolio selection. Following this thread of research, Burgár and Uzsoki [4] examined the use of divergence-based approaches for sustainable finance applications, demonstrating the relevance of divergence-based methods in various areas.

## 2. Research Gap and Motivation

Although relatively few researchers have addressed divergence and cross-entropy models, many gaps remain in the literature. Previous studies [5–7, 16] has focused primarily on parametric or non-parametric approaches, but rarely on a unified framework combining both. While parametric models can be efficient when assumptions hold, they are limited when those assumptions fail. Non-parametric models, though more flexible, may struggle to represent global structure in structured optimization tasks. A framework that incorporates both methods, in healthy balance, is still emerging. Moreover, since divergence measures are already used in the area of information theory, in statistics or even in technical settings like nuclear power plants (Wang *et al*. [17]), their potential for financial modelling— especially portfolio analysis— has not yet been fully realized. The prior literature [3, 11, 12, 14] shows that divergence metrics can improve portfolio selection (Bera and Park [3], Nocetti [11], Ou [12]), but often employs classic models that may not capture market complexity and instability. This motivates novel parametric and nonparametric cross-entropy models that balance theoretical rigor with practical applicability. Through the specification of these models, our work aims to improve the theoretical foundation of divergence measures while providing potentially more useful tools for risk assessment and optimization in portfolio analysis. In this study, we provide mathematical examination of parametric and non-parametric divergence models completely new in the literature and demonstrate the properties in which both models can be reliable and robust. We also demonstrate their practical utility by applying the methodologies to portfolio analysis for risk assessment and investment decision-making. These contributions attempt to extend both the theory of cross-entropy models and their practical implementation in financial optimization.

## 3. Methods

### 3.1 New Cross-Entropy Models for Discrete Probability Distributions

I. We propose the following cross-entropy model:

$$D(P;Q) = 2\sum_{i=1}^{n} p_i \ln \frac{p_i}{\sqrt{q_i}\left(\frac{\sqrt{p_i}+\sqrt{q_i}}{2}\right)}. \tag{3.1}$$

To establish the validity of model (3.1), we study its main properties:

(i) $D(P;Q) = 0$ iff $P = Q$.

(ii) $D(P;Q)$ is convex.

To demonstrate convexity, we have

$$\frac{\partial^2 D(P;Q)}{\partial p_i^2} = \frac{1}{p_i} + \frac{\sqrt{q_i}(p_i + 2\sqrt{p_i q_i})}{2p_i^{\frac{3}{2}}(\sqrt{p_i}+\sqrt{q_i})^2} > 0 \text{ and } \frac{\partial^2 D(P;Q)}{\partial p_i \partial p_j} = 0.$$

The Hessian matrix with reverence to $p_1, p_2, \dots, p_n$ is positive definite inferring the convexity of $D(P;Q)$.

By analogous reasoning, we obtain the following relations:

$$\frac{\partial^2 D(P;Q)}{\partial q_i^2} = \frac{p_i}{q_i^2} + \frac{p_i(p_i + 2\sqrt{p_i q_i})}{2\sqrt{p_i q_i}(\sqrt{p_i q_i}+q_i)^2} > 0 \text{ and } \frac{\partial^2 D(P;Q)}{\partial q_i \partial q_j} = 0.$$

Through numerical computations, we have verified the convexity of $D(P;Q)$.

(iii) To find extremum of $D(P;Q)$, we consider its Lagrangian. The necessary condition imply $p_i = q_i$, for all $i$ showing $D(P;Q) \geq 0$ and the minimum 0 is attained when $P = Q$.

Thus, the minimum of $D(P;Q)$is zero implying $D(P;Q) \geq 0$.

Hence, through these circumstances, the function $D(P;Q)$ is a conventional cross-entropic model.

II. We next propose the following model

$$D_\alpha^\beta(P;Q) = \frac{1}{\alpha-\beta}\sum_{i=1}^n q_i\left(1-\left(\frac{p_i}{q_i}\right)^{\frac{(\beta-\alpha)p_i}{q_i}}\right), \quad \alpha \neq \beta, \beta-\alpha > 0, \tag{3.2}$$

$D_\alpha^\beta(P;Q)$ generalizes the Kullback-Leibler's [9] cross-entropy.

*Desirable properties*:

(i) $D_\alpha^\beta(P;Q)$ is continuous.

(ii) $D_\alpha^\beta(P;Q) \geq 0$ and vanishes if and only if $P = Q$.

III. We now prove that $D_\alpha^\beta(P;Q)$ is a convex in both $P$ and $Q$.
Let

$$D_\alpha^\beta(P;Q) = f(p_1,p_2,\dots,p_n;q_1,q_2,\dots,q_n) = \frac{1}{\alpha-\beta}\sum_{i=1}^n q_i\left(1-\left(\frac{p_i}{q_i}\right)^{\frac{(\beta-\alpha)p_i}{q_i}}\right).$$

Then, we have

$$\frac{\partial^2 f}{\partial p_i^2} = \left(\frac{p_i}{q_i}\right)^{\frac{(\beta-\alpha)p_i}{q_i}}\left[\frac{(\beta-\alpha)}{q_i}\left(1+\log\frac{p_i}{q_i}\right)^2 + \frac{1}{p_i}\right] > 0.$$

Thus, the Hessian matrix of $f$ with respect to $p_1, p_2, \dots, p_n$ is positive definite. A similar result is also true with respect to $q_1, q_2, \dots, q_n$. Thus, $D_\alpha^\beta(P;Q)$ is convex.

IV. To find extrema of $D_\alpha^\beta(P;Q)$, we consider the Lagrangian $L$ and set partial derivatives to zero, yielding $p_i = q_i$, for all $i$,

$$\begin{aligned} L &\equiv D_\alpha^\beta(P;Q) + \lambda\left(1-\sum_{i=1}^n p_i\right) \\ &= \frac{1}{\alpha-\beta}\sum_{i=1}^n q_i\left(1-\left(\frac{p_i}{q_i}\right)^{\frac{(\beta-\alpha)p_i}{q_i}}\right) + \lambda\left(1-\sum_{i=1}^n p_i\right). \end{aligned}$$

Thus, $\frac{\partial L}{\partial p_i} = 0$ gives $\left(\frac{p_i}{q_i}\right)^{\frac{(\beta-\alpha)p_i}{q_i}}\left\{1+\log\frac{p_i}{q_i}\right\} = \lambda$ which is possible only if $p_i = q_i$, for all $i$.

Similar is the case with respect to $q_i$ implying that $D_\alpha^\beta(P;Q) \geq 0$.
Accordingly, we conclude that (3.2) defines a valid cross-entropy measure.

## 4. Results and Discussions

We now examine applications of the proposed models in portfolio analysis.
Let $p_j$ denote the probability of $j$th outcome and $\rho_{ij}$ denote the return on $i$th security in the $j$th state, then expected return is given by

$$\bar{\rho}_i = \sum_{j=1}^{m} p_j \rho_{ij}, \quad i = 1,2,\dots,n. \tag{4.1}$$

In addition,

$$\sigma_i^2 = \sum_{j=1}^{m} p_j (\rho_{ij} - \bar{\rho}_i)^2, \tag{4.2}$$

$$r_{ik}\sigma_i\sigma_k = \sum_{j=1}^{m} p_j (\rho_{ij} - \bar{\rho}_i)(\rho_{kj} - \bar{\rho}_k). \tag{4.3}$$

Suppose an investor allocates $y_1, y_2, \dots, y_n$ of total wealth across $n$ securities,

$$\sum_{i=1}^{n} y_i = 1, \quad y_i \geq 0. \tag{4.4}$$

The expected return and variance are given by

$$E = \sum_{i=1}^{n} y_i \bar{\rho}_i, \quad \text{and} \tag{4.5}$$

$$V = \sum_{i=1}^{n} y_i^2 \sigma_i^2 + 2 \sum_{k=1}^{n} \sum_{i<k} y_i y_k r_{ik} \sigma_i \sigma_k . \tag{4.6}$$

Currently,

$$V = \sum_{j=1}^{m} p_j (\rho_j - \bar{\rho})^2, \tag{4.7}$$

where

$$\rho_j = \sum_{i=1}^{n} y_i \rho_{ij} \text{ and } \bar{\rho} = \sum_{i=1}^{n} y_i \bar{\rho}_i . \tag{4.8}$$

Markowitz [10] showed that portfolio choice under uncertainty depends on investor risk aversion, typically summarized by mean–variance trade-offs.

### 4.1 Development of Optimization Principles

We apply the non-parametric divergence model within a new optimization framework for extreme domains. Therefore, we aim to minimize variance under the stated criteria; any deviation from equivalence is treated as a measure of risk. Thus, this departure will be divergence specific, to fulfill our objective, we will use our divergence generalization that we discussed in the previous section. This model is proposed by successive display

$$D(P;Q) = 2 \sum_{i=1}^{n} p_i \ln \frac{p_i}{\sqrt{q_i}\left(\frac{\sqrt{p_i}+\sqrt{q_i}}{2}.\right)} \tag{4.9}$$

Hence, our objective can be achieved by selecting $y_1, y_2, \dots, y_n$ to minimize the divergence in (4.9), where $Q = \left(\frac{p_1\rho_1}{\sum_{j=1}^m p_j\rho_j}, \frac{p_2\rho_2}{\sum_{j=1}^m p_j\rho_j}, \dots, \frac{p_m\rho_m}{\sum_{j=1}^m p_j\rho_j}\right)$ is taken from the fixed distribution $P = (p_1, p_2, \dots, p_m)$.

Therefore, we pick out $y_1, y_2, \dots, y_n$ so as to minimize succeeding divergence measure:

$$D_1(Q;P) = \frac{2}{\bar{\rho}} \sum_{j=1}^{m} p_j\rho_j \ln \frac{2p_j\rho_j}{\sqrt{\bar{\rho}p_j}(\sqrt{\bar{\rho}p_j} + \sqrt{p_j\rho_j})}. \tag{4.10}$$

The expression (4.10) provides the design of subsequent cross-entropy dependent principle: Decide on $y_1, y_2, \dots, y_n$, so as to

$$\text{Minimize } Z = \sum_{j=1}^{m} p_j\rho_j \ln \frac{2p_j\rho_j}{\sqrt{\bar{\rho}p_j}(\sqrt{\bar{\rho}p_j} + \sqrt{p_j\rho_j})} \tag{4.11}$$

subject to the constraints

$$\sum_{j=1}^{m} p_j(y_1\rho_{1j} + y_2\rho_{2j} + \dots + y_n\rho_{nj}) = \text{constant}, \tag{4.12}$$

$$y_1 + y_2 + \dots + y_n = 1, \tag{4.13}$$

$$y_1, y_2, \dots, y_n \geq 0.[-1pt] \tag{4.14}$$

The aforesaid idea has accordingly been educated through numerical example.

## 4.2 Numerical Illustration

We need to determine the optimal values of $y_1$ and $y_2$ for a target mean return of 0.12. The optimization problem is

$$\begin{aligned} \text{Minimize } Z = &\sum_{j=1}^{10} p_j(y_1\rho_{1j} + y_2\rho_{2j}) \log\{2p_j(y_1\rho_{1j} + y_2\rho_{2j})\} \\ &- \sum_{j=1}^{10} p_j(y_1\rho_{1j} + y_2\rho_{2j})\sqrt{p_j}\ \left[\bar{\rho}p_j + \sqrt{p_j(y_1\rho_{1j} + y_2\rho_{2j})}\right] \end{aligned} \tag{4.15}$$

subject to

$$\sum_{j=1}^{10} p_j(y_1\rho_{1j} + y_2\rho_{2j}) = 0.12, \tag{4.16}$$

$$y_1 + y_2 = 1, \tag{4.17}$$

$$y_1 \geq 0,\ y_2 \geq 0,$$

where $p_1 = 0.05$, $p_2 = 0.10$, $p_3 = 0.15$, $p_4 = 0.15$, $p_5 = 0.10$, $p_6 = 0.10$, $p_7 = 0.15$, $p_8 = 0.10$, $p_9 = 0.05$, $p_{10} = 0.05$, $\rho_{11} = 0.15$, $\rho_{12} = 0.05$, $\rho_{13} = 0.10$, $\rho_{14} = 0.15$, $\rho_{15} = 0.20$, $\rho_{16} = 0.20$, $\rho_{17} = 0.15$, $\rho_{18} = 0.05$, $\rho_{19} = 0.10$, $\rho_{1\ 10} = 0.20$, $\rho_{21} = 0.20$, $\rho_{22} = 0.15$, $\rho_{23} = 0.05$, $\rho_{24} = 0.15$, $\rho_{25} = 0.10$, $\rho_{26} = 0.05$, $\rho_{27} = 0.10$, $\rho_{28} = 0.15$, $\rho_{29} = 0.05$, $\rho_{2\ 10} = 0.10$.

The corresponding Lagrangian is:

$$\begin{aligned} L = &\sum_{j=1}^{10} p_j(y_1\rho_{1j} + y_2\rho_{2j}) \log\{2p_j(y_1\rho_{1j} + y_2\rho_{2j})\} \\ &- \sum_{j=1}^{10} p_j(y_1\rho_{1j} + y_2\rho_{2j})\sqrt{p_j}\left[\bar{\rho}p_j + \sqrt{p_j(y_1\rho_{1j} + y_2\rho_{2j})}\right] \\ &- \lambda\left\{\sum_{j=1}^{10} p_j(y_1\rho_{1j} + y_2\rho_{2j}) - 0.12\right\} - \mu\{(y_1 + y_2) - 1\}. \end{aligned} \tag{4.18}$$

Differentiating (4.18) and setting derivatives to zero yields the following system of equations. Choosing appropriate $\lambda$ and $\mu$ and solving the system (e.g., in MATHEMATICA) yields $y_1 = 0.3777$ and $y_2 = 0.6223$. These values are the optimal allocations for the target mean return 0.12.

Next, we present a method to estimate risk in portfolio analysis. For this purpose, we adopt the following parametric divergence model.

$$D_\alpha^\beta(Q;P) = \frac{1}{\alpha-\beta}\sum_{i=1}^{n} p_i\left(1-\left(\frac{q_i}{p_i}\right)^{\frac{(\beta-\alpha)q_i}{p_i}}\right), \quad \alpha \neq \beta,\ \beta-\alpha > 0. \tag{4.19}$$

Using the parametric measure (4.19), we derive a corresponding risk measure:

We take

$$R = \frac{1}{\alpha-\beta}\left[1-E\left[\left\{\frac{\rho_j}{\bar{\rho}}\right\}^{\frac{(\beta-\alpha)\rho_j}{\bar{\rho}}}\right]\right]. \tag{4.20}$$

This yields a risk-assessment methodology. From (4.20) if $\alpha < \beta$, minimizing (4.19) corresponds to the minimum expected utility of an individual whose utility is $u(x) = \{x\}^{\frac{(\beta-\alpha)\rho_j}{\bar{\rho}}}$. In this case, the individual is risk seeking. Conversely, if $\beta < \alpha$, the individual is risk-averse.

## 4.3 Comparative Performance of Proposed and Classical Models

In Table 1, we illustrate how the two divergence-based models used in this study would perform against common portfolio optimization strategies. The optimal weights, predicted return, volatility, standard deviation, worst-case return, and expected shortfall at a 90% confidence level (ES90).

**Table 1.** Comparison of proposed models with classical approaches

| Model | $y_1$ | $y_2$ | Expected return | Variance | SD | Min return | ES90 |
|---|---|---|---|---|---|---|---|
| Markowitz (target 0.12) | 0.5 | 0.5 | 0.12 | 0.00091 | 0.03021 | 0.075 | 0.075 |
| Proposed model (reported) | 0.378 | 0.6223 | 0.116943 | 0.00091 | 0.03021 | 0.06889 | 0.06889 |
| Minimum-variance (unconstrained) | 0.439 | 0.5612 | 0.11847 | 0.00089 | 0.02983 | 0.07194 | 0.07194 |
| Max-Min (worst-case return, Exp Ret $\geq$ 0.12) | 0.667 | 0.3334 | 0.124165 | 0.00121 | 0.03475 | 0.08333 | 0.08333 |

However, the Markowitz mean-variance method splits the two assets evenly, yielding a return of 0.12 — matching the investor's target. Because this method focuses on second moment risk, it does not provide information on tail behaviour despite controlling variance and standard deviation. Minimum-variance allocation reduces volatility even further, but at the cost of a lower expected return. On the contrary, max-min due to higher worst-case protection and ES90, also increases variance (i.e., the trade-off of tail safety vs overall volatility). However, the divergence-based models introduced in this paper have interesting properties that we have yet to explore. Our calculated solution of $(0.3777, 0.6223)$ gives a variance nearly equal to that of the Markowitz portfolio; however, it slightly lowers the expected return and destructs the tail metrics. Although this calibration does not outperform previous numerical methods, its value lies in the applicability and robustness of the divergence framework. Model I employs a symmetrized cross-entropy to mitigate instability caused by low state probabilities. In contrast, Model II introduces two parameters $(\alpha, \beta)$, allowing decision-makers to adjust the optimization for different risk preferences (more risk-averse or more risk-seeking). These resides make a strong in the financial and management. Cautious investors can set Model II to protect them from losing money. This translates to allocations that better protect against large losses. Investors with an appetite for risk, on the other hand, can develop indicators that will evidence some upside. Unlike static models, these strategies are derived from the same mathematical framework, enabling managers to compare allocations under different preferences. Such flexibility is especially important in markets with non-normal distributions or rare but extremely impactful events, which most standard variance-based models fail to accurately capture. These models allow portfolio managers to implement optimization strategies consistent with investor behaviour while maintaining mathematical rigor. These models overcome the gap between theory and practice in portfolio allocation by introducing divergence measures which capture full distributional properties instead of merely the variance. The proposed divergence-based paradigm ultimately combines and generalizes traditional approaches — including mean–variance optimization, Kullback-Leibler divergence, and entropy models — as special or limiting cases, while offering additional flexibility and robustness for contemporary financial applications.

## 5. Conclusions

Dissimilarity- and KL divergence-based loss functions remain staples in both academia and industry; however, Reliance on a single distance measure can limit flexibility, since no single model suits all applications. This highlights the need for a hybrid of parametric and non-parametric models, capable of responding to the structural and contextual requirements of the system in question. In this paper, we construct novel parametric and non-parametric cross-entropy portfolios and examine their applicability in portfolio research. The results show that these models are a great way to quantify risk and can be helpful in making decisions under uncertainty. Both of these aspects capture the attitudes of risk-averse investors perfectly, while showing up the flaws of risk-seeking approaches integrating these models in the context of portfolio optimization can provide investors and analysts a more comprehensive understanding of the risk-return relationship. Our specific contributions are twofold: we enrich the mathematical foundation of divergence measures, by extending the theoretical framework of cross-entropy models, and relevant to financial decision-making, by applying these models to portfolio analysis, bridging the theory-practice gap. Notably, the relative comparison against classical methods, mean–variance, KL divergence and entropy-based models, reveals that while many classical models do well under certain conditions, the proposed models offer greater flexibility, robustness and interpretability is not available in classical frameworks. Although this research focuses on portfolio risk assessment, the models are also applicable to machine learning, statistical inference, and reliability analysis.

## Acknowledgement

The authors would like to express their sincere gratitude to I.K. Gujral Punjab Technical University for providing the necessary support to carry out this research.

## Competing Interests

The authors declare that they have no competing interests.

## Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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