Research Article

On the Pendant Regular Domination Number of a Graph

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Abstract. This paper introduces and explores a novel domination parameter in graph theory, termed the pendant regular domination number. We initiate a foundational study by determining the exact values of this parameter for several standard and well-known graph families, including paths, cycles, stars, and complete graphs. In addition, few fundamental bounds are deduced and the behavior of the pendant regular domination number is examined under various graph operations such as the corona, join, and Cartesian product.

Keywords. Domination number, Pendant regular domination, Graph operations

Mathematics Subject Classification (2020). 05C38, 05C69, 05C76

1. Introduction

The concept of the domination number was first introduced by C. Berge [1] as a measure of external stability. The term domination for undirected graphs was later used by O. Ore [12] in 1962, who also developed the notions of minimal and minimum dominating sets. In 1972, R. M. Karp [7] proved that the set cover problem is $\mathcal{NP}$-complete, establishing a fundamental link between domination and computational complexity. Since then, the study of domination parameters and the development of efficient algorithms for them have found extensive

applications in solving network-related problems (Bouamama and Blum [3], and Raghavan and Zhang [13]).

Domination in graphs has significant applications across diverse areas of science and engineering, particularly in optimal resource placement, network monitoring, and system control. In wireless sensor networks, dominating sets model the minimum number of sensors required to efficiently monitor or control the entire system. In social networks, domination concepts help identify influential nodes for effective information dissemination and viral marketing (Leskovec *et al.* [8]). In bioinformatics, domination is applied to genetic regulatory networks to identify essential control genes or proteins (Haynes [6]). Similarly, in robotic navigation and facility location problems, domination-based strategies ensure adequate coverage and accessibility of service points (Garey and Johnson [4]). The study of pendant regular domination further extends these applications by modeling redundancy and uniform coverage through peripheral nodes (pendant vertices), thereby enhancing fault tolerance and energy efficiency in real-world systems.

In recent years, domination has continued to play a pivotal role in the design and optimization of *Wireless Sensor Networks* (WSN) and *Internet of Things* (IoT) systems. For example, Bindu *et al.* [2] applied domination-based clustering techniques in precision agriculture to optimize sensor placement and achieve energy-efficient coverage. Likewise, advanced methods integrating deep learning and graph neural networks leverage domination theory (Sivakumar *et al.* [16]) to identify minimal dominating sets of sensors, significantly prolonging network lifetime under energy constraints. Furthermore, specific domination parameters contribute to modeling stability and efficiency in network design: domination integrity (Mahde and Mathad [9]) evaluates robustness against failures by studying the effect of removing dominated vertices and their neighbors; minimum hub distance energy (Mathad and Mahde [10]) provides insights into efficiency and vulnerability in hub-centric systems such as transport and communication networks; and equitable two-degree domination (Sahal and Mathad [15]) ensures fair resource allocation and balanced load distribution in distributed systems. Collectively, these results underscore the practical significance of domination parameters in building stable, efficient, and resilient networks.

This paper focuses on studying simple finite undirected graphs. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. For a vertex $v \in V(G)$, the degree of v in G is denoted by $d_G(v)$ and is defined as the number of edges incident with v in G. For the vertices $u, v \in V(G)$, the distance between u and v is denoted by $d(u,v)$ and is defined as the length of the shortest path connecting u and v in G. The eccentricity of a vertex v is denoted by $e(v)$ and is maximum of $d(u,v)$ for all $u \in V(G)$. The *diameter* of G denoted by $d(G)$ or d is defined as $d(G) = \max\{e(v) : v \in V(G)\}$. A bipartite graph G is a graph whose vertex set V can be partitioned into two disjoint subsets V_1 and V_2 such that every edge of G joins a vertex V_1 to a vertex of V_2. If each vertex of V_1 is joined to every vertex of V_2, then G is called a complete bipartite graph. If V_1 and V_2 have m and n vertices respectively, then in a complete bipartite graph we write $G = K_{m,n}$. A star is a complete bipartite graph $K_{1,n}$. The *union* $G_1 \cup G_2$ of disjoint graphs G_1 and G_2 is the graph having vertex set $V_1 \cup V_2$ and the edge set $E_1 \cup E_2$. The *join* $G_1 + G_2$ is the graph consisting of $G_1 \cup G_2$ with all edges joining all vertices of V_1 with all vertices of V_2. The *corona* $G_1 \circ G_2$ is the graph obtained from the graphs G_1 and G_2 by taking one copy of G_1 and $|V(G_1)|$ copies of

G_2 and then joining each vertex of the ith copy of G_2 named (G_2, i), with the ith vertex of G_1 by an edge (Harary [5]).

A subset D of the vertex set V of a graph $G = (V, E)$ is called a dominating set of G, if $V \setminus D$ is adjacent to some vertex in D (Haynes *et al*. [6]). The cardinality of *Minimum Dominating Set* (MDS) is called domination number and it is denoted by $\gamma(G)$. The concept of regular domination was introduced to solve the problem of having equal amount of guard allocation and is mathematically defined as follows.

Definition 1.1 ([14])**.** A subset R of V is said to be *Regular Dominating Set* (RDS) of G if each vertex u that belongs to $V \setminus R$ is adjacent to some vertex in R and every vertex in $R \subset V$ has the same degree. Each RDS must contain atleast two vertices to make it well-defined. The cardinality of *Minimum Regular Dominating Set* (MRDS) is called regular domination number and it is denoted by $\gamma_R(G)$.

Definition 1.2 ([11])**.** A dominating set D of a graph G is said to be a *Pendant Dominating Set* (PDS) of G if induced sub graph of D contains at least one pendant vertex. The cardinality of *Minimum Pendant Dominating Set* (MPDS) is called pendant domination number and it is denoted by $\gamma_{pe}(G)$.

In this article, we introduce the concept of pendant regular domination, where the equally assigned guards in regular domination have at least one guard with a backup.

Definition 1.3. Let $G = (V, E)$ be a simple graph. A dominating set $D \subseteq V(G)$ is called a *Pendant Regular Dominating Set* (PRDS) if all vertices of D have the same degree in G, and the induced subgraph $G[D]$ contains at least one pendant vertex. The cardinality of *Minimum Pendant Regular Dominating Set* (MPRDS) is called pendant regular domination number and it is denoted by $\gamma_{PR}(G)$.
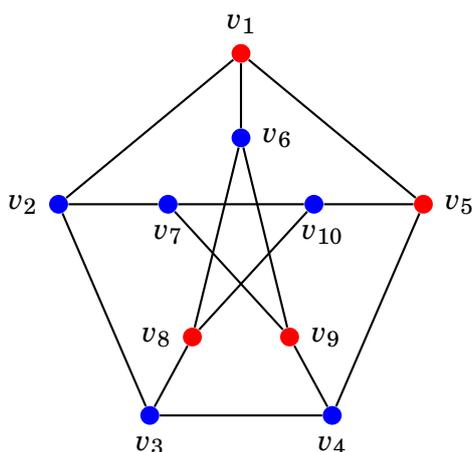
Figure 1. Peterson graph

The domination and regular domination is 3, where as the pendent regular domination of the Peterson graph is 4 as shown in Figure 1.

2. Main Results

Observation 2.1. *For every regular graph G, $\gamma_{pe}(G) = \gamma_{PR}(G)$.*

Proposition 2.2. *If D is a PRDS of G, then all vertices in D belong to the same degree class of the degree partition of $V(G)$.*

Proof. By definition, each vertex in D has the same degree in G. Hence D is a subset of one degree class in the degree partition of $V(G)$. $\qquad\square$

Proposition 2.3. *If D is a PRDS of G, then $|D| \geq 2$.*

Proof. Suppose $|D| = 1$, say $D = \{v\}$. Then $G[D]$ is a single isolated vertex with no edge, so it contains no pendant vertex. Hence $|D| \geq 2$. $\qquad\square$

Proposition 2.4. *Let D be a PRDS of G. Then $G[D]$ must contain at least one edge.*

Proof. Since $G[D]$ must contain a pendant vertex, there must exist a vertex $u \in D$ with $\deg_{G[D]}(u) = 1$. This is only possible if u is adjacent to another vertex $v \in D$. Thus $G[D]$ contains at least one edge. $\qquad\square$

Proposition 2.5. *The star $K_{1,n-1}$ $(n \geq 3)$ does not admit a PRDS.*

Proof. In $K_{1,n-1}$ the leaves all have degree 1 while the central vertex, say v has degree $n-1$. Any PRDS must consist of vertices with equal degree in G; thus a candidate PRDS must be either a subset of the leaves (degree 1) or the singleton $\{v\}$. The singleton center cannot be a PRDS because $G[\{v\}]$ has no pendant. Any nontrivial subset of leaves induces an edgeless subgraph (no edges among leaves), hence $G[D]$ contains no pendant vertex. Therefore, $K_{1,n-1}$ admits no PRDS. $\qquad\square$

Theorem 2.6 ([11])**.** *Let G be a cycle or path with n vertices, then we have*

$$\gamma_{pe}(G) = \begin{cases} \frac{n}{3} + 1, & \text{if } n \equiv 0 \pmod 3; \\ \frac{n}{3}, & \text{if } n \equiv 1 \pmod 3; \\ \left\lceil \frac{n}{3} \right\rceil + 1, & \text{if } n \equiv 2 \pmod 3. \end{cases}$$

Theorem 2.7. *For a complete graph K_n, $\gamma_{PR}(K_n) = 2$, for $n \geq 2$.*

Proof. Since every regular dominating set contains at least two vertices in it, any two random vertices of K_n forms an MRDS, whose induced sub graph has two pendant vertices. Thus, we have $\gamma_{PR}(K_n) = 2$. $\qquad\square$

Theorem 2.8. *For a cycle C_n or path P_n with n vertices, we have*

$$\gamma_{PR}(P_n) = \gamma_{PR}(C_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil + 1, & \text{if } n \equiv 0, 2 \pmod 3; \\ \frac{n}{3}, & \text{if } n \equiv 1 \pmod 3. \end{cases}$$

Proof. By direct implication of Theorem 2.6, we obtain the result. $\qquad\square$

Theorem 2.9. *For any wheel graph W_n ($n \geq 4$), we have*

$$\gamma_{PR}(W_n) = \begin{cases} \left\lceil \frac{n-1}{3} \right\rceil, & \text{if } n \equiv 2 \ (\mathrm{mod} \ 3); \\ \left\lceil \frac{n-1}{3} \right\rceil + 1, & \text{if } n \equiv 0, 1 \ (\mathrm{mod} \ 3). \end{cases}$$

Proof. As a wheel graph W_n has one universal vertex and remaining $(n-1)$ vertices are on its cycle C_{n-1}. It follows that the MPRDS of C_{n-1} dominates the universal vertex in W_n and from Theorem 2.8, we yield $\gamma_{PR}(W_n) = \gamma_{PR}(C_{n-1})$. □

Observation 2.10. *For a complete bipartite graph $K_{m,n}$, the PRDS exists only in the case when $m = n$ and in this case $\gamma_{PR}(K_{m,n}) = 2$.*

Observation 2.11. *For any double star graph $S_{m,n}$, the PRDS exists only when $m = n$ and the MPRDS is of cardinality 2.*

Definition 2.1 ([14]). The lollipop graph, represented by the symbol $L_{n,m}$ is a graph having a bridge between a complete graph K_n and a path graph P_m.

Theorem 2.12. *For a Lollipop graph $L_{3,m}$ with $m \geq 2$, we have*

$$\gamma_{PR}(L_{3,m}) = \begin{cases} \left\lceil \frac{m}{3} \right\rceil + 1, & \text{if } m \equiv 1 \ (\mathrm{mod} \ 3); \\ \left\lceil \frac{m}{3} \right\rceil + 2, & \text{if } m \equiv 0, 2 \ (\mathrm{mod} \ 3). \end{cases}$$

Proof. Let the vertex set of $L_{3,m}$ be $\{v_1, v_2, v_3, u_1, u_2, \ldots, u_m\}$ where $\deg(v_3) = 3$, $\deg(u_m) = 1$, and $\deg(v_1) = \deg(v_2) = 2 = \deg(u_k)$, $1 \leq k \leq m-1$. Thus, the MRDS can contain only vertices of degree 2. Now we construct an MPRDS by considering one vertex of K_3, which is of degree 2 say v_1 and MRDS of P_m as per Theorem 2.8. Therefore, we obtain the required result. □

Definition 2.2 ([14]). A cone graph $C_{m,n}$, is obtained by joining of a cycle graph C_m on m vertices and an empty graph $\overline{K_n}$ on n vertices.

Theorem 2.13. *For any Cone graph $C_{m,n}$ with $m \geq 4$ and $n \geq 3$,*

$$\gamma_{PR}(C_{m,n}) = \begin{cases} \left\lceil \frac{m}{3} \right\rceil, & \text{if } n \equiv 1 \ (\mathrm{mod} \ 3); \\ \left\lceil \frac{m}{3} \right\rceil + 1, & \text{if } n \equiv 0, 2 \ (\mathrm{mod} \ 3). \end{cases}$$

Proof. In $C_{m,n}$, degree of every vertex in C_m is $n+2$, and degree of the remaining vertices is m. To generate an PRDS, we have two options; one by considering only the vertices of $\overline{K_n}$ and the other by considering only the vertices of C_m. The first option does not produce an PRDS as the induced sub graph does not contain any pendant vertex and hence, we need to construct an MPRDS using only the vertices of C_m. This can be done by considering the MRDS of C_m. Thus by Theorem 2.8, the required result follows. □

Definition 2.3 ([14]). A fan graph $F_{m,n}$ is obtained by linking an empty graph $\overline{K_m}$ on m vertices and a path graph P_n on n vertices.

Theorem 2.14. *For a Fan graph $F_{m,n}$ with $m \geq 2$ and $n \geq 4$,*

$$\gamma_{PR}(F_{m,n}) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil, & \text{if } n \equiv 1 (\mathrm{mod} \ 3); \\ \left\lceil \frac{n}{3} \right\rceil + 1, & \text{if } n \equiv 0, 2 (\mathrm{mod} \ 3). \end{cases}$$

Proof. Analogous to the proof of $C_{m,n}$, we can deduce this result for $F_{m,n}$. □

Definition 2.4 ([14])**.** Windmill graph $W_{m,n}$ is a graph constructed by joining m copies of complete graph K_n with a common vertex K_1.

Theorem 2.15. *For any Windmill graph $W_{m,n}$ with $m, n \geq 2$, $\gamma_{PR}(W_n) = m + 1$.*

Proof. We observe that every vertex in the m copies of K_n in $W_{m,n}$ is of degree n, and we cannot consider the common vertex in PRDS. So, we construct a PRDS using the vertices of m copies of K_n, by choosing one random vertex in each copies of K_n we get the MRDS and further to form an PRDS, additionally we choose at least one pendant vertex from any one copy of K_n in $W_{m,n}$. Hence, we obtain $\gamma_{PR}(W_n) = m + 1$. □

Theorem 2.16. *For any graph G of order $n \geq 2$, the pendant regular domination number satisfies*

$$1 \leq \gamma_{PR}(G) \leq n.$$

Theorem 2.17. *If G admits a PRDS D and the common degree of vertices of D in G is d, then*

$$\max\left\{2, \gamma(G), \left\lceil \frac{n}{d+1} \right\rceil\right\} \leq |D|.$$

Consequently,

$$\max\left\{2, \gamma(G), \left\lceil \frac{n}{\Delta+1} \right\rceil\right\} \leq \gamma_{PR}(G).$$

Proof. Let D be a PRDS with common degree d in G and put $k = |D|$.

Case 1. $k \geq 2$. The induced graph $G[D]$ contains a pendant vertex, so $G[D]$ has at least one edge; hence $k \geq 2$.

Case 2. $k \geq \gamma(G)$. By definition D dominates G, therefore its size cannot be smaller than the domination number.

Case 3. $k \geq \lceil n/(d+1) \rceil$. Every vertex $v \in D$ has degree d in G, so its closed neighbourhood $N_G[v]$ contains exactly $d + 1$ vertices (itself plus its d neighbors). As D dominates G, the family $\{N_G[v] : v \in D\}$ covers $V(G)$. Hence

$$k(d+1) \geq n,$$

which gives $k \geq \lceil n/(d+1) \rceil$.

Since $d \leq \Delta$ the displayed inequality with d implies the weaker universal form with Δ, completing the proof. □

The next theorem shows that, in a globally regular network, enforcing a backup guard (a pendant inside the chosen set) costs at most one extra guard beyond an optimal dominating deployment. Both equality cases occur in natural families (so the bound is tight).

Theorem 2.18. *Let G be an r-regular graph, $r \geq 1$. If G admits a PRDS then*

$$\gamma(G) \leq \gamma_{PR}(G) \leq \gamma(G) + 1.$$

Proof. The left inequality is immediate since any PRDS is a dominating set.

For the upper bound let S be a minimum dominating set in G and assume S is minimal (no proper subset of S dominates). Two cases occur:

Case 1. $G[S]$ contains a pendant vertex. Then S itself satisfies the PRDS conditions because all vertices of S have degree r in G (global regularity) and $G[S]$ contains a vertex of degree 1 in the induced subgraph; hence $\gamma_{PR}(G) \leq |S| = \gamma(G)$.

Case 2. $G[S]$ contains no pendant vertex. By minimality of S, for each $v \in S$ there exists a private neighbor $p(v) \in V \setminus S$ adjacent to v and to no other vertex of S. Choose some $v \in S$ and pick $p = p(v)$. Let $D := S \cup \{p\}$. Then

(i) D dominates G because S already did.

(ii) Every vertex of D has degree r in G, so the equal-degree condition is satisfied.

(iii) In $G[D]$, vertex v is adjacent to p and to no other vertex of D (since p is private to v and $G[S]$ had no pendant), hence $\deg_{G[D]}(v) = 1$ and v is a pendant of $G[D]$.

Thus D is a PRDS of size $|S| + 1 = \gamma(G) + 1$, proving the upper bound. $\square$

Now we explore the behavior of pendant regular domination numbers under few graph operations. These results provide valuable insights into the properties of pendant regular domination and have implications for its applications in graph theory and related fields.

Theorem 2.19. *For any join $P_n + P_m$ with $n, m \geq 4$,*

$$\gamma_{PR}(P_n + P_m) = \begin{cases} 2, & \text{if } n = m; \\ \left\lceil \frac{\min(n,m)}{3} \right\rceil + 1, & \text{if } n, m \equiv 0, 2 \pmod 3; \\ \left\lceil \frac{\min(n,m)}{3} \right\rceil, & \text{if } n, m \equiv 1 \pmod 3. \end{cases}$$

Proof. As per the definition of join of graphs, each vertices of P_n is adjacent to every vertex of P_m and the degree of each vertex of P_n in $P_n + P_m$ is $n + 1$.

Case 1. When $m = n$, the induced subgraph containing one random vertex of P_n and one random vertex of P_m (as each vertex is of same degree) forms an MPRDS.

Case 2. When $m \neq n$, we construct an PRDS either by considering only the vertices of P_n or only the vertices of P_m, as any vertex of P_n (or P_m) dominates all vertices of P_m (or P_n). Next, to dominate the vertices of P_n (or P_m), we consider the MPRDS of P_n (or P_m) from Theorem 2.8, which concludes the results. $\square$

Theorem 2.20. *Let G be non-regular graph of order n and H be a complete graph K_m of order m with $m \geq 2$ then $\gamma_{PR}(G \circ H) = n + 1$.*

Proof. By considering one random vertex from each n copies of K_m we get MRDS of $G \circ H$. Now to make this an PRDS we need to consider atleast one more vertex from any one of the K_m. Thus, we have $\gamma_{PR}(G \circ H) = n + 1$. $\square$

Theorem 2.21. *For any paths P_n and P_m,*

$$\gamma_{PR}(P_n \circ P_m) = \begin{cases} n \left\lceil \frac{m}{3} \right\rceil, & \text{if } n, m \equiv 1 \pmod 3; \\ n \left\lceil \frac{m}{3} \right\rceil + 1, & \text{if } n, m \equiv 0, 2 \pmod 3. \end{cases}$$

Proof. Since degree of each vertex of P_n is not equal in $P_n \circ P_m$, we can not form MPRDS of $P_n \circ P_m$ using only the vertices of P_n. So, we consider the MPRDS of P_m in each of its n copies to generate an MPRDS of $P_n \circ P_m$. Hence, by incorporating Theorem 2.8 we yield the result. $\square$

3. Conclusion

This study introduced and explored the concept of pendant regular domination in graphs. We determined its values for several standard graph families, derived basic bounds, and examined its behavior under key graph operations.

The notion of pendant regular domination naturally models scenarios where resources such as guards, sensors, or facilities are placed at vertices of equal degree, thereby ensuring fairness and balanced workload. The additional requirement that the induced subgraph contains at least one pendant vertex provides a built-in backup mechanism, making this model particularly useful in applications like network monitoring, facility location, bioinformatics, and cybersecurity, where both uniformity and redundancy are essential.

Beyond these initial results, pendant regular domination offers promising directions for further research in graph domination theory and its applications. The future scope of this work is outlined below:

(1) Characterizing the class of graphs G for which $\gamma_{PR}(G) = \gamma(G)$.

(2) Analyzing how $\gamma_{PR}(G)$ behaves under graph operations like edge addition/deletion, vertex removal, and subdivision.

(3) Studying the nature of MPRDS for Cartesian, direct, strong, and disjunctive products.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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