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Research Article

A Note on Multiplicative Fuzzy on BP-Algebras

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Abstract. Multiplicative fuzzy sets are defined by combining the membership values of the elements in a fuzzy set. This can be considered a generalization of fuzzy sets. Multiplicative fuzzy sets are useful in areas like quantum mechanics. In this paper, we define multiplicative fuzzy BP-algebras using the multiplication operation on membership functions and investigate some of their properties. Multiplicative fuzzy topological BP-algebras are also discussed.

Keywords. Absorbing subset, BP-Absorbing subset, Graded set, Multiplicative Fuzzy BP-Algebra

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1. Introduction

Imai and Iseki [11] presented a pair of important categories of hypothetical algebras: BCK and BCI algebras in 1966. Among these algebras, BCK-algebras are recognized as a proper subclass within the wider class of BCI-algebras. Later, Dudeck and Zhang [8] proposed the concept of BCC-ideals in the framework of BCC-algebras, while Foster [10] studied fuzzy topological groups. Following this, Meng *et al*. [15] introduced fuzzy implicative ideals of BCK-algebras. Akram and Dar [5] investigated the fuzziness in d-algebras. Building on this, Dudek and Jun [7] extended the theory by developing the fuzzification of BCC-ideals. Lee and Ryu [13] studied topological BCK-algebras, while Ahmed and Ahmed [2] studied Fuzzy BCK-algebras. Annalakshmi and Chandramouleeswaran [6], Jansi and Thiruveni [12] respectively extended this fuzzification to the topological system of TM-algebras and BF-algebras. In our earlier paper, we investigated the topologies on BP-algebras (Abisha and Kandaraj [1]). In this research, we present the concept of multiplicative fuzzy BP-algebras as the natural extension of fuzzy subalgebras within BP-algebras. We examine a range of fundamental properties associated with these structures, particularly their homomorphic images and inverse images. Since Graded sets offer a suitable foundation for generalizing classical topological ideas into fuzzy topological spaces, we explore the integration of fuzzy topological structures with multiplicative fuzzy BP-algebras. This fusion gives rise to what we refer to as multiplicative fuzzy topological BP-algebras.

2. Preliminaries

Definition 2.1 ([3]). Let X be a non-null set equipped with a bivariate function $*$ and a fixed element 0. The algebraic structure $(X,*,0)$ is known as a BP-algebra if it fulfills the following three fundamental conditions:

(i) $x*x=0$: This axiom states that the binary operation applied to any element with itself yields the constant 0. It reflects an idempotent-like property that serves as a key characteristic of BP-algebras.

(ii) $x*(x*y)=y$: It indicates a symmetry related to the operation's inverse and it means that the operation $*$, when applied in the form $x*(x*y)$, behaves like a form of inverse or complementation, effectively cancelling out the first.

(iii) $(x*z)*(y*z)=x*y$, for any $x,y,z\in X$.

This identity ensures that the effect of "subtracting" the same element z from both x and y does not change their relative difference. It's a kind of invariance under a common operation with z.

Proposition 2.2 ([3]). *If $(X,*,0)$ is a BP-algebra, then several useful properties can be derived from its axioms.*

For any $x,y\in X$, the following hold:

(i) $0*(0*x)=x$: *This is a recovery property, starting from x, applying the operation with 0 twice brings x.*

(ii) $x*(x*y)=y$: *This reaffirms the second axiom, emphasizing its importance as a fundamental identity in BP-algebras.*

(iii) $x*0=x$: *This indicates that the constant 0 behaves like a right identity under the operation $*$.*

(iv) $x*y=0\Longrightarrow y*x=0$: *This is a symmetry condition on the operation result 0. It implies a mutual equivalence in some sense.*

(v) $x=y\Longrightarrow 0*x=0*y$: *This shows that the operation with 0 is injective. If applying 0 to two different elements gives the same result, the elements must be equal.*

(vi) $(x*y)=(x*z)*(y*z)$: *This restates the third axiom, reinforcing the invariance of difference under the same transformation.*

(vii) $0*x=x\Longrightarrow x*y=y*x$: *If x is fixed under operation with 0, then the operation becomes symmetric for that element with all others.*

Proposition 2.3 ([3]). *If the BP-algebra* $(X,*,0)$ *satisfies the condition* $(x*y)*z = x*(z*y)$ *for all* $x,y,z \in X$*, then* $0*x = x$ *for every* $x \in X$.

This additional condition strengthens the structure, suggesting that every element x is preserved when operated on by 0 from the left, indicating a form of left identity behavior for 0 under a special associatively.

Theorem 2.4 ([3]). *In a BP-algebra*$(X,*,0)$*, if for any* $x,y \in X, y*x = 0$ *and* $x*y = 0$*, then* $x = y$.

This theorem confirms that the operation $*$ distinguishes elements precisely when both directions of the operation are defined.

Definition 2.5 ([8]). Let $S \neq \varphi$ be a subset of a BP-algebra X. Then S is called a BP-subalgebra of X if for all $x,y \in S$, $x*y \in S$.

This definition ensures that the subset S is closed under the binary operation $*$ and thus retains the structure of the BP-algebra.

Definition 2.6 ([8]). Let $(X,*,0)$ be a BP-algebra and let I be a non-empty subset of X. Then I is said to be an Absorbing subset (Ideal) of X if it satisfies the following conditions:

(i) $0 \in I$: The Absorbing subset must contain the distinguished constant element 0.

(ii) $x*y \in I$ and $y \in I \Longrightarrow x \in I$: This condition ensures the absorption or closure operation, showing that the presence of y and $x*y$ implies the presence of x.

Definition 2.7 ([8]). Let $(X,*,0)$ be a BP-algebra and I be a non-empty subset of X. Then I is called a BP-Absorbing subset (BP-Ideal) of X if it satisfies the following conditions:

(1) $0 \in I$: The neutral element must be in the BP-Absorbing subset.

(2) $(x*y)*z \in I$ and $y \in I \Longrightarrow x*z \in I$: This condition extends the absorption behavior to a more generalized form, involving three elements and nested operations.

Lemma 2.8 ([8]). *In a BP-algebra* X*, any BP-Absorbing subset* I *is an Absorbing subset of* X.

This lemma connects the concepts of BP-Absorbing subsets and Absorbing subsets, showing that the stronger condition of being a BP-Absorbing subset automatically satisfies the more general definition of an Absorbing subset.

Remark 2.9 ([9]). Every BP-Absorbing subset in a BP-algebra is also a BP-subalgebra, but not all BP-subalgebras need to be a BP-Absorbing subsets.
Similarly, every Absorbing subset of a BP-algebra is BP-subalgebra, but not all BP-subalgebras qualify as Absorbing subsets.

Definition 2.10 ([4]). Let $(X,*,0)$ be a BP-algebra. Define a binary relation "$\leq$" by $x \leq y$ if $x*y = 0$. This relation is known as a BP-order on X.
It can be shown that this relation forms a partially ordered set X.

Definition 2.11 ([14]). Let X be a non-empty set. A mapping $\mu: X \to [0,1]$ is called a fuzzy set (or graded set) in X. The complement of μ is denoted by $\overline{\mu}(x) = 1 - \mu(x)$, for all $x \in X$.

3. Multiplicative Fuzzy on *BP*-algebras

Definition 3.1. A graded set X in a BP-algebra B with membership function μ_X is called a fuzzy BP-subalgebra if $\mu_X(a * b) \geq \min\{\mu_X(a), \mu_X(b)\}$, for all $a, b \in B$.
This ensures that the membership value of the product $a * b$ is at least as much as the smaller of the membership values of a and b. This aligns with the idea that the operation stays "within" the fuzzy substructure.

Definition 3.2. A graded set (X, μ_X) in B is called a multiplicative fuzzy BP-algebra if it satisfies the inequality $\mu_X(a * b) \geq \mu_X(a) \cdot \mu_X(b)$, for all $a, b \in B$.
Instead of using the minimum, this uses multiplication of membership values. It is a stricter condition and ensures more "gradation" of inclusion based on fuzzy logic.

Example 3.3. This example demonstrates that multiplicative fuzzy BP-algebras can exist without being fuzzy BP-subalgebras, showing the independence of definitions.

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	1	0	0
4	4	3	4	3	0

Let (X, μ_X) be a graded set in B defined by $\mu_X(4) = 0.4$ and $\mu_X(a) = 0.8$, for all $a \neq 4$.
It is effortless to check that (X, μ_X) is a multiplicative fuzzy BP-algebra in B.
Let (Y, μ_Y) be a fuzzy set in B defined by

$$\mu_Y(0) = 0.38,\ \mu_Y(1) = 0.4,\ \mu_Y(2) = 0.3,\ \mu_Y(3) = 0.6 \text{ and } \mu_Y(4) = 0.2.$$

By routine calculation, we can show that (Y, μ_Y) is a multiplicative fuzzy BP-algebra in B.
Note that every fuzzy BP-subalgebra is a multiplicative fuzzy BP-algebra, but the converse may not be true.
For example, the multiplicative fuzzy BP-algebra (Y, μ_Y) in Example 3.3 is not a fuzzy BP-subalgebra since $\mu_Y(1 * 3) = \mu_Y(0) = 0.38 < 0.4 = \min\{\mu_Y(1), \mu_Y(3)\}$.

Proposition 3.4. *If (X, μ_X) is a multiplicative fuzzy BP-algebra in B, then $\mu_X(0) \geq (\mu_X(a))^2$, for all $a \in B$.*

Proof. Since $a * a = 0$, for all $a \in B$, we have

$$\mu_X(0) = \mu_X(a * a) \geq \mu_X(a) \cdot \mu_X(a) = (\mu_X(a))^2, \quad \text{for all } a \in B.$$

Using the identity $a * a = 0$,this gives a lower bound for the membership value of 0 in terms of any other element's membership. □

Proposition 3.5. *Let* (X,μ_X) *be a multiplicative fuzzy BP-algebra in B. If there exists a sequence* $\{a_n\}$ *in B such that* $\lim_{n\to\infty}(\mu_X(a_n))^2 = 1$*, then* $\mu_X(0) = 1$.

Proof. Using Proposition 3.4, we have

$$\mu_X(0) \geq (\mu_X(a_n))^2, \quad \text{for each } n \in \mathbb{N}.$$

Since

$$\begin{aligned} 1 &\geq \mu_X(0) \\ &\geq \lim_{n\to\infty}(\mu_X(a_n))^2 \\ &= 1. \end{aligned}$$

Hence, $\mu_X(0) = 1$. □

Theorem 3.6. *If* (X,μ_X) *and* (Y,μ_Y) *are multiplicative fuzzy BP-algebras in B, then* $(X \cap Y,\mu_{X\cap Y})$ *is a multiplicative fuzzy BP-algebra.*

Proof. Let $a,b \in B$. Then

$$\begin{aligned} (\mu_{X\cap Y})(a*b) &= \min\{\mu_X(a*b),\mu_Y(a*b)\} \\ &\geq \min\{\mu_X(a)\cdot\mu_X(b),\mu_Y(a)\cdot\mu_Y(b)\} \\ &\geq (\min\{\mu_X(a),\mu_Y(a)\})\cdot(\min\{\mu_X(b),\mu_Y(b)\}) \\ &= \mu_{X\cap Y}(a)\cdot\mu_{X\cap Y}(b). \end{aligned}$$

Hence, $(X\cap Y,\mu_{X\cap Y})$ is a multiplicative fuzzy BP-algebra.

This proves that the class of multiplicative fuzzy BP-algebras is *closed under intersection*, preserving the structure under set-theoretic operations. □

Remark 3.7. A graded set (X,μ_X) in B is a fuzzy BP-subalgebra of B if and only if the nonempty level subset $S(X;u) = \{a \in B/\mu_X(a) \geq u\}$ is a BP-subalgebra of B for every $u \in [0,1]$. But, we know that there is a multiplicative fuzzy BP-algebra such that for some $u \in [0,1]$, $S(X;u)$ is not a BP-subalgebra of B as seen in the following example.

Not all multiplicative fuzzy BP-algebras have level subsets that satisfy subalgebra properties, highlighting a *limitation* of the structure.

Example 3.8. Consider the BP-algebra $B = \{0,1,2,3,4\}$ given in Example 3.3.

Let (X,μ_X) be a graded set in B defined by $\mu_X(0) = \mu_X(2) = \mu_X(3) = 0.8$, $\mu_X(1) = 0.7$, and $\mu_X(4) = 0.4$. Then $(X,\mu_X)$is a multiplicative fuzzy BP-algebra.

But $S(X;0.8) = \{0,2,3\}$ is not a BP-subalgebra of B since $3*2 = 1 \notin \{0,2,3\}$.

This gives a *counterexample* to the general assumption that level subsets of multiplicative fuzzy BP-subalgebras form subalgebras.

Theorem 3.9. *If (X,μ_X) is a multiplicative fuzzy BP-algebra in B, then $S(X;1)$ is either empty or a BP-subalgebra of B.*

Proof. Let $a,b \in S(X;1)$. Then $\mu_X(a) = 1 = \mu_X(b)$ which implies

$$\mu_X(a*b) \geq \mu_X(a)\cdot\mu_X(b) = 1.$$

Hence $\mu_X(a*b) = 1$ (that is $a*b \in S(X;1)$).

Consequently, $S(X;1)$ is a BP-subalgebra of B.

Only the full-membership elements (value = 1) form a subalgebra, ensuring crisp substructure forms a fuzzy algebra. □

Theorem 3.10. *Let $f: B \to C$ be a homomorphism of BP-algebras. If (Y,μ_Y) is a multiplicative fuzzy BP-algebra in C, then the inverse image $(f^{-1}(Y),\mu_{f^{-1}(Y)})$ of $(Y,\mu_Y)$is a multiplicative fuzzy BP-algebra in B.*

Proof. For any $a_1,a_2 \in B$, we have

$$\begin{aligned}\mu_{f^{-1}(Y)}(a_1*a_2) &= \mu_Y(f(a_1*a_2))\\ &= \mu_Y(f(a_1)*f(a_2))\\ &= \mu_Y(f(a_1)\cdot\mu_Y(f(a_2))\\ &= \mu_{f^{-1}(Y)}(a_1)\cdot\mu_{f^{-1}(Y)}(a_2).\end{aligned}$$

Hence, $(f^{-1}(Y),\mu_{f-1(Y)})$ is a multiplicative fuzzy BP-algebra in B.

Fuzzy structure is preserved *backward* through homomorphisms, allowing fuzzy properties to be inherited. □

Theorem 3.11. *Let $f: B \to C$ be an onto homomorphism of BP-algebras. If (X,μ_X) is a multiplicative fuzzy BP-algebra in B, then the image $(f(X),\mu_{f(X)})$ of (X,μ_X) is a multiplicative fuzzy BP-algebra in C.*

Proof. For any $b_1,b_2 \in C$, let $G_1 = f^{-1}(b_1)$, $G_2 = f^{-1}(b_2)$ and $G_{1*2} = f^{-1}(b_1*b_2)$.

Consider the set

$$\begin{aligned}G_1*G_2 &= \{a \in B/a = x_1*x_2, x_1 \in G_1, x_2 \in G_2\}\\ &= \{a \in B/a = x_1*x_2, x_1 \in G_1, x_2 \in G_2\}.\end{aligned}$$

To prove: $G_1*G_2 \subseteq G_{1*2}$.

If $a \in G_1*G_2$, then $a = a_1*a_2$, for some $a_1 \in G_1$ and $a_2 \in G_2$.

Hence $f(a) = f(a_1*a_2) = f(a_1)*f(a_2) = b_1*b_2$ and so $a \in f^{-1}(b_1*b_2) = G_{1*2}$.

Now we have

$$\begin{aligned}\mu_{f(X)}(b_1*b_2) &= \sup_{a\in f^{-1}(b_1*b_2)} \mu_X(a)\\ &= \sup_{a\in G_{1*2}} \mu_X(a)\\ &\geq \sup_{a\in G_1*G_2} \mu_X(a)\end{aligned}$$

$$\geq \sup_{a_1\in G_1, a_2\in G_2} \mu_X(a_1 * a_2)$$
$$\geq \sup_{a_1\in G_1, a_2\in G_2} \mu_X(a_1)\cdot\mu_X(a_2).$$

Since $[0,1]\times[0,1]\to[0,1]$ is continuous, for every $\varepsilon>0$ there exists $\lambda>0$ such that if

$$a_i \geq \sup_{a_i\in G_i} \mu_X(a_i)-\lambda, \quad \text{for } i=1,2.$$

Then

$$\overline{a}_1\cdot\overline{a}_2 \geq \sup_{a_1\in G_1} \mu_X(a_1)\cdot \sup_{a_2\in G_2} \mu_X(a_2)-\delta.$$

Choose $x_1\in G_1$ and $x_2\in G_2$ such that

$$\mu_X(x_1)\geq \sup_{a_1\in G_1}\mu_X(a_1)-\lambda$$

and

$$\mu_X(x_2)\geq \sup_{a_2\in G_2}\mu_X(a_2)-\lambda.$$

Then

$$\mu_X(x_1)\cdot\mu_X(x_2)\geq \sup_{a_1\in G_1}\mu_X(a_1)\cdot \sup_{a_2\in G_2}\mu_X(a_2)-\delta$$

and so

$$\mu_{f(X)}(b_1*b_2)\geq \sup_{a_1\in G_1, a_2\in G_2}\mu_X(a_1)\cdot\mu_X(a_2)$$
$$\geq \sup_{a_1\in G_1}\mu_X(a_1)\cdot \sup_{a_2\in G_2}\mu_X(a_2)$$
$$=\mu_{f(X)}(b_1)\cdot\mu_{f(X)}(b_2).$$

Hence, $(f(X),\mu_{f(X)})$ is a multiplicative fuzzy BP-algebra in C. □

Theorem 3.12. *Let (X,μ_X) be a fuzzy set in B and let (R,μ_R) be a left fuzzy relation on (X,μ_X). If (R,μ_R) is a multiplicative fuzzy BP-algebra on $B\times B$, then (X,μ_X) is a multiplicative fuzzy BP-algebra in B.*

Proof. Let (R,μ_R) be a multiplicative fuzzy BP-algebra in $B\times B$, and let $a,b,s,t\in B$. Then

$$\mu_X(a*b)=\mu_R((a*b)\cdot(s*t))$$
$$=\mu_R((a\cdot s)*(b\cdot t))$$
$$\geq\mu_R(a\cdot s)\cdot\mu_R(b\cdot t)$$
$$=\mu_X(a)\cdot\mu_X(b).$$

This follows that (X,μ_X) is a multiplicative fuzzy BP-algebra in B. □

Theorem 3.13. *Let (R,μ_R) be a fuzzy relation on B and let (Y,μ_Y) be a fuzzy set in B given by $\mu_Y(a)=\inf\{\mu_R(a,b),\mu_R(b,a)/b\in B\}$, for all $a\in B$. If (R,μ_R) is a multiplicative fuzzy BP-algebra in $B\times B$ satisfying the condition $\mu_R(a,0)=1=\mu_R(0,a)$ for all $a\in B$, then (Y,μ_Y) is a multiplicative fuzzy BP-algebra in B.*

Proof. Suppose $a,b,c \in B$, we have

$$\begin{aligned}\mu_R(a*b,c) &= \mu_R(a*b,c*0)\\ &= \mu_R((a,c)*(b,0))\\ &\geq \mu_R(a,c)\cdot\mu_R(b,0)\\ &= \mu_R(a,c)\end{aligned}$$

and

$$\begin{aligned}\mu_R(c,a*b) &= \mu_R(c*0,a*b)\\ &= \mu_R((c,a)*(0,b))\\ &\geq \mu_R(c,a)\cdot\mu_R(0,b)\\ &= \mu_R(c,a).\end{aligned}$$

It follows that

$$\begin{aligned}\mu_R(a*b,c)\cdot\mu_R(c,a*b) &\geq \mu_R(a,c)\cdot\mu_R(c,a)\\ &\geq \{\mu_R(a,c)\cdot\mu_R(c,a)\}\cdot\{\mu_R(b,c)\cdot\mu_R(c,b)\}.\end{aligned}$$

Hence

$$\begin{aligned}\mu_R(a,b) &= \inf\{\mu_R(a*b,c)\cdot\mu_R(c,a*b)/c \in B\}\\ &\geq \inf\{\mu_R(a,c)\cdot\mu_R(c,a)/c\in B\}\cdot\inf\{\mu_R(b,c)\cdot\mu_R(c,b)/c\in B\}\\ &= \mu_R(a)\cdot\mu_R(b).\end{aligned}$$

Therefore, (Y,μ_Y) is a multiplicative fuzzy BP-algebra in B. □

Conclusion

The research introduces the concept of multiplicative fuzzy BP-algebras as a natural extension of fuzzy subalgebras within BP-algebras. This work explores the integration of fuzzy topological structure with multiplicative fuzzy BP-algebras, leading to the definition of multiple fuzzy topological BP-algebras. We have examined the fundamental properties of these structures, particularly the relation to homomorphic images and inverse images. In the future, we can explore computational methods and algorithms related to these structures.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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