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Research Article

# A New Modified Integral Transform

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**Abstract.** We introduce a new modified integral transform – a comprehensive extension encompassing the classical Laplace transform and its variants developed in recent decades. We establish its fundamental properties, including existence, linearity, scaling, shifting (both first and second), differentiation, integration, periodicity, and convolution. As a unifying framework, simplifies and generalizes various known integral transforms. We demonstrate its effectiveness through solutions to ordinary and partial differential equations, Volterra integral equations, partial integro-differential equations, and systems of ODEs, supported by illustrative examples.

**Keywords.** Laplace transform, New modified integral transform, Fractional order integral equations, Integral equation, Differential equations

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## 1. Introduction

Integral transforms are mathematical operations that transfer functions from one form to another, frequently to make solving differential equations or performing other operations easier, and the original answer can be retrieved via the inverse integral transform and is effectively used in a variety of domains, including signal processing, differential equations and applied mathematics.

The integral transform of function $f(t)$ is denoted by $F(s)$ and is given by the integral

$$F(s)=\int_a^b K(s,t)f(t)dt,$$

where $K(s,t)$ is called as kernel of the transform and limits $(a,b)$ are specified for particular integral transform.

The Laplace transform was among the pioneering transforms which is used to solve mathematical equations. Subsequently, several integral transforms emerged in different domains, named after the mathematicians who devised them, such as the Mellin transform, Hankel transform, Fourier transform, Elzaki transform, Kamal transform, Sumudu transform, Natural transform, Aboodh transform, Formable transform, Gupta transform, and Rohit transform, etc. (see, Aboodh [1], Andrews and Shivamoggi [2], Ansari [3], Belgacem and Karaballi [4], Belgacem and Silambarasan [5], Elzaki [6], Gupta *et al*. [8], Gupta [9], Kamal and Sedeeg [11]) and are effectively used for solving linear and nonlinear boundary value problems and initial value problems in the modern era. These transformations are primarily used in financial mathematics, artificial engineering, quantum calculus, financial mathematics, bioengineering, CFD, Abel's integral equations, and bio-mathematics (see also, Rudolf and Vessella [12], and Zhao [13]).

In recent years, several integral transforms in the class of Laplace transform have been developed by researchers of which a new general integral transform referred to as the Jafari transform and Jafari-Yang transform introduced by Jafari [10] (see also, El-Mesady [7]),

$$F(s)=p(s)\int_0^\infty f(t)e^{-q(s)t}dt$$

covers most of the exiting transforms.

The main objective of this paper is to provide a modification of this novel general integral transform.

## 2. Preliminaries

This section presents a broadly modified integral transform that includes the majority, though not all, of the integral transforms associated with the Laplace transform family.

**Definition 2.1.** Let $f(t)$ be an integrable function defined for $t\geq 0$. We define the new modified integral transform by

$$\mathcal{T}_a\{f(t),t\to p\}=\mathcal{F}(p)=\phi(p)\int_0^\infty f(t)a^{-\psi(p)t}dt, \tag{2.1}$$

provided the integral exists for some $\psi(p)$, where $a\in(1,\infty)$ and $\phi(p)$, $\psi(p)$ are the regular complex functions such that $\phi(p)\neq 0$, for all $p\in\mathbb{C}$.

We have not yet tackled the issue of which class of functions truly possesses the new modified integral transform. Let us explore the conditions that determine the existence of the new modified integral transform.

**Definition 2.2** ([2])**.** Suppose that $f$ is a piecewise continuous function with the further property that there exists a real number $c_0$ such that

$$\lim_{t\to\infty}|f(t)|e^{-ct}=\begin{cases}0, & c>c_0,\\ \text{no limit}, & c<c_0.\end{cases} \tag{2.2}$$

A function $f$ satisfying this condition is said to be of exponential order $c_0$, also written $O(e^{c_0t})$. Equation (2.2) may or may not be satisfied if $c=c_0$.

**Theorem 2.1** (Existence Theorem). *If $f$ is piecewise continuous on $t \geq 0$ and is $O(e^{c_0 t})$, then $f(t)$ has a new modified integral transform $\mathcal{F}(p)$ in the half-plane $\mathrm{Re}(\psi(p)) > \frac{c_0}{\log a}$. Moreover, integral of new modified integral transform converges both absolutely and uniformly for $\mathrm{Re}(\psi(p)) \geq \frac{c_2}{\log a} > \frac{c_0}{\log a}$.*

*Proof.* To establish that a given function $f$ has a new modified integral transform $\mathcal{F}(p)$, we must show that integral of new modified integral transform

$$\mathcal{F}(p) = \phi(p) \int_0^\infty f(t) a^{-\psi(p)t} dt, \tag{2.3}$$

converges. This will be the case provided

$$|\mathcal{F}(p)| \leq |\phi(p)| \int_0^\infty |f(t) e^{-(\psi(p)t)\log a}| dt = |\phi(p)| \int_0^\infty |f(t)| e^{-ct} dt < \infty, \tag{2.4}$$

where $c = \mathrm{Re}(\psi(p)) \log a$. Let $f$ be piecewise continuous on $t \geq 0$ and of $O(e^{c_0 t})$, and let $c_1$ be a number such that $c_0 < c_1 < c$. Because $f(t) = O(e^{c_0 t})$, it follows that for any given small positive constant $\epsilon$, there exists some $t_0$ such that

$$|f(t)| e^{-c_1 t} < \epsilon, \quad \text{when } t > t_0. \tag{2.5}$$

We now write

$$|\phi(p)| \int_0^\infty |f(t)| e^{-ct} dt = |\phi(p)| \int_0^{t_0} |f(t)| e^{-ct} dt + |\phi(p)| \int_{t_0}^\infty |f(t)| e^{-ct} dt, \tag{2.6}$$

where the first integral with finite limits exists because $f$ is piecewise continuous. Furthermore, the second integral satisfies

$$\begin{aligned} |\phi(p)| \int_{t_0}^\infty |f(t)| e^{-ct} dt &= |\phi(p)| \int_{t_0}^\infty e^{-(c-c_1)t} |f(t)| e^{-c_1 t} dt \\ &< \epsilon |\phi(p)| \int_{t_0}^\infty e^{-(c-c_1)t} dt \\ &= \epsilon |\phi(p)| \frac{e^{-(c-c_1)t_0}}{(c-c_1)}. \end{aligned} \tag{2.7}$$

But this is true for $c = \mathrm{Re}(\psi(p)) \log a > c_1$, i.e., $\mathrm{Re}(\psi(p)) > \frac{c_1}{\log a}$ and, thus, we have established conditions under which the integral of new modified integral transform converges absolutely in the half-plane $\mathrm{Re}(\psi(p)) > \frac{c_0}{\log a}$. It can also be shown that the integral of new modified integral transform converges uniformly for $\mathrm{Re}(\psi(p)) \geq \frac{c_2}{\log a} > \frac{c_0}{\log a}$, where $c_2$ is any real number satisfying $c_0 < c_2 \leq c$. □

## 3. The Transforms of Some Typical Functions

The new modified integral transform of numerous functions can be derived through the standard formal integration of the integral (2.1).

**1. Function $f(t) = 1$:**

**Solution.** By the definition of new modified integral transform, we have the following

$$\begin{aligned} \mathcal{T}_a\{f(t), t \to p\} = \mathcal{F}(p) &= \phi(p) \int_0^\infty f(t) a^{-\psi(p)t} dt \\ &= \phi(p) \int_0^\infty a^{-\psi(p)t} dt \end{aligned}$$

$$= \phi(p) \int_0^\infty e^{-\psi(p)t\log a} dt$$
$$= \phi(p) \frac{e^{-\psi(p)t\log a}}{-\psi(p)\log a}\Big|_0^\infty .$$

Thus, the integral diverges for $\mathrm{Re}(\psi(p))\log a \le 0$, while for $\mathrm{Re}(\psi(p))\log a > 0$ or $\mathrm{Re}(\psi(p)) > 0$, we have

$$\mathcal{T}_a\{1, t \to p\} = \mathcal{F}(p) = \frac{\phi(p)}{\psi(p)\log a}.$$

Similarly, we can derive the following transforms:

**Table 1.** Table of new modified integral transform

| Function $f(t)$ | New modified integral transform $\mathcal{F}(p)$ | |
|---|---|---|
| $k$ | $k\frac{\phi(p)}{\psi(p)\log a}$, | $\mathrm{Re}(\psi(p)) > 0$ |
| $t$ | $\frac{\phi(p)}{(\psi(p)\log a)^2}$, | $\mathrm{Re}(\psi(p)) > 0$ |
| $t^n$ | $\frac{n!\phi(p)}{(\psi(p)\log a)^{n+1}}$, | $\mathrm{Re}(\psi(p)) > 0$, $n \in \mathbb{N}$ |
| $t^b$ | $\frac{\Gamma(b+1)\phi(p)}{(\psi(p)\log a)^{b+1}}$, | $\mathrm{Re}(\psi(p)) > 0$, $b > -1$ |
| $e^{bt}$ | $\frac{\phi(p)}{(\psi(p)\log a)-b}$, | $\mathrm{Re}(\psi(p)) > \frac{b}{\log a}$ |
| $a^{bt}$ | $\frac{\phi(p)}{(\psi(p)-b)\log a}$, | $\mathrm{Re}(\psi(p)) > b$ |
| $\sin(bt)$ | $\frac{\phi(p)b}{(\psi(p)\log a)^2+b^2}$, | $\mathrm{Re}(\psi(p)) > 0$ |
| $\cos(bt)$ | $\frac{\phi(p)\psi(p)\log a}{(\psi(p)\log a)^2+b^2}$, | $\mathrm{Re}(\psi(p)) > 0$ |
| $\sinh(bt)$ | $\frac{\phi(p)b}{(\psi(p)\log a)^2-b^2}$, | $\mathrm{Re}(\psi(p)) > \frac{\lvert b\rvert}{\log a}$ |
| $\cosh(bt)$ | $\frac{\phi(p)\psi(p)\log a}{(\psi(p)\log a)^2-b^2}$, | $\mathrm{Re}(\psi(p)) > \frac{\lvert b\rvert}{\log a}$ |

## 4. Basic Operational Properties

Computing new modified integral transform through its integral definition can be a laborious task and is not always required. By applying various operational properties of integral transforms, many additional transforms can be derived once a few have been obtained directly from the defining integral.

**Theorem 4.1** (Linearity Property). *If $\mathcal{F}(p)$ and $\mathcal{G}(p)$ are the new modified integral transform, respectively, of $f(t)$ and $g(t)$, then for any constants $C_1$ and $C_2$,*

$$\mathcal{T}_a\{C_1 f(t) + C_2 g(t), t \to p\} = C_1\mathcal{F}(p) + C_2\mathcal{G}(p).$$

*Proof.* Using definition and a simple consequence of the linearity property of integrals, we have

$$\mathcal{T}_a\{C_1 f(t) + C_2 g(t), t \to p\} = \phi(p)\int_0^\infty [C_1 f(t) + C_2 g(t)] a^{-\psi(p)t} dt$$
$$= \phi(p)\int_0^\infty C_1 f(t) a^{-\psi(p)t} dt + \phi(p)\int_0^\infty C_2 g(t) a^{-\psi(p)t} dt$$
$$= C_1\phi(p)\int_0^\infty f(t) a^{-\psi(p)t} dt + C_2\phi(p)\int_0^\infty g(t) a^{-\psi(p)t} dt$$
$$= C_1\mathcal{F}(p) + C_2\mathcal{G}(p).$$
□

**Theorem 4.2** (Change of Scale Property). *If $\mathcal{F}(p)$ is the new modified integral transform of $f(t)$, then*

$$\mathcal{T}_a\{f(bt), t \to p\} = \frac{1}{b}\mathcal{F}(p)\Big|_{\psi(p)\to\frac{\psi(p)}{b}}, \quad b > 0.$$

*Proof.* For $b > 0$ and making the variable change $u = bt$, we have

$$\begin{aligned}\mathcal{T}_a\{f(bt), t \to p\} &= \phi(p)\int_0^\infty f(bt)a^{-\psi(p)t}dt \\ &= \frac{1}{b}\phi(p)\int_0^\infty f(u)a^{-\psi(p)\frac{u}{b}}du \\ &= \frac{1}{b}\left[\phi(p)\int_0^\infty f(u)a^{-\frac{\psi(p)}{b}u}du\right] \\ &= \frac{1}{b}\mathcal{F}(p)\Big|_{\psi(p)\to\frac{\psi(p)}{b}}, \quad b > 0.\end{aligned}$$

□

**Theorem 4.3** (First Shifting Property). *If $\mathcal{F}(p)$ is the new modified integral transform of $f(t)$, then*

$$\mathcal{T}_a\{a^{bt}f(t), t \to p\} = \mathcal{F}(p)\big|_{\psi(p)\to(\psi(p)-b)}.$$

*Proof.* Using definition of new modified integral transform, we get

$$\begin{aligned}\mathcal{T}_a\{a^{bt}f(t), t \to p\} &= \phi(p)\int_0^\infty a^{bt}f(t)a^{-\psi(p)t}dt \\ &= \phi(p)\int_0^\infty f(t)a^{-(\psi(p)-b)t}dt \\ &= \mathcal{F}(p)\big|_{\psi(p)\to(\psi(p)-b)}.\end{aligned}$$

□

**Theorem 4.4** (Second Shifting Property). *If $\mathcal{F}(p)$ is the new modified integral transform of $f(t)$, then*

$$\mathcal{T}_a\{f(t-b)H(t-b), t \to p\} = a^{-\psi(p)b}\mathcal{F}(p),$$

*where $H(t-b)$ is the Heaviside unit function.*

*Proof.* Using definition of new modified integral transform, we get

$$\begin{aligned}\mathcal{T}_a\{f(t-b)H(t-b), t \to p\} &= \phi(p)\int_0^\infty f(t-b)H(t-b)a^{-\psi(p)t}dt \\ &= \phi(p)\int_b^\infty f(t-b)a^{-\psi(p)t}dt. \end{aligned} \tag{4.1}$$

Changing the variable by the substitution $u = t - b$ in eq. (4.1), we have

$$\begin{aligned}\mathcal{T}_a\{f(t-b)H(t-b), t \to p\} &= \phi(p)\int_0^\infty f(u)a^{-\psi(p)(u+b)}du \\ &= \phi(p)\int_0^\infty a^{-\psi(p)b}f(u)a^{-\psi(p)u}du \\ &= a^{-\psi(p)b}\left[\phi(p)\int_0^\infty f(u)a^{-\psi(p)u}du\right] \\ &= a^{-\psi(p)b}\mathcal{F}(p).\end{aligned}$$

This is also called translation property, and it can also be expressed as

$$\mathcal{T}_a\{f(t)H(t-b),t\to p\}=a^{-\psi(p)b}\mathcal{T}_a\{f(t+b),t\to p\},$$

which may be a more useful form in certain applications. □

**Theorem 4.5** (Differentiation Property). *If $f,f',\dots,f^{(n-1)}$ are all continuous functions on $t\ge 0$, $f^{(n)}$ is piecewise continuous on $t\ge 0$, and if all are of exponential order $c_0$, then for $n=1,2,3,\dots$,*

$$\mathcal{T}_a\{f^{(n)}(t),t\to p\}=(\psi(p)\log a)^n\mathcal{F}(p)-\sum_{i=0}^{n-1}\phi(p)(\psi(p)\log a)^{n-1-i}f^{(i)}(0),$$

*where $\mathcal{F}(p)$ is the new modified integral transform of $f(t)$.*

*Proof.* Using definition of new modified integral transform and by parts integration, we get

$$\begin{aligned}\mathcal{T}_a\{f'(t),t\to p\}&=\phi(p)\int_0^\infty f'(t)a^{-\psi(p)t}dt\\&=\phi(p)\left[a^{-\psi(p)t}f(t)\Big|_0^\infty dt-\int_0^\infty f(t)a^{-\psi(p)t}(-\psi(p)\log a)dt\right]\\&=\phi(p)\left[-f(0)+(\psi(p)\log a)\int_0^\infty f(t)a^{-\psi(p)t}dt\right]\\&=-\phi(p)f(0)+(\psi(p)\log a)\phi(p)\int_0^\infty f(t)a^{-\psi(p)t}dt\\&=-\phi(p)f(0)+(\psi(p)\log a)\mathcal{F}(p)\\&=(\psi(p)\log a)\mathcal{F}(p)-\phi(p)f(0).\end{aligned}\tag{4.2}$$

Similarly, if $f$ and $f'$ are continuous and $f''$ is piecewise continuous on $t\ge 0$, and if all three functions are of exponential order $c_0$, we can use (4.2) to get

$$\mathcal{T}_a\{f''(t),t\to p\}=(\psi(p)\log a)\mathcal{T}_a\{f'(t),t\to p\}-\phi(p)f'(0),$$

which simplifies to

$$\begin{aligned}\mathcal{T}_a\{f''(t),t\to p\}&=(\psi(p)\log a)[(\psi(p)\log a)\mathcal{F}(p)-\phi(p)f(0)]-\phi(p)f'(0)\\&=(\psi(p)\log a)^2\mathcal{F}(p)-\phi(p)(\psi(p)\log a)f(0)-\phi(p)f'(0).\end{aligned}\tag{4.3}$$

By repeated application of (4.2) and (4.3), we arrive at the following general result.

$$\mathcal{T}_a\{f^{(n)}(t),t\to p\}=(\psi(p)\log a)^n\mathcal{F}(p)-\sum_{i=0}^{n-1}\phi(p)(\psi(p)\log a)^{n-1-i}f^{(i)}(0),$$

for $n=1,2,3,\dots$. □

**Theorem 4.6** (Integration Property). *If $f$ is piecewise continuous on $t\ge 0$, and is of exponential order $c_0$, then*

$$\mathcal{T}_a\left\{\int_0^t f(u)du,t\to p\right\}=\frac{\mathcal{F}(p)}{(\psi(p)\log a)},\quad \mathrm{Re}(\psi(p))>0,$$

*where $\mathcal{F}(p)$ is the new modified integral transform of $f(t)$.*

*Proof.* Let us define a function

$$g(t)=\int_0^t f(u)du,$$

which is a continuous function since $f$ is piecewise continuous; furthermore, $g(0)=0$ and $g'(t)=f(t)$. Hence, $g$ satisfies the conditions of differentiable property of transform, and therefore, it leads to

$$\begin{aligned}\mathcal{F}(p) &= \mathcal{T}_a\{f(t), t \to p\} \\ &= \mathcal{T}_a\{g'(t), t \to p\} \\ &= (\psi(p)\log a)\mathcal{T}_a\{g(t), t \to p\} - \phi(p)g(0) \\ &= (\psi(p)\log a)\mathcal{T}_a\left\{\int_0^t f(u)du, t \to p\right\} - \phi(p)\times 0 \\ &= (\psi(p)\log a)\mathcal{T}_a\left\{\int_0^t f(u)du, t \to p\right\}\end{aligned}$$

implies

$$\mathcal{T}_a\left\{\int_0^t f(u)du, t \to p\right\} = \frac{\mathcal{F}(p)}{(\psi(p)\log a)}, \quad \text{Re}(\psi(p)) > 0. \tag{4.4}$$

□

**Theorem 4.7** (Derivatives of Transform). *If $f$ is piecewise continuous on $t \geq 0$, and is of exponential order $c_0$, then*

$$\mathcal{T}_a\{t^n f(t), t \to p\} = \frac{(-1)^n}{(\log a)^n}\frac{\phi(p)}{\psi'(p)}\underbrace{\left(\frac{1}{\psi'(p)}\left(\frac{1}{\psi'(p)}\left(\cdots\left(\frac{1}{\psi'(p)}\right.\right.\right.\right.}_{(n-1)\ times}\left(\frac{\mathcal{F}(p)}{\phi(p)}\right)'\underbrace{\left.\left.\left.\left.\right)'\cdots\right)'\right.\right.}_{(n-1)\ times},$$

*where $\mathcal{F}(p)$ is the new modified integral transform of $f(t)$.*

*Proof.* Using definition of new modified integral transform and by parts integration, we get

$$\mathcal{F}(p) = \phi(p)\int_0^\infty f(t)a^{-\psi(p)t}dt. \tag{4.5}$$

From eq. (4.5), we have

$$\frac{\mathcal{F}(p)}{\phi(p)} = \int_0^\infty f(t)a^{-\psi(p)t}dt. \tag{4.6}$$

Now, we take derivative on both sides of eq. (4.6) with respect to $p$, it leads

$$\begin{aligned}\left(\frac{\mathcal{F}(p)}{\phi(p)}\right)' &= \int_0^\infty f(t)a^{-\psi(p)t}[t\log a(-\psi'(p))]dt \\ &= -\psi'(p)\log a\int_0^\infty tf(t)a^{-\psi(p)t}dt \\ &= \frac{-\psi'(p)\log a}{\phi(p)}\phi(p)\int_0^\infty tf(t)a^{-\psi(p)t}dt\end{aligned}$$

implies

$$-\frac{\phi(p)}{\psi'(p)\log a}\left(\frac{\mathcal{F}(p)}{\phi(p)}\right)' = \phi(p)\int_0^\infty tf(t)a^{-\psi(p)t}dt.$$

Thus

$$\mathcal{T}_a\{tf(t), t \to p\} = \frac{(-1)}{\log a}\frac{\phi(p)}{\psi'(p)}\left(\frac{\mathcal{F}(p)}{\phi(p)}\right)'. \tag{4.7}$$

Continued differentiation of (4.7) leads to

$$\mathcal{T}_a\{t^n f(t), t \to p\} = \frac{(-1)^n}{(\log a)^n}\frac{\phi(p)}{\psi'(p)}\underbrace{\left(\frac{1}{\psi'(p)}\left(\frac{1}{\psi'(p)}\left(\cdots\left(\frac{1}{\psi'(p)}\right.\right.\right.\right.}_{(n-1)\text{ times}}\left(\frac{\mathcal{F}(p)}{\phi(p)}\right)'\underbrace{\left.\left.\left.\left.\right)'\cdots\right)'\right.\right.}_{(n-1)\text{ times}}.$$

□

**Theorem 4.8** (Integration of Transform)**.** *If $f$ is piecewise continuous on $t \geq 0$, and is of exponential order $c_0$ and $\frac{f(t)}{t}$ has new modified integral transform, then*

$$\mathcal{T}_a\left\{\frac{f(t)}{t}, t \to p\right\} = \phi(p)\log a \int_p^\infty \mathcal{F}(p)\frac{\psi'(p)}{\phi(p)}\,dp,$$

*provided $\lim_{t\to 0}\frac{f(t)}{t}$ exists and where $\mathcal{F}(p)$ is the new modified integral transform of $f(t)$.*

*Proof.* Define the function $\frac{f(t)}{t} = g(t)$. Then, we have

$$f(t) = tg(t). \tag{4.8}$$

Using definition of new modified integral transform with application of Theorem 4.7, we get

$$\mathcal{T}_a\{f(t)\} = \mathcal{T}_a\{tg(t)\},$$
$$\mathcal{F}(p) = -\frac{\phi(p)}{\psi'(p)\log a}\left(\frac{\mathcal{G}(p)}{\phi(p)}\right)',$$
$$-\mathcal{F}(p)\psi'(p)\log a = \mathcal{G}'(p) - \frac{\mathcal{G}(p)}{\phi(p)}\phi'(p).$$

Rewrite the above equation to have

$$\mathcal{G}'(p) - \frac{\phi'(p)}{\phi(p)}\mathcal{G}(p) = -\mathcal{F}(p)\psi'(p)\log a. \tag{4.9}$$

This is linear non-homogeneous differential equation of first order in $\mathcal{G}(p)$ whose corresponding integrating factor is $\frac{1}{\phi(p)}$. Therefore, the solution is given by

$$\left[\mathcal{G}(p)\frac{1}{\phi(p)}\right]' = -\frac{\mathcal{F}(p)\psi'(p)\log a}{\phi(p)}. \tag{4.10}$$

Integrating both the sides of (4.10) from $\infty$ to $p$ with respect to $p$, we have

$$\int_\infty^p \left[\mathcal{G}(s)\frac{1}{\phi(s)}\right]' ds = -\int_\infty^p \frac{\mathcal{F}(s)\psi'(s)\log a}{\phi(s)}\,ds. \tag{4.11}$$

Hence, we obtain

$$\mathcal{G}(p) = \mathcal{T}_a\left\{\frac{f(t)}{t}\right\} = \phi(p)\log a \int_p^\infty \mathcal{F}(s)\frac{\psi'(s)}{\phi(s)}\,ds,$$

which simplifies to

$$\mathcal{T}_a\left\{\frac{f(t)}{t}, t \to p\right\} = [\phi(p)\log a]\int_p^\infty \mathcal{F}(s)\frac{\psi'(s)}{\phi(s)}\,ds. \tag{4.12}$$

□

**Note.** The constant of integration is taken such that $\lim_{|p|\to\infty}\mathcal{G}(p) = 0$.

**Definition 4.1** ([2])**.** A function $f$ is called periodic if there exists a constant $T > 0$ for which $f(t+T) = f(t)$ for all $t \geq 0$. The smallest value of $T$ for which the property holds is called the fundamental period, or simply, the period.

The following theorem concerning periodic functions:

**Theorem 4.9** (Periodic Property)**.** *Let $f$ be piecewise continuous on $t \geq 0$ and of $O(e^{c_0 t})$. If $f$ is also periodic with period $T$, then*

$$\mathcal{T}_a\{f(t), t \to p\} = \frac{\phi(p)}{1 - a^{-\psi(p)T}} \int_0^T f(t) a^{-\psi(p)t}\, dt.$$

*Proof.* We can write the new integral transform as

$$\mathcal{T}_a\{f(t), t \to p\} = \phi(p) \int_0^T f(t) a^{-\psi(p)t}\, dt + \phi(p) \int_T^\infty f(t) a^{-\psi(p)t}\, dt.$$

By substituting $t = u + T$ in second integral, we get

$$\begin{aligned}\mathcal{T}_a\{f(t), t \to p\} &= \phi(p) \int_0^T f(t) a^{-\psi(p)t}\, dt + \phi(p) \int_0^\infty f(u+T) a^{-\psi(p)(u+T)}\, du \\ &= \phi(p) \int_0^T f(t) a^{-\psi(p)t}\, dt + \phi(p) a^{-\psi(p)T} \int_0^\infty f(u) a^{-\psi(p)u}\, du \\ &= \phi(p) \int_0^T f(t) a^{-\psi(p)t}\, dt + a^{-\psi(p)T} \mathcal{T}_a\{f(t), t \to p\}\, dt.\end{aligned}$$

Solving for $\mathcal{T}_a\{f(t), t \to p\}$ yields

$$(1 - a^{-\psi(p)T}) \mathcal{T}_a\{f(t), t \to p\} = \phi(p) \int_0^T f(t) a^{-\psi(p)t}\, dt$$

implies

$$\mathcal{T}_a\{f(t), t \to p\} = \frac{\phi(p)}{1 - a^{-\psi(p)T}} \int_0^T f(t) a^{-\psi(p)t}\, dt.$$

□

**Definition 4.2** ([2])**.** The Laplace convolution integral of two functions $f$ and $g$ is denoted by $(f * g)(t)$ and defined as

$$(f * g)(t) = \int_0^t f(t-u) g(u)\, du.$$

The following theorem presents a result concerning convolution integral:

**Theorem 4.10** (Convolution Theorem)**.** *If $f$ and $g$ are piecewise continuous functions on $t \geq 0$ and are of $O(e^{c_0 t})$, and if $\mathcal{F}(p)$ and $\mathcal{G}(p)$ are the new modified integral transforms, respectively, of $f(t)$ and $g(t)$, then*

$$\mathcal{T}_a\{(f * g)(t), t \to p\} = \frac{\mathcal{F}(p)\mathcal{G}(p)}{\phi(p)}.$$

*Proof.* In order to derive the convolution theorem, let us begin by recalling the definition of convolution integral and transform

$$\begin{aligned}\mathcal{T}_a\{(f * g)(t), t \to p\} &= \phi(p) \int_0^\infty (f * g)(t) a^{-\psi(p)t}\, dt \\ &= \phi(p) \int_{t=0}^\infty \left\{ \int_{u=0}^t f(t-u) g(u)\, du \right\} a^{-\psi(p)t}\, dt. \end{aligned} \tag{4.13}$$

Changing the order of integration in (4.13), we get

$$\mathcal{T}_a\{(f * g)(t), t \to p\} = \phi(p) \int_{u=0}^\infty g(u) \left\{ \int_{t=u}^\infty f(t-u) a^{-\psi(p)t}\, dt \right\} du. \tag{4.14}$$

Put $t-u=z$, one can have $dt=dz$ and $t=u \implies z=0$, and $t=\infty \implies z=\infty$. Hence, we have

$$\begin{aligned}
\mathcal{T}_a\{(f*g)(t),t\to p\} &= \phi(p)\int_{u=0}^{\infty} g(u)\left\{\int_{z=0}^{\infty} f(z)a^{-\psi(p)(u+z)}\,dz\right\}du \\
&= \int_{u=0}^{\infty} g(u)a^{-\psi(p)u)}\left\{\phi(p)\int_{z=0}^{\infty} f(z)a^{-\psi(p)z}\,dz\right\}du \\
&= \int_{u=0}^{\infty} g(u)a^{-\psi(p)u}\mathcal{F}(p)\,du \\
&= \mathcal{F}(p)\frac{1}{\phi(p)}\left\{\phi(p)\int_{u=0}^{\infty} g(u)a^{-\psi(p)u}\right\}du \\
&= \frac{\mathcal{F}(p)}{\phi(p)}\mathcal{G}(p) \\
&= \frac{\mathcal{F}(p)\mathcal{G}(p)}{\phi(p)},
\end{aligned} \tag{4.15}$$

which proves the required result. □

## 5. Example

In this section, we present several problems to illustrate the applications of the transform:

### 5.1 Solution of Ordinary Differential Equation With Initial Conditions

**Example 5.1.** Solve the following ordinary differential equation of third-order.

$$y'''+2y''+2y'+3y=\sin t+\cos t \tag{5.1}$$

with initial conditions

$$y(0)=y''(0)=0,\quad y'(0)=1 \tag{5.2}$$

**Solution.** By taking new modified integral transform on both sides of equation (5.1), we get

$$\begin{aligned}
&(\psi(p)\log a)^3\mathcal{Y}(p)-\phi(p)(\psi(p)\log a)^2y(0)-\phi(p)(\psi(p)\log a)y'(0)-\phi(p)y''(0)\\
&+2((\psi(p)\log a)^2\mathcal{Y}(p)-\phi(p)(\psi(p)\log a)y(0)-\phi(p)y'(0))+2((\psi(p)\log a)\mathcal{Y}(p)-\phi(p)y(0))+3\mathcal{Y}(p)\\
&=\frac{\phi(p)}{(\psi(p)\log a)^2+1}+\frac{\phi(p)\psi(p)\log a}{(\psi(p)\log a)^2+1}.
\end{aligned}$$

The use of equation (5.2) leads to the solution for $\mathcal{Y}(p)$ as

$$\mathcal{Y}(p)=\frac{\phi(p)[(\psi(p)\log a)^3+2(\psi(p)\log a)^2+2(\psi(p)\log a)+3]}{((\psi(p)\log a)^2+1)((\psi(p)\log a)^3+2(\psi(p)\log a)^2+2(\psi(p)\log a)+3)}$$

or

$$\mathcal{Y}(p)=\frac{\phi(p)}{(\psi(p)\log a)^2+1}.$$

Inverting gives the solution

$$y(t)=\sin t.$$

**Example 5.2.** Solve the Volterra integral equation

$$y(t)=t+\int_0^t y(u)\sin(t-u)\,du. \tag{5.3}$$

**Solution.** By taking new modified integral transform on both sides of equation (5.3), we get

$$\mathcal{Y}(p) = \frac{\phi(p)}{(\psi(p)\log a)^2} + \frac{1}{\phi(p)}\mathcal{T}_a\{y(t)\}\mathcal{T}_a\{\sin t\}$$
$$= \frac{\phi(p)}{(\psi(p)\log a)^2} + \frac{\mathcal{Y}(p)}{(\psi(p)\log a)^2+1}$$

or

$$\mathcal{Y}(p) = \frac{\phi(p)((\psi(p)\log a)^2+1)}{(\psi(p)\log a)^4}$$
$$= \frac{\phi(p)}{(\psi(p)\log a)^2} + \frac{\phi(p)}{(\psi(p)\log a)^4}.$$

Inversion yields the solution

$$y(t) = t + \frac{t^3}{6}.$$

### 5.2 Solution of Partial Integro-Differential Equation using New Modified Integral Transform

In this case, the function to be transformed involves more than one independent variable. Hence, it is convenient to use a special notation to specify the variable undergoing transformation. For example, the new modified integral transforms of $u(x,t)$ with respect to $t$ is defined

$$\mathcal{T}_a\{u(x,t), t \to p\} = \phi(p)\int_0^\infty u(x,t)a^{-\psi(p)t}\,dt = \mathcal{U}(x,p). \tag{5.4}$$

Similarly, we have that

$$\mathcal{T}_a\{u_t(x,t), t \to p\} = (\psi(p)\log a)\mathcal{U}(x,p) - \phi(p)u(x,0), \tag{5.5}$$
$$\mathcal{T}_a\{u_{tt}(x,t), t \to p\} = (\psi(p)\log a)^2\mathcal{U}(x,p) - \phi(p)(\psi(p)\log a)u(x,0) - \phi(p)u_t(x,0) \tag{5.6}$$

and

$$\mathcal{T}_a\{u_x(x,t), t \to p\} = \phi(p)\int_0^\infty u_x(x,t)a^{-\psi(p)t}\,dt$$
$$= \frac{\partial}{\partial x}\phi(p)\int_0^\infty u(x,t)a^{-\psi(p)t}\,dt$$
$$= \mathcal{U}_x(x,p). \tag{5.7}$$

The above results are used to solve the partial differential equations.

**Example 5.3.** Solve

$$u_{tt} = u_x + 2\int_0^t (t-s)u(x,s)\,ds - 2e^x \tag{5.8}$$

with initial conditions

$$u(x,0) = e^x, \quad u_t(x,0) = 0 \tag{5.9}$$

and boundary condition

$$u(0,t) = \cos t. \tag{5.10}$$

**Solution.** By taking new modified integral transform on both sides of eq. (5.8) with respect to $t$, we get

$$(\psi(p)\log a)^2\mathcal{U}(x,p) - \phi(p)(\psi(p)\log a)u(x,0) - \phi(p)u_t(x,0)$$

$$= \frac{d}{dx}\mathcal{U}(x,p) + 2\frac{1}{\phi(p)}\mathcal{U}(x,p)\frac{\phi(p)}{(\psi(p)\log a)^2} - 2e^x\frac{\phi(p)}{\psi(p)\log a}.$$

Now by applying initial conditions, we get

$$\frac{d}{dx}\mathcal{U}(x,p) + \left[\frac{2}{(\psi(p)\log a)^2} - (\psi(p)\log a)^2\right]\mathcal{U}(x,p) = \frac{2e^x\phi(p)}{\psi(p)\log a} - \phi(p)(\psi(p)\log a)e^x. \quad (5.11)$$

This is linear differential equation in $\mathcal{U}(x,p)$,

$$\text{Integrating Factor} = e^{\left[\frac{2}{(\psi(p)\log a)^2} - (\psi(p)\log a)^2\right]x}.$$

The general solution of equation (5.11) is

$$\mathcal{U}(x,p) = \frac{\phi(p)(\psi(p)\log a)e^x}{(\psi(p)\log a)^2+1} + Ce^{-\left[\frac{2}{(\psi(p)\log a)^2} - (\psi(p)\log a)^2\right]x}, \quad (5.12)$$

where $C$ is constant of integration. Since from equation (5.10), we get

$$\mathcal{U}(0,p) = \frac{\phi(p)(\psi(p)\log a)}{(\psi(p)\log a)^2+1}$$

which by using equation (5.12), we get $C = 0$ and

$$\mathcal{U}(x,p) = \frac{\phi(p)(\psi(p)\log a)e^x}{(\psi(p)\log a)^2+1}.$$

Inversion yields the solution of equation (5.8) as,

$$u(x,t) = e^x\cos t.$$

**Example 5.4.** Solve

$$xu_x = u_{tt} + x\sin t + \int_0^t \sin(t-s)u(x,s)\,ds \quad (5.13)$$

with initial conditions

$$u(x,0) = 0, \quad u_t(x,0) = x \quad (5.14)$$

and boundary condition

$$u(1,t) = t. \quad (5.15)$$

**Solution.** By taking new modified integral transform on both sides of eq. (5.13) with respect to $t$, we get

$$x\frac{d}{dx}\mathcal{U}(x,p) = (\psi(p)\log a)^2\mathcal{U}(x,p) - \phi(p)(\psi(p)\log a)u(x,0) - \phi(p)u_t(x,0)$$
$$+ x\frac{\phi(p)}{(\psi(p)\log a)^2+1} + \frac{1}{\phi(p)}\frac{\phi(p)}{(\psi(p)\log a)^2+1}\mathcal{U}(x,p).$$

Now by applying initial conditions, we get

$$x\frac{d}{dx}\mathcal{U}(x,p) = (\psi(p)\log a)^2\mathcal{U}(x,p) - \phi(p)x + x\frac{\phi(p)}{(\psi(p)\log a)^2+1} + \frac{\mathcal{U}(x,p)}{(\psi(p)\log a)^2+1}. \quad (5.16)$$

The general solution of equation (5.16) is

$$\mathcal{U}(x,p) = Cx^{\left[(\psi(p)\log a)^2 + \frac{1}{(\psi(p)\log a)^2+1}\right]} + x\frac{\phi(p)}{(\psi(p)\log a)^2}, \quad (5.17)$$

where $C$ is constant of integration. Since from eq. (5.15), we get

$$\mathcal{U}(1,p)=\frac{\phi(p)}{(\psi(p)\log a)^2}$$

which by using equation (5.17), we get $C=0$ and

$$\mathcal{U}(x,p)=x\frac{\phi(p)}{(\psi(p)\log a)^2}.$$

Inversion yields the solution of eq. (5.13) as,

$$u(x,t)=xt.$$

## 5.3 Solution of Partial Differential Equation With Initial and Boundary Conditions

**Example 5.5** (First-Order Initial-Boundary Value Problem)**.** Solve the equation

$$u_t+u_x+u=0 \tag{5.18}$$

with initial and boundary conditions

$$u(x,0)=0, \tag{5.19}$$

$$u(0,t)=\sin t. \tag{5.20}$$

**Solution.** By taking new modified integral transform on both sides of eqs. (5.18) and (5.20) with respect to $t$, and by using equation (5.19), we get

$$\frac{d}{dx}\mathcal{U}(x,p)+(\psi(p)\log a+1)\mathcal{U}(x,p)=0, \tag{5.21}$$

$$\mathcal{U}(0,p)=\frac{\phi(p)}{(\psi(p)\log a)^2+1}. \tag{5.22}$$

Using the integrating factor $a^{\psi(p)x}e^x$, the solution of eq. (5.21) is

$$\mathcal{U}(x,p)=Ca^{-\psi(p)x}e^{-x}$$

where $C$ is constant of integration. Since from eq. (5.22), $C=\frac{\phi(p)}{(\psi(p)\log a)^2+1}$ for bounded solution. Consequently,

$$\mathcal{U}(x,p)=\frac{\phi(p)}{(\psi(p)\log a)^2+1}a^{-\psi(p)x}e^{-x}.$$

By using the Second shifting property of new modified integral transform, inversion yields the solution of eq. (5.18) as

$$u(x,t)=e^{-x}\sin(t-x)H(t-x).$$

**Example 5.6** (Second-Order Initial-Boundary Value Problem)**.** Solve the equation

$$u_t=u_{xx} \tag{5.23}$$

with initial and boundary conditions

$$u(x,0)=3\sin 2\pi x, \tag{5.24}$$

$$u(0,t)=0, \tag{5.25}$$

$$u(1,t)=0, \quad \text{where } \ 0<x<1,\, t>0. \tag{5.26}$$

**Solution.** By taking new modified integral transform on both sides of eqs. (5.23), (5.25) and (5.26) with respect to $t$, and by using eq. (5.24), we get

$$\frac{d^2}{dx^2}\mathcal{U}(x,p)-(\psi(p)\log a)\mathcal{U}(x,p)=-3\phi(p)\sin 2\pi x, \tag{5.27}$$

$$\mathcal{U}(0,p)=0 \quad\text{and}\quad \mathcal{U}(1,p)=0. \tag{5.28}$$

The general solution of eq. (5.27) is

$$\mathcal{U}(x,p)=C_1 e^{(\psi(p)\log a)x}+C_2 e^{-(\psi(p)\log a)x}+\frac{3\phi(p)\sin 2\pi x}{4\pi^2+\psi(p)\log a},$$

where $C_1$ and $C_2$ are constants of integration. Since from eq. (5.28), $C_1=0$ and $C_2=0$ for bounded solution. Consequently,

$$\mathcal{U}(x,p)=3\frac{3\phi(p)\sin 2\pi x}{4\pi^2+\psi(p)\log a}.$$

Inversion yields the solution of equation (5.23) as

$$u(x,t)=3e^{-4\pi^2 t}\sin 2\pi x.$$

## 5.4 Solution of System of Ordinary Differential Equations

**Example 5.7** (System of First-Order Ordinary Differential Equations)**.** Solve the system

$$x'=3x+4y, \tag{5.29}$$

$$y'=2x+y, \tag{5.30}$$

with initial conditions

$$x(0)=1,\quad y(0)=0. \tag{5.31}$$

**Solution.** By taking new modified integral transform on both sides of eqs. (5.29) and (5.30) with respect to $t$, we get

$$(\psi(p)\log a-3)\mathcal{X}(p)-4\mathcal{Y}(p)=\phi(p),$$

$$-2\mathcal{X}(p)+(\psi(p)\log a-1)\mathcal{Y}(p)=0.$$

After solving these equations, we get

$$\begin{aligned}\mathcal{X}(p)&=\frac{\phi(p)(\psi(p)\log a-1)}{(\psi(p)\log a-3)(\psi(p)\log a-1)-8}\\ &=\frac{\phi(p)(\psi(p)\log a-1)}{(\psi(p)\log a)^2-4(\psi(p)\log a)-5}\\ &=\frac{\phi(p)(\psi(p)\log a-1)}{(\psi(p)\log a-5)(\psi(p)\log a+1)}\\ &=\frac{\frac{2}{3}\phi(p)}{(\psi(p)\log a-5)}+\frac{\frac{1}{3}\phi(p)}{(\psi(p)\log a+1)}.\end{aligned}$$

Hence, the inversion yields,

$$x(t)=\frac{2}{3}e^{5t}+\frac{1}{3}e^{-t}$$

and from eq. (5.29), we get

$$y(t)=\frac{1}{3}e^{5t}-\frac{1}{3}e^{-t}.$$

**Example 5.8** (System of Second-Order Ordinary Differential Equations)**.** Solve the system

$$x''=\frac{1}{2}(-x+y), \tag{5.32}$$

$$y''=\frac{1}{2}(x-y), \tag{5.33}$$

with initial conditions

$$x(0)=0,\quad x'(0)=2,\quad y(0)=0,\quad y'(0)=0. \tag{5.34}$$

**Solution.** By taking new modified integral transform on both sides of eqs. (5.32) and (5.33) with respect to $t$, we get

$$\left((\psi(p)\log a)^2+\frac{1}{2}\right)\mathcal{X}(p)-\frac{1}{2}\mathcal{Y}(p)=2\phi(p),$$

$$-\frac{1}{2}\mathcal{X}(p)+\left((\psi(p)\log a)^2+\frac{1}{2}\right)\mathcal{Y}(p)=0.$$

After solving these equations, we get

$$\begin{aligned}\mathcal{X}(p)&=\frac{2\phi(p)\left((\psi(p)\log a)^2+\frac{1}{2}\right)}{\left((\psi(p)\log a)^2+\frac{1}{2}\right)^2-\frac{1}{4}}\\&=\frac{2\phi(p)\left((\psi(p)\log a)^2+\frac{1}{2}\right)}{(\psi(p)\log a)^2\left((\psi(p)\log a)^2+1\right)}\\&=\frac{\phi(p)}{(\psi(p)\log a)^2}+\frac{\phi(p)}{(\psi(p)\log a)^2+1}.\end{aligned}$$

Hence, the inversion yields

$$x(t)=t+\sin t$$

and from eq. (5.32), we get

$$y(t)=t-\sin t.$$

## 5.5 Evaluation of Definite Integrals Using New Modified Integral Transform

**Example 5.9.** Evaluate the integral

$$\int_0^\infty \frac{\sin t}{t}\,dt.$$

**Solution.** We have

$$\mathcal{T}_a\{\sin t\}=\frac{\phi(p)}{(\psi(p)\log a)^2+1},$$

$$\mathcal{T}_a\left\{\frac{\sin t}{t}\right\}=\phi(p)\log a\int_p^\infty \frac{\psi'(p)}{(\psi(p)\log a)^2+1}\,dp. \tag{5.35}$$

Changing the variable by the substitution $\psi(p)\log a=x$ in eq. (5.35), we have

$$\mathcal{T}_a\left\{\frac{\sin t}{t}\right\}=\frac{\phi(p)\log a}{\log a}\int_{\psi(p)\log a}^\infty \frac{1}{x^2+1}\,dx$$

$$= \phi(p)[\tan^{-1} x]^{\infty}_{\psi(p)\log a}$$
$$= \phi(p)\left[\frac{\pi}{2} - \tan^{-1}(\psi(p)\log a)\right].$$

Now by definition of new modified integral transform

$$\phi(p)\int_0^\infty a^{-\psi(p)t}\frac{\sin t}{t}\,dt = \phi(p)\left[\frac{\pi}{2} - \tan^{-1}(\psi(p)\log a)\right].$$

By setting $\psi(p) = 0$, in above result, we obtain

$$\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}.$$

**Example 5.10.** Evaluate the integral

$$g(t) = \int_0^\infty \frac{\sin tx}{x(a^2+x^2)}\,dx.$$

**Solution.** By taking new modified integral transform on both sides with respect to $t$ and changing the order of integration, we get

$$\begin{aligned}
\mathcal{T}_a\{g(t)\} &= \int_0^\infty \frac{\mathcal{T}_a\{\sin tx\}}{x(a^2+x^2)}\,dx \\
&= \phi(p)\int_0^\infty \frac{1}{(a^2+x^2)((\psi(p)\log a)^2+x^2)}\,dx \\
&= \frac{\phi(p)}{((\psi(p)\log a)^2+a^2)}\int_0^\infty \left[\frac{1}{(a^2+x^2)} - \frac{1}{((\psi(p)\log a)^2+x^2)}\right]dx \\
&= \frac{\phi(p)}{((\psi(p)\log a)^2+a^2)}\frac{\pi}{2}\left[\frac{1}{a} - \frac{1}{\psi(p)\log a}\right] \\
&= \frac{\pi}{2a}\frac{\phi(p)}{(\psi(p)\log a + a)}\frac{1}{\psi(p)\log a} \\
&= \frac{\pi\phi(p)}{2a}\left[\frac{\frac{1}{-a}}{\psi(p)\log a + a} + \frac{\frac{1}{a}}{\psi(p)\log a}\right] \\
&= \frac{\pi}{2}\left[\frac{\phi(p)}{\psi(p)\log a} - \frac{\phi(p)}{\psi(p)\log a + a}\right].
\end{aligned} \tag{5.36}$$

Inversion yields the solution

$$f(t) = \frac{\pi}{2}(1 - e^{-at}).$$

## 6. Conclusion

The new modified integral transform is introduced as a powerful tool for solving both differential and integral equations. This transform extends the class of Laplace transforms by encompassing a wide range of cases through different choices of $\phi(p)$ and $\psi(p)$. We established the existence theorem for the newly defined transform and derived nearly all of its fundamental properties, closely analogous to those of the Laplace transform. In addition, transforms of elementary functions are presented, and suitable examples are provided to illustrate and validate the results.

However, the theory of residues and the inversion theorem cannot be applied to new modified integral transform in the same direct manner as with Laplace transform, because the kernel is generalized (depends on general functions $\phi(p)$ and $\psi(p)$), the analytic structure is complex, and the necessary contour integral representation may not exist or be too complicated.

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The authors declare that they have no competing interests.

### Authors' Contributions

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## References

**[1]** K. S. Aboodh, The new integral transformation "Aboodh transform", *Global Journal of Pure and Applied Mathematics* **9**(1) (2013), 35 – 43.

**[2]** L. C. Andrews and B. K. Shivamoggi, *Integral Transforms for Engineers*, SPIE Press, 353 pages (1999).

**[3]** A. Ansari, The generalized Laplace transform and fractional differential equations of distributed orders, *Differential Equations and Control Processes* **3** (2012), 128 – 138, URL: https://diffjournal.spbu.ru/pdf/alireza.pdf.

**[4]** F. B. M. Belgacem and A. A. Karaballi, Sumudu transform fundamental properties investigations and applications, *Journal Applied Mathematics and Stochastic Analysis* **2006** (2006), article ID 091083, DOI: 10.1155/JAMSA/2006/91083.

**[5]** F. B. M. Belgacem and R. Silambarasan, Theory of natural transform, *Journal of Mathematics in Engineering, Science and Aerospace* **3**(1) (2012), 105 – 135.

**[6]** T. M. Elzaki, The new integral transform "Elzaki transform", *Global Journal of Pure and Applied Mathematics* **7**(1) (2011), 57 – 64.

**[7]** A. I. El-Mesady, Y. S. Hamed and A. M. Alsharif, Jafari transformation for solving a system of ordinary differential equations with medical application, *Fractal and Fractional* **5**(3) (2021), 130, DOI: 10.3390/fractalfract5030130.

**[8]** R. Gupta, R. Gupta and D. Verma, Propounding a new integral transform: Gupta transform with applications in science and engineering, *International Journal of Scientific Research in Multidisciplinary Studies* **6**(3) (2020), 14 – 19.

**[9]** R. Gupta, On novel integral transform: Rohit Transform and its application to boundary value problems, *ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences* **4**(1) (2020), 8 – 13.

**[10]** H. Jafari, A new general integral transform for solving integral equations, *Journal of Advanced Research* **32** (2021), 133 – 138, DOI: 10.1016/j.jare.2020.08.016.

**[11]** A. Kamal and H. Sedeeg, The new integral transform "Kamal transform", *Advances in Theoretical and Applied Mathematics* **11**(4) (2016), 451 – 458.

**[12]** G. Rudolf and S. Vessella, Relations between Abel transform and other integral transforms, in: *Abel Integral Equation*, Lecture Notes in Mathematics series, Vol. 1461, Springer, Berlin — Heidelberg, (1991), DOI: 10.1007/BFb0084672.

[13] J. Zhao, Robust image watermarking algorithm based on radon and analytic Fourier-Mellin transforms, *The Open Automation and Control Systems Journal* **7** (2015), 1071 – 1074, DOI: 10.2174/1874444301507011071.