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Research Article

A Study on Bipolar Soft *b*-Metric Spaces

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Abstract. In this work, bipolar soft b-metric space is introduced, which is constructed from b-metric space and several soft point sets. Also, using this framework, several fixed point theorems illustrated using bipolar soft contractive mappings and offer significant clarifications on this idea through medical applications.

Keywords. b-Metric spaces, Contraction mappings, Bipolar soft metric spaces

Mathematics Subject Classification (2020). 47H10, 54H25

1. Introduction

In contemporary mathematical analysis, the creation of generalized metric spaces has been essential, especially for the investigation of fixed point theory, decision-making, and optimization issues. Since Maurice Fréchet [12] initially codified traditional metric spaces in 1906, numerous authors have expanded this classical result in a number of ways to support increasingly intricate structures (Das and Samanta [7], Dhage [8], Gunduz (Aras) and Posul [13], Matthews [16], Mustafa and Sims [18], Mutlu and Gürdal [19], Öztunç *et al*. [20]). Bakhtin's [3] introduction of the b-metric space, which generalizes the classical triangle inequality by adding a constant factor, is one example of this expansion. The b-metric spaces have been widely used in nonlinear analysis since Czerwik [6] conducted additional research on them.

At the same time, Molodtsov's [17] invention of soft set theory offered a potent mathematical tool for managing imprecision and uncertainty in data. Researchers created soft metric spaces — which provide a more flexible description of complicated systems— by fusing metric spaces with soft set theory. Bipolar soft b-metric spaces are the result of the recent incorporation of the idea of bipolarity, which takes into account both positive and negative elements of information, into these frameworks. A more comprehensive framework for handling real-world issues involving complimentary or conflicting factors is offered by this new structure.

The basic characteristics of bipolar soft b-metric spaces, such as their completeness, convergence, and contractive mappings, are examined in this research. We expand on the work of prominent mathematicians, e.g., Kannan [14], whose contraction mappings have been extensively investigated in generalized metric spaces, and S. Banach [4] whose renowned Banach contraction principle forms the basis of fixed point theory (Abbas *et al*. [1], Ali *et al*. [2], Dhawan and Grewal [9, 10], Dhawan and Tripti [11]). Additionally, we investigate the use of bipolar soft b-metric spaces in decision-making issues, highlighting their applicability in domains including data analysis, optimization, and artificial intelligence.

This study's importance stems from its capacity to integrate and expand on current mathematical frameworks while offering useful applications for managing uncertainty. A new method is presented that improves theoretical comprehension and practical usability by introducing bipolarity and soft set theory into b-metric spaces.

2. Preliminaries

This part gives the study's background information and goes over some of the main concepts from soft set theory.

Definition 2.1 ([15])**.** A pair (Λ, M) is a soft set over Υ, where Λ is a mapping described by $\Lambda : \Upsilon \to P(\Upsilon)$. In other words, the soft set can be defined as a parameterized family of set Υ subsets. $\Lambda(\varrho)$ can be viewed as the set of e-elements or e-approximate elements of the soft set (Λ, M) for any e in the set.

Definition 2.2 ([15])**.** If $\forall\ \varrho \in M$, $H(\varrho) \subseteq D(\varrho)$, then (H, M) is a soft subset of $(D, M)$for two soft sets (H, M) and (D, M) over Υ. The relationship's emblem is $(D, M) \subseteq (H, M)$.
In a similar vein, if (D, M) is a soft subset of (H, M), then (H, M) is known as a soft superset of (D, M).
The relationship's emblem is $(D, M) \supseteq (H, M)$. If (H, M) is a soft subset of (D, M) and (H, M) is a soft subset of (H, M), then two soft sets (H, M) and (D, M) over Υ are said to be soft equal.

Definition 2.3 ([15])**.** The soft set (Γ, M) where $\forall\ \varrho \in M$, $\Gamma(\varrho) = H(\varrho) \cap D(\varrho)$ is the intersection of two soft sets (H, M) and (D, M) over Υ.
The other words, $(H, M) \cap (D, M) = (\Gamma, M)$.

Definition 2.4 ([15])**.** The soft set (Γ, M) where $\forall\ \varrho \in M$, $\Gamma(\varrho) = H(\varrho) \cup D(\varrho)$ is the union of two soft sets (H, M) and (D, M) over Υ.
The other words, $(H, M) \cup (D, M) = (\Gamma, M)$.

Definition 2.5 ([5])**.** If for every $\varrho \in M$, $\Lambda(\varrho) = \emptyset$, then a soft set (Λ, M) over Υ is a null soft set, denoted by Φ.

Definition 2.6 ([5]). If $\Lambda(\varrho) = \Upsilon$ for all $\varrho \in M$, then a soft set (Λ, M) over Υ is an absolute soft set represented by Υ.

Definition 2.7 ([5]). For any $\varrho \in M$, the difference (Γ, M) between two soft sets $(\mathcal{H}, M)$ and (D, M) over Υ is denoted by $(H, M)\backslash(D, M)$.

Definition 2.8 ([17]). A soft set (Λ, M) is represented by its complement, $(\Lambda, M)^c$, which is defined as $(\Lambda, M)^c = (\Lambda^c, M)$ where $\mho^c : M \to P(\Upsilon)$ is a mapping given by $\mho^c(\varrho) = \Upsilon \backslash \mho(\varrho)$ for all $\varrho \in M$ and $\mho^c$ is known as the soft complement function of $\mho$.

Definition 2.9 ([17]). Assume that the soft set (Λ, M) is over Υ. A soft point, represented by (u, M), is defined as the soft set (Λ, M) if, for any element $\varrho \in M$, $\mho(\varrho) = \{u\}$ and $\mho(\varrho') = \emptyset$ for all $\varrho' \in M - \{\varrho\}$.

Definition 2.10 ([5]). Assume that the soft topological space over Υ is (Υ, ψ, M). If there is a soft open set (H, M) such that $u_k \in (H, M) \subseteq u_k \in (\Lambda, M)$, then a soft set $(\Lambda, M) \subseteq (\Upsilon, M)$ is referred to as a soft neighborhood of the soft point $u_k \in (\Lambda, M)$.

Definition 2.11 ([15]). Let Λ represent the set of all real numbers, $B(\Lambda)$ represent the set of all bounded non-empty subsets of Λ, and let M represent a set of parameters.
A mapping $\mho : M \to B(\Lambda)$ is then referred to as a soft real set. The symbol for it is (Λ, M). (Λ, M) is referred to as a soft real number and represented by t, ϖ, m etc., if it is a singleton soft set. In this case, t, ϖ, and m will stand for a certain class of soft real numbers such that, for every ϱ in M, $t(\varrho) = \varrho$.
0 and 1 are the soft real numbers, where $0(\varrho) = 0$, $1(\varrho) = 1$ for all $\varrho \in M$, respectively.

Definition 2.12 ([15]). Assuming t and ϖ to be two soft real values, the following claims are true:

(i) $t \leq \varpi$, if $t(\varrho) \leq \varpi(\varrho)$ for all $\varrho \in M$,

(ii) $t \geq \varpi$, if $t(\varrho) \geq \varpi(\varrho)$ for all $\varrho \in M$,

(iii) $t < \varpi$, if $t(\varrho) < \varpi(\varrho)$ for all $\varrho \in M$,

(iv) $t > \varpi$, if $t(\varrho) > \varpi(\varrho)$ for all $\varrho \in M$.

Definition 2.13 ([5]). Let $\mathring{A}(\Upsilon)$ and $\mathring{A}(\Lambda)$ be two non-empty sets and $\varpi : \mathring{A}(D) \times \mathring{A}(\Lambda) \to [0, \infty)$ be a function which satisfies the following:

(1) If $\varpi(\vartheta_k, \varrho_{k'}) = 0$, then $\vartheta_k = \varrho_{k'}$ for all $(\vartheta_k, \varrho_{k'}) \in \mathring{A}(D) \times \mathring{A}(\Lambda)$.

(2) If $\zeta_k = \varrho_{k'}$, then $\varpi(\vartheta_k, \varrho_{k'}) = 0$ for all $(\vartheta_k, \varrho_{k'}) \in \mathring{A}(D) \times \mathring{A}(\Lambda)$.

(3) $\varpi(\vartheta_k, \varrho_{k'}) = \varpi(\varrho_{k'}, \vartheta_k)$ for all $(\vartheta_k, \varrho_{k'}) \in \mathring{A}(D) \cap \mathring{A}(\Lambda)$.

(4) $\varpi(\vartheta^1_{k_1}, \varrho^2_{k'_2}) \leq [\varpi(\vartheta^1_{k_1}, \varrho^1_{k'_1}) + \varpi(\vartheta^2_{k_2}, \varrho^1_{k'_1}) + \varpi(\vartheta^2_{k_2}, \varrho^2_{k'_2})]$ for all $\vartheta^1_{k_1}, \vartheta^2_{k_2} \in \mathring{A}(D)$, $\varrho^1_{k'_1}, \varrho^2_{k'_2} \in \mathring{A}(\Lambda)$.

Then ϖ is called a bipolar metric on $(\mathring{A}(D), \mathring{A}(\Lambda))$ and the pair $(\mathring{A}(D), \mathring{A}(\Lambda), d)$ is called a bipolar metric space.

Definition 2.14 ([5]). Let $\Lambda(M)^*$ be the set of all non-negative soft real numbers, and let $\mathring{A}(D), \mathring{A}(\Lambda)$ be two non-empty soft sets of soft points on U and Y, respectively. Let ϖ be a function between $\varpi : \mathring{A}(D) \times \mathring{A}(\Lambda) \to \Lambda(M)^*$. Take into account these attributes:

(BO1) If $\varpi(\vartheta_k, \varrho_{k'}) = 0$, then $\vartheta_k = \varrho_{k'}$ for all $(\vartheta_k, \varrho_{k'}) \in \mathring{A}(D) \times \mathring{A}(\Lambda)$.

(BO2) If $\vartheta_k = \varrho_{k'}$, then $\varpi(\vartheta_k, \varrho_{k'}) = 0$ for all $(\vartheta_k, \varrho_{k'}) \in \mathring{A}(D) \times \mathring{A}(\Lambda)$.

(BO3) $\varpi(\vartheta_k, \varrho_{k'}) = \varpi(\varrho_{k'}, \vartheta_k)$ for all $(\vartheta_k, \varrho_{k'}) \in \mathring{A}(D) \cap \mathring{A}(\Lambda)$.

(BO4) $\varpi(\vartheta^1_{k_1}, \varrho^2_{k'_2}) \le [\varpi(\vartheta^1_{k_1}, \varrho^1_{k'_1}) + \varpi(\vartheta^2_{k_2}, \varrho^1_{k'_1}) + \varpi(\vartheta^2_{k_2}, \varrho^2_{k'_2})]$ for all $\vartheta^1_{k_1}, \vartheta^2_{k_2} \in \mathring{A}(D)$, $\varrho^1_{k'_1}, \varrho^2_{k'_2} \in \mathring{A}(\Lambda)$.

(i) D is bipolar soft pseudo-semimetric on $(\mathring{A}(D), \mathring{A}(\Lambda))$, if (BO2) and (BO3) holds.

(ii) If condition (BO4) is satisfied and ϖ is a bipolar soft pseudo-semimetric then it is a bipolar soft pseudo-metric.

(iii) If condition (BO1) is satisfied and ϖ is a bipolar soft pseudo-metric then it is referred to as a bipolar soft metric.

Then, $(\mathring{A}(D), \mathring{A}(\Lambda), \varpi, \Lambda(M)^*)$ is a bipolar soft metric space.

3. Contractive Mapping on Bipolar Soft Metric Spaces

We presented the idea of a bipolar soft b-metric space. Next, proven a few fixed point theorems and defined key terms, including whole bipolar soft b-metric space, bipolar soft bisequence, and Cauchy bisequence.

Definition 3.1. Let $\Lambda(M)^*$ be the set of all non-negative soft real numbers, and let $\mathring{A}(D), \mathring{A}(\Lambda)$ be two non-empty soft sets of soft points on Υ and Y, respectively. Let $\varpi : \mathring{A}(D) \times \mathring{A}(\Lambda) \to \Lambda(M)^*$ be a function, and let $b \ge 1$ be any constant. If

(BO4) $\varpi(\vartheta^1_{k_1}, \varrho^2_{k'_2}) \le b[d_\eta(\vartheta^1_{k_1}, \varrho^1_{k'_1}) + \varpi(\vartheta^2_{k_2}, \varrho^1_{k'_1}) + \varpi(\vartheta^2_{k_2}, \varrho^2_{k'_2})]$, for all $\vartheta^1_{k_1}, \vartheta^2_{k_2} \in \mathring{A}(D)$, $\varrho^1_{k'_1}, \varrho^2_{k'_2} \in \mathring{A}(\Lambda)$.

(i) If condition (BO4) is satisfied and ϖ is a bipolar soft b pseudo-semimetric then it is a bipolar soft pseudo-metric.

(ii) If condition (BO1) is satisfied and ϖ is a bipolar soft b pseudo-metric then it is referred to as a bipolar soft b-metric.

Then $(\mathring{A}(D), \mathring{A}(\Lambda), \varpi, \Lambda(M)^*)$ is a bipolar soft b-metric space.

Example 3.2. Let $X = \Lambda$ and $Y = [-5, 5]$ and $M = G$ be a non-empty set of parameters. Next, let us establish a mapping,

$$\varpi : \mathring{A}(D) \times \mathring{A}(\Lambda) \to \Lambda(M)^*$$

by

$$\varpi(\vartheta_j, \varrho_l) = |\vartheta^2 - \varrho^2| + |j - l|$$

for all $\vartheta_j \in \mathring{A}(D)$ and $\varrho_l \in \mathring{A}(\Lambda)$.

On pair $\mathring{A}(D) \times \mathring{A}(\Lambda)$, it is evident that ϖ is a bipolar soft metric.

Definition 3.3. Let $p : (\Upsilon_1 \cup V_1, M) \to (\Upsilon_2 \cup V_2, M)$ be a soft function. Let $(\Upsilon_1, V_1, d_{\eta_1}, M)$ and $(\Upsilon_2, V_2, d_{\eta_2}, M)$ be two bipolar soft pseudo-semimetric spaces. The p soft mapping from $(\Upsilon_1, V_1, d_{\eta_1}, M)$ to $(\Upsilon_2, V_2, d_{\eta_2}, M)$ is referred to as a covariant soft mapping if $p(\Upsilon_1) \subseteq (\Upsilon_2)$ and $p(V_1) \subseteq (V_2)$ are satisfied.

Theorem 3.4. *Let $p:(\Upsilon,V,\varpi,M)\to(\Upsilon,V,\varpi,M)$ be a contraction mapping for a bipolar soft b-metric. For the soft function, $p:(\Upsilon,V,\varpi,M)\to(\Upsilon,V,\varpi,M)$ a single soft fixed point exists.*

Proof. Given that p is a mapping of bipolar soft contraction, there is a $\varrho\in(0,1)$ such that

$$\varpi(p(\vartheta_k),p(\varrho_{k'}))\le\varrho\varpi(\vartheta_k,\varrho_{k'}),\quad \text{for all } (\vartheta_k,\varrho_{k'})\in\mathring{A}(\Upsilon)\times\mathring{A}(V).$$

Let $\vartheta^0_{k_0}\in\mathring{A}(\Upsilon)$, $\varrho^0_{k'_0}\in\mathring{A}(V)$.

For each $r\in N$, define

$$p(\vartheta^r_{k_r})=\vartheta^{r+1}_{k_{r+1}},\quad p(\varrho^r_{k'_r})=\varrho^{r+1}_{k'_{r+1}}.$$

Then, $\{(\vartheta^r_{k_r},\varrho^r_{k'_r})\}$ is a soft bisequence on (Υ,V,ϖ,M). Then, for each positive integer r and w, we have

$$\begin{aligned}\varpi(\vartheta^r_{k_r},\varrho^r_{k'_r})&=\varpi(p(\vartheta^{r-1}_{k_{r-1}}),p(\varrho^{r-1}_{k'_{r-1}}))\\&\le\varrho\varpi(\vartheta^{r-1}_{k_{r-1}},\varrho^{r-1}_{k'_{r-1}})\\&\ \vdots\\&\le\varrho^r\varpi(\vartheta^0_{k_0},\varrho^0_{k'_0}).\end{aligned}$$

Also define

$$\mho=\varpi(\vartheta^0_{k_0},\varrho^0_{k'_0})+\varpi(\vartheta^0_{k_0},\varrho^0_{k'_0}).$$

Then

$$\begin{aligned}\varpi(\vartheta^r_{k_r},\varrho^{r+1}_{k'_{r+1}})&=\varpi(p(\varrho^{r-1}_{k'_{r-1}}),p(\varrho^r_{k'_r}))\\&\le\varrho\varpi(\vartheta^{r-1}_{k_{r-1}},\varrho^r_{k'_r})\\&\ \vdots\\&\le\varrho^r\varpi(\vartheta^0_{k_0},\varrho^1_{k'_1}).\end{aligned}$$

Thus,

$$\begin{aligned}\varpi(\vartheta^{r+w}_{k_{r+w}},\varrho^r_{k'_r})&\le\varpi(\vartheta^{r+w}_{k_{r+w}},\varrho^{r+1}_{k'_{r+1}})+\varpi(\vartheta^r_{k_r},\varrho^{r+1}_{k'_{r+1}})+\varpi(\vartheta^r_{k_r},\varrho^r_{k'_r})\\&\le\varpi(\vartheta^{r+w}_{k_{r+w}},\varrho^{r+1}_{k'_{r+1}})+\varrho^r(\varpi(\vartheta^0_{k_0},\varrho^1_{k'_1})+\varpi(\vartheta^0_{k_0},\varrho^0_{k'_0}))\\&\le\varpi(\vartheta^{r+w}_{k_{r+w}},\varrho^{r+1}_{k'_{r+1}})+\varrho^r\mho\\&\le\varpi(\vartheta^{r+w}_{k_{r+w}},\varrho^{r+2}_{k'_{r+2}})+\varpi(\vartheta^{r+1}_{k_{r+1}},\varrho^{r+2}_{k'_{r+2}})+\varpi(\vartheta^{r+1}_{k_{r+1}},\varrho^{r+1}_{k'_{r+1}})+\varrho^r\mho\\&\le\varpi(\vartheta^{r+w}_{k_{r+w}},\varrho^{r+2}_{k'_{r+2}})+(\varrho^{r+1}+\varrho^r)\mho\\&\ \vdots\\&\le\varpi(\vartheta^{r+w}_{k_{r+w}},\varrho^{r+w}_{k'_{r+w}})+(\varrho^{r+w-1}+\cdots+\varrho^{r+1}+\varrho^r)\mho\\&\le(\varrho^{r+w}+\cdots+\varrho^{r+1}+\varrho^r)\mho\\&\le\varrho^r\ \mho\sum_{l=0}^{\infty}\varrho^l\end{aligned}$$

$$= \frac{\varrho^r \mho}{1-\varrho}$$
$$= \wp_n.$$

Similarly, we obtain

$$\varpi(\vartheta^r_{k_r}, \varrho^{r+w}_{k'_{r+w}}) \le \wp_n.$$

Since $\wp_n \to 0$,then for each $\varepsilon > 0$, there exists $r_0 \in N$ such that

$$\wp_{r_0} < \frac{\varepsilon}{3}$$

Then

$$\begin{aligned} \varpi(\vartheta^n_{k_n}, \varrho^m_{k'_m}) &\le \varpi(\vartheta^n_{k_n}, \varrho^{m_0}_{k'_{m_0}}) + \varpi(\vartheta^{n_0}_{k_{n_0}}, \varrho^{m_0}_{k'_{m_0}}) + \varpi(\vartheta^{n_0}_{k_{n_0}}, \varrho^m_{k'_m}) \\ &\le 3\wp_{n_0} \\ &< \varepsilon \end{aligned}$$

and hence $(\vartheta^n_{k_n}, \varrho^m_{k'_m})$ is a Cauchy soft bisequence.
Since (Υ, V, ϖ, M) is complete bipolar soft b-metric space, $\{(\vartheta^n_{k_n}, \varrho^m_{k'_m})\}$ converges.
So, $z_k \in \mathring{A}(\Upsilon) \cap \mathring{A}(V)$ is a soft point to which soft bisequence bicoverges,

$$p(\varrho^n_{k'_n}) = \varrho^{n+1}_{k'_{n+1}} \to \mu_k \in \mathring{A}(\Upsilon) \cap \mathring{A}(V).$$

Hence, there exists a unique soft limit for $p\{(\varrho^n_{k'_n})\}$.
Now, using soft continuous for $p\{p(\varrho^n_{k'_n})\} \to p(\mu_k)$.
So, $p(\mu_k) = \mu_k$.
Therefore, ϖ has a soft fixed point μ_k.
If ϖ has a soft fixed point φ_k, then

$$p(\varphi_k) = \varphi_k$$
$$\Longrightarrow \quad \varphi_k \in \mathring{A}(\Upsilon) \cap \mathring{A}(V)$$

We get

$$\begin{aligned} \varpi(\mu_k, \varphi_k) &= \varpi(p(\mu_k), p(\varphi_k)) \\ &\le \varrho \varpi(\mu_k, \varphi_k), \end{aligned}$$

which implies $\varpi(\mu_k, \varphi_k) = 0$, so $\varphi_k = \mu_k$. □

4. Application: Multi-Criteria Decision-Making in Healthcare Resource Allocation

Problem Context

In healthcare, resource allocation (e.g., hospital bed distribution, medical staff assignment) involves both positive and negative criteria:

- *Positive factors*: Availability of doctors, efficiency of treatment, proximity to patients.
- *Negative factors*: Cost, waiting time, hospital congestion.

Modeling with Bipolar Soft Metric Spaces

Let

- χ be the set of hospitals $\iota_1, \iota_2, \dots, \iota_n$,
- each hospital is evaluated using a bipolar soft metric function d_s,
- the soft set is defined as (Γ^+, Γ^-), where
  - $\Gamma^+(\iota_i)$ represents the *positive membership* (e.g., quality of service),
  - $\Gamma^-(\iota_i)$ represents the *negative membership* (e.g., high cost).

Using BSMS, the government can determine which hospital provides the best trade-off between quality and cost by computing distances between hospitals using a bipolar soft metric.

Example in Bipolar Soft Metric Spaces

Let (χ, ϖ) be a bipolar soft metric space, where χ is the set of hospitals. Define the bipolar soft metric:

$$\varpi((\vartheta, \Gamma^+, \Gamma^-), (\varrho, \beta^+, \beta^-)) = \max\{|\vartheta - \varrho|, |\Gamma^+ - \beta^+|, |\Gamma^- - \beta^-|\}.$$

Consider three hospitals H_1, H_2, H_3 with the following evaluations:

Hospital	χ (Index)	Γ^+ (Quantity)	Γ^- (Cost)
ι_1	1	0.9	0.2
ι_2	2	0.7	0.4
ι_3	3	0.5	0.6

We compute the bipolar soft metric distances:

$$\varpi(\iota_1, \iota_2) = \max\{|1-2|, |0.9-0.2|, |0.2-0.4|\} = \max\{1, 0.2, 0.2\} = 1,$$

$$\varpi(\iota_2, \iota_3) = \max\{|2-3|, |0.7-0.5|, |0.4-0.6|\} = \max\{1, 0.2, 0.2\} = 1.$$

Since, H_1 has the highest positive membership ($\Gamma^+ = 0.9$) and the lowest negative membership ($\Gamma^- = 0.2$) it is the best choice for allocating resources.

5. Conclusion

Determining the distance between two points in a set has been a problem since ancient times in a number of fields, including mathematics. In addition to measuring distance, determining the separation between many locations within distinct sets is another mathematical challenge that arises in scientific research and everyday life. For tackling these issues and solving these types of distant jobs, the bipolar metric is an essential starting point. One particular example of a soft set is the idea of soft point, which is thought to be a significant and motivating tool in mathematics.

This innovative metric structure, which is produced by combining the structure of bipolar metric and soft points, is extremely important given the significance of both soft sets and bipolar metric spaces. The idea of bipolar soft metric space has been introduced in this work. Bipolar metric spaces and soft points of soft sets serve as the foundation for this theory. After that, few fixed point theorems are demonstrated and build bipolar soft b-metric space, which is based on soft points of soft sets. Being a new generalization of metric spaces, this new design is useful and offers numerous opportunities for further research into this idea.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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