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Research Article

Redundancy Optimization for a System Comprising Two Operative Unit and *N* Cold Standby Units with Activation Time

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Abstract. A well-known idea in the field of reliability engineering is the use of standby redundancy to raise a system's reliability and maximize profitability, ensuring the sustainability of economies and industries. To accomplish this, a reliability model is developed that incorporates a finite number (N) of cold standby units, in addition to two operational units, taking into account the activation time required to transition a standby unit to an operational state. Upon failure of either operational unit, a cold standby unit is activated to assume its responsibilities, functioning with equivalent effectiveness. The system operates with two working units at a time and if in any state there is only one operative unit available, then the system works at reduced capacity. The determination of the required number of standby units involves establishing cut-off points based on factors such as revenue, failure rates, installation costs, activation rates, etc. Computational analyses have been conducted to facilitate this process, incorporating Markov processes and various performability measures. The regeneration point technique is employed to optimize the number of standby units.

Keywords. Redundancy, Optimization, Two operative unit, Activation time, N cold standby units, Profit analysis, Regenerative point technique

Mathematics Subject Classification (2020). 90B25, 91B70, 62K20, 65K10, 62N05

1. Introduction

Cold-standy redundancy is a widely employed design strategy in many operational systems, particularly in applications where reliability is paramount. In a cold-standby configuration, redundant units are not susceptible to failure while in a dormant state. This approach has proven effective in space exploration and satellite systems, where high reliability is crucial for both repairable and non-repairable systems. For instance, space-based inertial reference units must maintain accuracy over extended mission durations without opportunities for repair, making cold-standby redundancy an essential design consideration. Cold-standby redundancy is also employed in various other systems to attain high reliability, such as textile manufacturing systems (Pandey *et al*. [15]), and carbon recovery systems used in fertilizer plants (Kumar *et al*. [12]).

Substantial studies have been conducted to analyze the reliability of redundant systems, including those comprising identical and non-identical units. Various methods and methodologies for evaluating reliability have been extensively discussed by Birolini [5] and Srinivasan and Subramanian [22], providing a comprehensive foundation for reliability analysis.

Several researchers, such as Goel *et al*. [7] and Yamashiro [24], have contributed to the analysis and optimization of redundancy problems. However, the majority of these studies focus on the analysis of single-unit systems, rather than more complex multi-unit systems. Gupta *et al*. [8, 9] analyzed a two dissimilar-unit multi-component cold standby system with correlated failures and repairs. The modeling and profitability analysis of a repairable system with one operational unit, three cold standby units, and a single repairman are covered by Ahmad *et al*. [1]. Kumar and Goel [10] discussed a two-unit cold standby system while considering the ideas of priority with a single repairman, inspection, deterioration, and preventive maintenance. After repair, the unit is referred to as a degraded unit and operates at a reduced capacity. Kumar *et al*. [11] examine a 2-unit standby system with three modes: functioning, activation, and failure. The standby unit requires time to activate, and the failed unit must wait for repair during this period if the operative unit fails entirely or partially. Liu *et al*. [13] presented a three-component repairable system with unreliable switching failure using a multiple vacation policy for the repairman to utilize human resources fully. Yang and Tsao [25] use matrix-analytic to calculate the steady-state availability of a repairable system with M primary components, S spare components, and single repairman service. Coit [6] examines systems with non-constant failure rates and imperfect switching, identifying optimal design configurations for non-repairable series-parallel systems using cold-standby redundancy. Ardakan and Hamadani [2] investigated the redundancy allocation problem for a series-parallel system and proposed a novel technique that combines classic active and cold-standby strategies. Singh and Agrafiotis [21] investigated a standby system with two identical units that undergo preventative maintenance at a predefined interval, during which it is stopped temporarily. The maintenance process is carried out by a single repair facility that adheres to the triple rule, which encompasses preventative maintenance, repair, and replacement procedures.

Wang *et al*. [23] proposed a comprehensive model to optimize the selection of components, the number of redundant components, and the frequency of periodic inspections, enabling the effective implementation of a maintenance strategy that combines periodic inspections and preventive maintenance for cold-standby components. Roy and Gupta [19] studied the impact of adding a cold standby component to a coherent system to improve its mean residual life function. Ma *et al*. [14] introduced innovative stochastic reliability models for discrete-time cold standby repairable systems, taking into account the complexities of an unreliable repair facility and a retrial mechanism.

Batra and Taneja [3, 4] formulated reliability models and conducted optimization to determine the appropriate quantity of standby units in systems featuring either one or two operative units. The goal was to attain the desired level of system reliability. Parveen *et al*. [16–18] establish a reliability model considering one operative unit and N cold/warm/hot standby unit with the concept of activation and switching time and optimize the number of standby units for $N = 1,2$ and 3. In the literature, limited research has been reported on redundancy optimization for systems comprising two components, where two are active, and the other is in cold standby. Additionally, when addressing the optimization of cold standby units, the crucial factor of activation time to render standby units operational has been omitted, even though it is a vital consideration for numerous standby systems. To address this gap, in this paper, a reliability model is investigated by considering two operational units and N cold standby units while also considering the activation time required to transition a cold standby unit into an operational state. Generalized results have been obtained for different measures of system effectiveness, aiming to maximize the expected utilization of standby units and optimize their deployment.

2. Assumptions and System Characterizations

(1) At first, N units are kept in standby mode while two are operating.

(2) When a working unit fails, a backup unit is turned on. This backup unit takes some time to start working properly, known as activation time.

(3) The activation of the standby unit, when initiated, is finished before the repair of the failure unit is completed.

(4) If at any stage there remains only one unit operative, the system is said to work at reduced capacity.

(5) There is only one repairman with the system.

(6) The unit becomes as good as new on its repair.

(7) Failure times and activation time repair times are assumed to follow exponential Distribution.

3. Notations

The notations for various rates and states are:

λ	Failure rate of operative unit
γ	Activation rate of cold standby unit
β	Rate of repair of operative unit
Cs	Cold standby unit
CS_N	N units are cold standby

Op Operative unit
CS_a Cold standby is being activated for operation
F_{wr} A failed operational unit is awaiting repair
F_r Operative unit under repair
NF_r N operative units are failed

For the notation $\phi_i(t)$, $Q_{ij}(t)$, $M_i(t)$, $q_{ij}(t)$, $AF_i(t)$, $AR_i(t)$, $V_i(t)$, $AT_i(t)$, $B_i(t)$, one may refer [4].

4. System Configuration

Redundancy optimization is used in order to increase system availability and dependability. So, the analyzed system comprises $N+2$ units, with two unit in operative mode and the remaining N units in cold standby mode, awaiting activation. The system is designed in a way that it needs two unit to be in working condition, if at any stage there remains only one unit operative, the system is said to work at reduced capacity. Additionally, in cases where the operative unit experiences a complete failure, the standby unit is activated to take over as operative, resulting in the system transitioning to a down state. The system fails if all units fail before a given fixed time.

5. Transition Densities and Mean Sojourn Times

The transition between states of the system is shown in Figure 1, there is a total $3(N+1)$ number of states in the model. The states $2,5,8,\dots,3N-1$ are down states, state $3N+2$ is failed, and all remaining states are up states.

Representations of the System States:

State 0: $(2Op, CS_N)$;
State 1: $(Op, F_r, CS_a, CS_{N-1})$;
State 2: (Op, F_r, Op, CS_{N-1});
State 3: $(Op, 1F_{wr}, F_r, CS_a, CS_{N-2})$;
State 4: $(Op, 1F_{wr}, F_r, Op, CS_{N-2})$;
State 5: $(Op, 2F_{wr}, F_r, CS_a, CS_{N-3})$;
State 6: $(2F_{wr}, F_r, Op, Op, CS_{N-3})$;

$\vdots$

State $2N-3$: $((N-2)F_{wr}, F_r, Op, CS_a, Cs)$;
State $2N-2$: $(Op, Op, (N-2)F_{wr}, F_r, CS_1)$;
State $2N-1$: $((N-1)F_{wr}, F_r, Op, CS_a)$;
State $2N$: $((N-1)F_{wr}, F_r, Op, Op)$;
State $2N+1$: (Op, NF_{wr}, F_r);
State $2N+2$: $((N+1)F_{wr}, F_r)$;

The densities $q_{ab}(t)$ for transit from state a to b are given by

$$q_{01}(t) = 2\lambda e^{-(2\lambda)t},$$

$$q_{2k-2,2k-1}(t) = 2\lambda e^{-(2\lambda+\beta)t}, \quad 1 < k \le N+1,$$

$$q_{2k-1,2k}(t) = \gamma e^{-\gamma t}, \quad 1 \le k \le N,$$

$$q_{2N+1.2N+2}(t) = \gamma e^{-(\beta+\gamma)t},$$

$$q_{2N+1,2N}(t) = \beta e^{-(\beta+\gamma)t},$$

$$q_{2k,2k-2}(t) = \beta e^{-(\beta+2\lambda)t}, \quad 1 \le k \le N,$$

$$q_{2N+2,2N+1}(t) = \beta e^{-\beta t}.$$

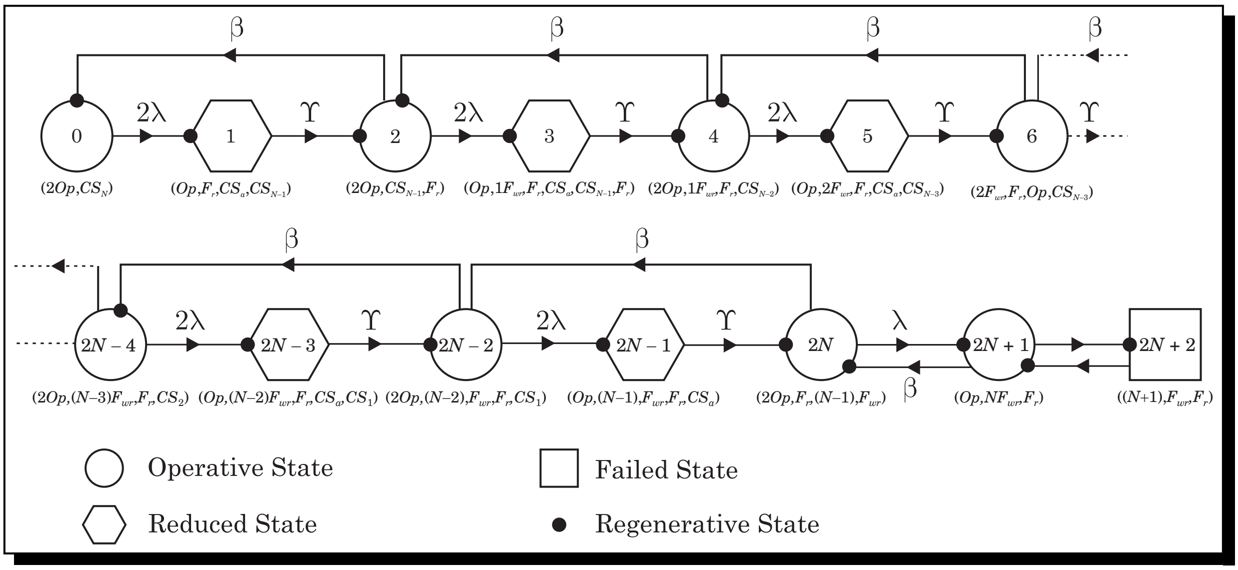

Figure 1. State transition diagram

Thus, $p_{ab} = \lim_{s\to 0} q^*_{ab}(s)$ are given as:

$$p_{01} = 1, \qquad p_{2k-2,2k-1} = \frac{2\lambda}{2\lambda+\beta}, \quad 1 < k \le N+1,$$

$$p_{2k-1,2k} = 1, \quad 1 \le k \le N, \qquad p_{2N+1,2N+2} = \frac{\lambda}{\beta+\lambda},$$

$$p_{2N+1,2N} = \frac{\beta}{\beta+\gamma}, \qquad p_{2k,2k-2} = \frac{\beta}{\beta+2\lambda}, \quad 1 \le k \le N,$$

$$p_{2N+2,2N+1} = 1.$$

Consequently, we can verify that

$$p_{2j-2,2j-1} + p_{2i,2i-2} = 1, \quad 1 < j \le N+1, \ 1 \le i \le N,$$

$$p_{2N+1,2N+2} + p_{2N+1,2N} = 1.$$

Thus, Mean Sojourn Time (μ_a) are:

If T_a denotes the stay time of the system in state 'a', then using the following mathematical relationship between μ_a and T_a,

$$\mu_a = \int_0^\infty P[T_a > t]dt, \qquad \mu_0 = \frac{1}{2\lambda},$$

$$\mu_{2k} = \frac{1}{2\lambda+\beta}, \quad 1 < k \le N, \qquad \mu_{2k-1} = \frac{1}{\gamma}, \quad 1 \le k \le N,$$

$$\mu_{2N+1} = \frac{1}{\beta+\gamma}, \qquad \mu_{2N+2} = \frac{1}{\beta}.$$

Commencing from state 'a' as the initial point, the unconditional mean times $m_{ab} = E(q_{ab}(t)) = \int_0^\infty t q_{ab}(t)dt$ are computed as

$$m_{01} = \frac{1}{2\lambda}, \qquad m_{2k-2,2k-1} = \frac{2\lambda}{(2\lambda+\beta)^2}, \quad 1 < k \le N+1,$$

$$m_{2k-1,2k} = \frac{1}{\gamma}, \quad 1 \le k \le N, \qquad m_{2N+1,2N+2} = \frac{\gamma}{(\beta+\gamma)^2},$$

$$m_{2N+1,2N} = \frac{\beta}{(\beta+\gamma)^2}, \qquad m_{2k,2k-2} = \frac{\beta}{(\beta+2\lambda)^2}, \quad 1 \le k \le N,$$

$$m_{2N+2,2N+1} = \frac{1}{\beta}.$$

It may be verified that

$$m_{01} = \mu_0,$$

$$m_{2k-2,2k-1} + m_{2i,2i-2} = \mu_{2i}, \quad 1 < j \le N+1,\ 1 \le i \le N,$$

$$m_{2k-1,2k} = \mu_{2k-1}, \quad 1 \le i \le N,$$

$$m_{2N+1,2N} + m_{2N+1,2N+2} = \mu_{2N+1},$$

$$m_{2N+2,2N+1} = \mu_{2N+2}.$$

6. System Performance Metrics

6.1 MTSF

By treating failure as an absorbing state, the following results are obtained

$$\phi_0(t) = Q_{01}(t) \circledS \phi_1(t), \tag{6.1}$$

$$\phi_{2i}(t) = Q_{2i,2i-2}(t) \circledS \phi_{2i-2}(t) + Q_{2i,2i+1}(t) \circledS \phi_{2i+1}(t), \quad 1 \le i \le N, \tag{6.2}$$

$$\phi_{2i-1}(t) = Q_{2i-1,2i}(t) \circledS \phi_{2i}(t), \quad 1 \le i \le N, \tag{6.3}$$

$$\phi_{2N+1}(t) = Q_{2N+1,2N}(t) \circledS \phi_{2N}(t) + Q_{2N+1,2N+2}(t). \tag{6.4}$$

Solving the above equations using the Laplace transform, and Upon inversion, the function $\phi_0^{(N)}(t)$ can be derived, and subsequently, the *Mean Time to System Failure* (MTSF) is determined as

$$\begin{aligned}
\phi_0^{(N)} &= \lim_{s\to 0} \frac{1-\phi_0^{**(N)}(s)}{s} = \lim_{s\to 0} \frac{1-\frac{T^{(N)}(s)}{L^{(N)}(s)}}{s} = \lim_{s\to 0} \frac{L^{(N)}(s)-T^{(N)}(s)}{s.L^{(N)}(s)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{form} \\
&= \lim_{s\to 0} \frac{L^{(N)'}(s)-T^{(N)'}(s)}{s\cdot L^{(N)'}(s)+L^{(N)}(s)} = \frac{L^{(N)'}-T^{(N)'}}{L^{(N)}} \quad \text{(Applying L-Hospital Rule)} \\
&= \frac{T_0^{(N)}}{L_0^{(N)}},
\end{aligned} \tag{6.5}$$

where

$$L_0^{(N)} = p_{3N+1,3N+2} \prod_{i=1}^{N} p_{3i-2,3i-1} p_{3i,3i+1} \tag{6.6}$$

and $T_0^{(N)}$ in determinant form can be evaluated using the MATLAB or MATHEMATICA software.

6.2 Availability at Full Capacity

Based on the definition of $AF_i(t)$ and the possible transitions, we get

$$AF_0(t) = M_{01}(t) + q_{01}(t) \circledS AF_1(t), \tag{6.7}$$

$$AF_{2i}(t) = M_{2i,2i-2}(t) + q_{2i,2i-2}(t) \circledS \phi_{2i-2}(t) + q_{2i,2i+1}(t) \circledS AF_{2i+1}(t), \quad 1 \le i \le N, \tag{6.8}$$

$$AF_{2i-1}(t) = q_{2i-1,2i}(t) \circledS AF_{2i}(t), \quad 1 \le i \le N, \tag{6.9}$$

$$AF_{2N+1}(t) = q_{2N+1,2N}(t) \circledS AF_{2N}(t) + Q_{2N+1,2N+2}(t) \circledS AF_{2N+2}(t), \tag{6.10}$$

$$AF_{2N+2}(t) = q_{2N+2,2N+1}(t) \circledS AF_{2N+1}(t), \tag{6.11}$$

where

$$M_0(t) = e^{-2\lambda t}, \quad M_{2i}(t) = e^{-(2\lambda+\beta)t}, \quad 1 \le i \le N. \tag{6.12}$$

Solving the above equations using the Laplace transform, upon inversion, the function $AF_0^{(N)}(t)$ is obtained, from which the system availability can be directly determined as

$$AF_0^{(N)} = \lim_{s\to 0} sA_0^*(s) = \lim_{s\to 0} \frac{sT_{1f}^{(N)}(s)}{L_{1f}^{(N)}(s)} = \frac{T_{1f}^{(N)}}{L_{1f}^{(N)}}, \tag{6.13}$$

where $T_1^{(N)}$ and $L_1^{(N)}$ are given as

$$\begin{aligned} T_{1f}^{(N)} &= \mu_0 p_{2N+1,2N} \prod_{i=1}^{N} p_{2i,2i-2} + \mu_2 p_{2N+1,2N} \prod_{i=2}^{N} p_{2i,2i-2} + \mu_4 p_{23} p_{2N+1,2N} \prod_{i=3}^{N} p_{2i,2i-2} + \cdots \\ &\quad + \mu_{2N-2} p_{23} p_{2N+1,2N} p_{2N,2N-2} \prod_{i=3}^{N-1} p_{2i-2,2i-1} + \mu_{2N} p_{23} p_{2N+1,2N} \prod_{i=3}^{N} p_{2i-2,2i-1} \end{aligned} \tag{6.14}$$

and

$$\begin{aligned} L_{1f}^{(N)} &= (\mu_0+\mu_1) p_{2N+1,2N} \prod_{i=1}^{N} p_{2i,2i-2} + (\mu_2 + \mu_3 p_{23}) p_{2N+1,2N} \prod_{i=2}^{N} p_{2i,2i-2} \\ &\quad + (\mu_4 + \mu_5 p_{45}) p_{23} p_{2N+1,2N} \prod_{i=3}^{N} p_{2i,2i-2} + \cdots \\ &\quad + (\mu_{2N-2} + \mu_{2N-1} p_{2N,2N-1}) p_{23} p_{2N+1,2N} p_{2N,2N-2} \prod_{i=3}^{N-1} p_{2i-2,2i-1} \\ &\quad + \mu_{2N} p_{2N+1,2N} \prod_{i=2}^{N} p_{2i-2,2i-1} + \mu_{2N+1} \prod_{i=2}^{N+1} p_{2i-2,2i-1} \\ &\quad + \mu_{2N+2} p_{2N+1,2N+2} \prod_{i=2}^{N+1} p_{2i-2,2i-1}. \end{aligned} \tag{6.15}$$

6.3 Availability at Reduced Capacity

Based on the definition of $AR_i(t)$ and the possible transitions, we get

$$AR_0(t) = q_{01}(t) \circledS AR_1(t), \tag{6.16}$$

$$AR_{2i}(t) = q_{2i,2i-2}(t) \circledS AR_{2i-2}(t) + q_{2i,2i+1}(t) \circledS AR_{2i+1}(t), \quad 1 \le i \le N, \tag{6.17}$$

$$AR_{2i-1}(t) = M_{2i-1,2i}(t) + q_{2i-1,2i}(t) \circledS AR_{2i}(t), \ 1 \le i \le N, \tag{6.18}$$

$$AR_{2N+1}(t) = M_{2N+1,2N}(t) + q_{2N+1,2N}(t) \circledS AR_{2N}(t) + q_{2N+1,2N+2}(t) \circledS AR_{2N+2}(t), \tag{6.19}$$

$$AR_{2N+2}(t) = q_{2N+2,2N+1}(t) \circledS AR_{2N+1}(t) \tag{6.20}$$

where

$$M_{2i-1}(t) = e^{-\gamma t}, \quad M_{3N+1}(t) = e^{-(\beta+\gamma)t}, \quad 1 \le i \le N. \tag{6.21}$$

Solving the above equations using the Laplace transform, and the function $AR_0^{(N)}(t)$ can be obtained after inversion,

$$AR_0^{(N)} = \lim_{s\to 0} sA_0^*(s) = \lim_{s\to 0} \frac{sT_{1r}^{(N)}(s)}{L_{1f}^{(N)}(s)} = \frac{T_{1r}^{(N)}}{L_{1f}^{(N)}}, \tag{6.22}$$

$$\begin{aligned} T_{1r}^{(N)} &= \mu_1 p_{2N+1,2N} \prod_{i=1}^{N} p_{2i,2i-2} + \mu_3 p_{23} p_{2N+1,2N} \prod_{i=2}^{N} p_{2i,2i-2} \\ &\quad + \mu_5 p_{23} p_{2N+1,2N} \prod_{i=3}^{N} p_{2i,2i-2} + \cdots + \mu_{2N-1} p_{2N+1,2N} p_{2N,2N-2} \prod_{i=2}^{N} p_{2i-2,2i-1} \\ &\quad + \mu_{2N+1} \prod_{i=2}^{N+1} p_{2i-2,2i-1} \end{aligned} \tag{6.23}$$

and $L_{1f}^{(N)}$ is already calculated.

6.4 Other Measures

6.4.1 Expected Busy Period

$$B_0^{(N)} = \lim_{s\to 0} \frac{sT_2^{(N)}(s)}{L_1^{(N)}(s)} = \frac{T_2^{(N)}}{L_1^{(N)}}. \tag{6.24}$$

6.4.2 Expected Number of Visits for Repair

$$V_0^{(N)} = \lim_{s\to 0} sV_0^{**}(s) = \lim_{s\to 0} \frac{sT_3^{N}(s)}{L_1^{(N)}(s)} = \frac{T_3^{(N)}}{L_1^{(N)}}. \tag{6.25}$$

6.4.3 Expected Activation Time

$$AT_0^{(N)} = \lim_{s\to 0} \frac{sT_4^{(N)}(s)}{L_1^{(N)}(s)} = \frac{T_4^{(N)}}{L_1^{(N)}}, \tag{6.26}$$

where

$$\begin{aligned} T_2^{(N)} &= \mu_1 p_{2N+1,2N} \prod_{i=1}^{N} p_{2i,2i-2} + (\mu_2 + \mu_3 p_{23}) p_{2N+1,2N} \prod_{i=2}^{N} p_{2i,2i-2} \\ &\quad + (\mu_4 + \mu_5 p_{45}) p_{23} p_{2N+1,2N} \prod_{i=3}^{N} p_{2i,2i-2} + \cdots \\ &\quad + (\mu_{2N-2} + \mu_{2N-1} p_{2N,2N-1}) p_{23} p_{2N+1,2N} p_{2N,2N-2} \prod_{i=3}^{N-1} p_{2i-2,2i-1} \\ &\quad + \mu_{2N} p_{2N+1,2N} \prod_{i=2}^{N} p_{2i-2,2i-1} + \mu_{2N+1} \prod_{i=2}^{N+1} p_{2i-2,2i-1} \\ &\quad + \mu_{2N+2} p_{2N+1,2N+2} \prod_{i=2}^{N+1} p_{2i-2,2i-1}, \end{aligned} \tag{6.27}$$

$$T_3^{(N)} = p_{2N+1,2N} \prod_{i=1}^{N} p_{2i,2i-2}, \tag{6.28}$$

$$T_4^{(N)} = \mu_1 p_{2N+1,2N} \prod_{i=1}^{N} p_{2i,2i-2} + \mu_3 p_{23} p_{2N+1,2N} \prod_{i=2}^{N} p_{2i,2i-2}$$
$$+ \mu_5 p_{23} p_{2N+1,2N} \prod_{i=3}^{N} p_{2i,2i-2} + \cdots + \mu_{2N-1} p_{2N+1,2N} p_{2N,2N-2} \prod_{i=2}^{N} p_{2i-2,2i-1} \tag{6.29}$$

and $L_{1f}^{(N)}$ is already defined.

7. Profitability Analysis

The profit function is

$$\text{Profit}(P_N) = C_{01} AF_0^{(N)} + C_{02} AR_0^{(N)} - C_1 B_0^{(N)} - C_2 V_0^{(N)} - C_3 AT_0^{(N)} - N.IC_0, \tag{7.1}$$

where C_{01} and C_{02} represent revenue, C_1, C_2, C_3 and IC_0 denote repair cost, repairman visit cost, and activation cost, installation cost per unit of time, respectively.

8. Numerical Findings and Graphical Analysis

Reliability metrics are computed for a system comprising $N+2$ units, employing arbitrary parameter values for illustrative purposes. Graphical study has been carried out taking some particular values of parameters involved.

(i) With $\gamma = 0.01$ and $\beta = 0.02$, the numerical results for the MTSF for different values of λ are presented in Table 1.

Table 1. MTSF with respect to varied (λ) and different values of N

λ	MTSF			
	$N=1$	$N=2$	$N=3$	$N=4$
0.1	126.8700	244.8117	359.6531	474.4888
0.2	113.5237	221.2948	328.3633	435.4313
0.3	109.0208	213.9595	318.5988	423.2380
0.4	106.7648	210.3821	313.8348	417.2875
0.5	105.4107	208.2641	311.0136	413.7630
0.6	104.5081	206.8637	309.1480	411.4322
0.7	103.8634	205.8691	307.8228	409.7765
0.8	103.3800	205.1263	306.8330	408.5397
1.0	103.0040	204.5504	306.0655	407.5807

According to Table 1, it is observed that increasing the failure rate of operative unit leads to a decrease in MTSF, whereas the addition of standby units exhibits an almost linear relationship, resulting in an increase in MTSF.

(ii) For $\gamma = 0.2$, $\beta = 0.3$ availability with respect to λ when $N = 1,2,3$ and 4.

Table 2. Availability of the system with respect to varied (λ) ($N = 1,2,3$ and 4)

λ	Availability			
	$N = 1$	$N = 2$	$N = 3$	$N = 4$
0.1	0.9436	0.9685	0.9803	0.9869
0.2	0.8093	0.848	0.8664	0.8765
0.3	0.6769	0.7102	0.7230	0.7284
0.4	0.5693	0.5940	0.6017	0.6042
0.5	0.4856	0.5033	0.5080	0.5092
0.6	0.4206	0.4335	0.4364	0.4369
0.7	0.3694	0.3790	0.3808	0.3811
0.8	0.3286	0.3358	0.3370	0.3372
1.0	0.2954	0.3009	0.3017	0.3019

Table 2 illustrates that availability decreases with an increasing failure rate but rises as addition of standby units.

(iii) For $\beta = 0.01$, $\lambda = 0.01$, Figure 2 depicts the profit variation with respect to γ for different values of N.

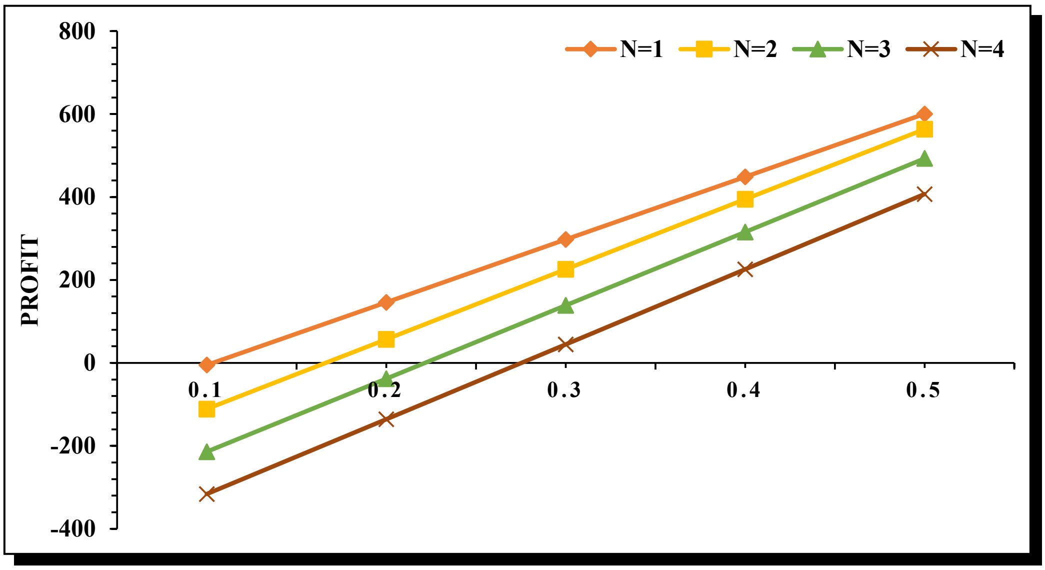

Figure 2. Profit versus Activation rate (γ) for $N = 1,2,3$ and 4

From Figure 2, one may observe that profit increases with an increase in activation rate and also with a decrease in the number of standby units.

(iv) For $\beta = 0.3$, $\gamma = 0.2$, $C_{01} = 2000$, $C_{02} = 1000$, $C_1 = 300$, $C_2 = 200$, $C_3 = 300$, $IC_0 = 100$, Figure 3 depicts the profit variation with respect to λ for different values of N.

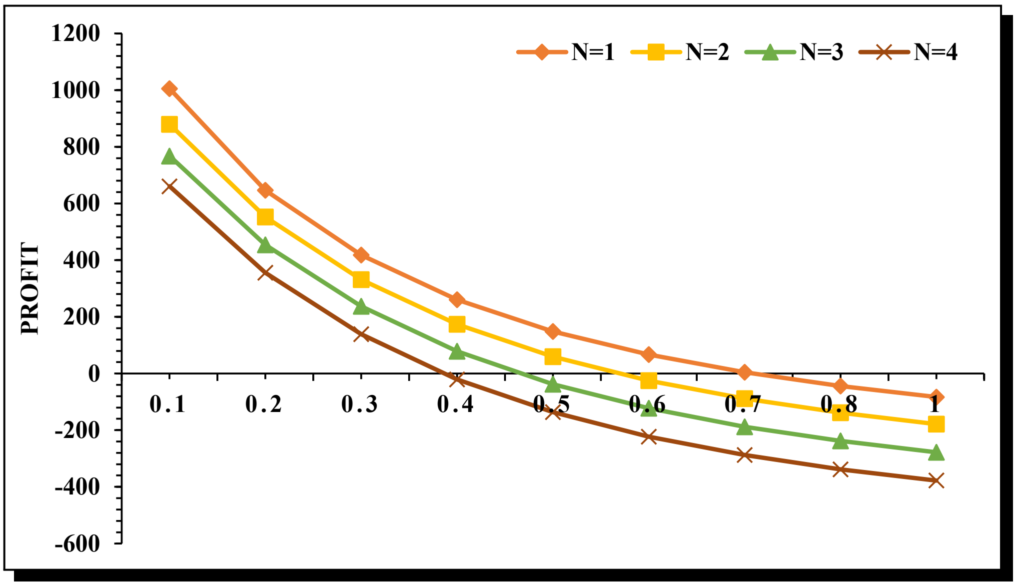

Figure 3. Profit versus λ for $N = 1,2,3$ and 4

From Figure 3, it has been observed that profit decreases with an increase in failure rate and also with addition of standby units.

(v) For $\beta = 0.3$, $\lambda = 0.1$, $\gamma = 0.2$, $C_{01} = 1000$, $C_{02} = 600$, $C_1 = 300$, $C_2 = 200$, $IC_0 = 100$ as indicated in Figure 4.

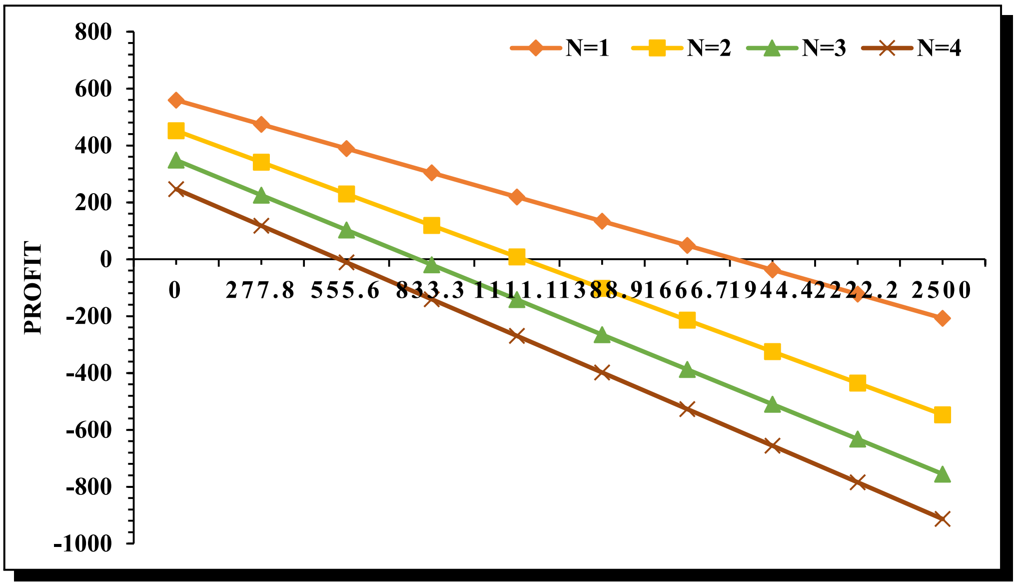

Figure 4. Profit versus Activation cost for $N = 1,2,3$ and 4

Figure 4 illustrates the behavior of profit with the activation cost for different numbers of standbys ($N = 1,2,3$ and 4). With an increase in activation cost, the profit of the system decreases. Also, it has been seen that when the number of standby increases, the profit falls.

(vi) For $\beta = 0.3$, $\lambda = 0.1$, $\gamma = 0.2$, $C_{01} = 1800$, $C_{02} = 600$, $C_1 = 300$, $C_3 = 300$, $IC_0 = 150$ as indicated in Figure 5.

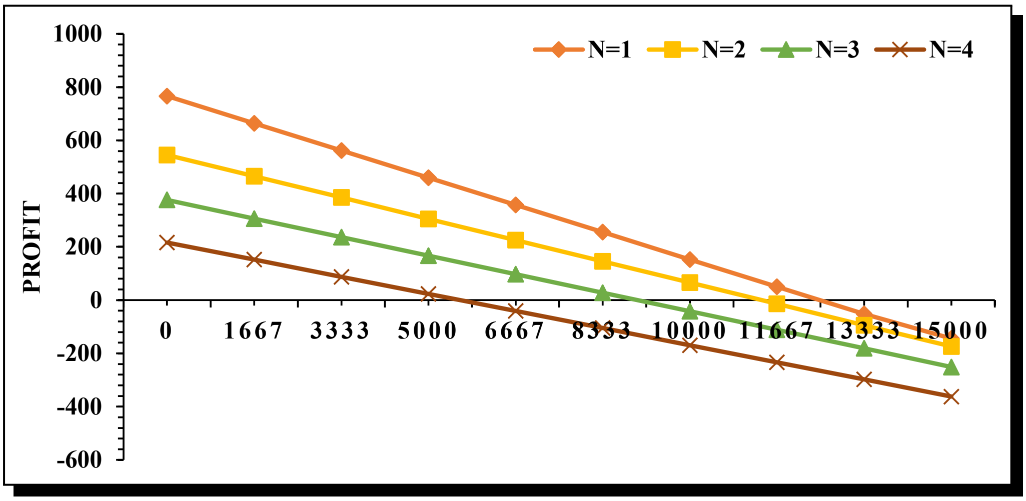

Figure 5. Profit versus C_2 for $N = 1,2,3$ and 4

Figure 5 shows how profit behaves in relation to the repairman's cost per visit for various numbers of standbys ($N = 1,2,3$, and 4). The profit of the system decreases as the cost per visit of the repairman values increases. Additionally, it has been shown that the profit decreases as the number of standbys rises.

(vii) Figure 6 illustrates the variation in profit for different values of (C_{01}) and $N = 1,2,3,4$ and with fixed values for $\beta = 0.3$, $\lambda = 0.1$, $\gamma = 0.2$, $C_{02} = 600$, $C_1 = 300$, $C_2 = 200$, $C_3 = 300$, $IC_0 = 100$.

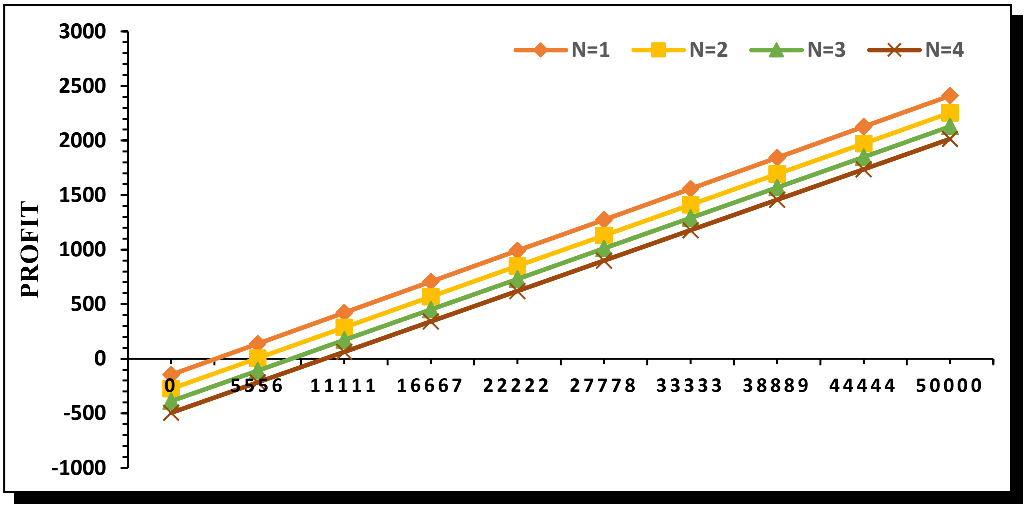

Figure 6. Profit versus C_{01} for $N = 1,2,3$ and 4

From Figure 6 it can be shown that with an increase in revenue (C_{01}) values, the profit of the system rises and decreases with increase the number of standby units.

(viii) Figure 7 shows the change in profit for varied (C_{02}) and $N = 1,2,3$ and with fixed values for $\beta = 0.3$, $\lambda = 0.1$, $\gamma = 0.2$, $C_{01} = 800$, $C_1 = 300$, $C_2 = 200$, $C_3 = 300$, $IC_0 = 100$.

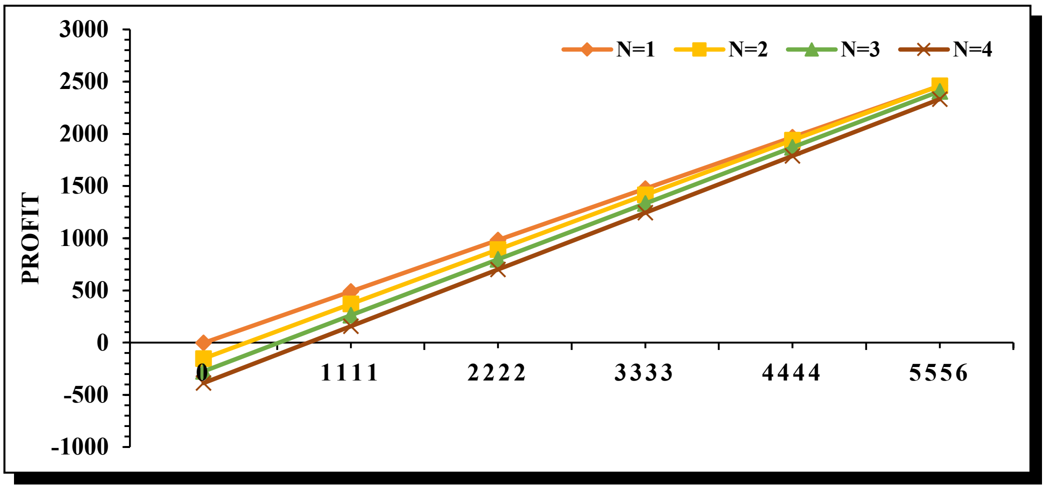

Figure 7. Profit versus C_{02} for $N = 1,2,3$ and 4

From Figure 7 it can be shown that with an increase in revenue (C_{02}) values, the profit of the system rises and decreases with an increase the number of standby units.

(ix) For $\beta = 0.3$, $\lambda = 0.1$, $\gamma = 0.2$, $C_{01} = 2000$, $C_{02} = 600$, $C_1 = 300$, $C_2 = 200, C_3 = 300$ as indicated in Figure 8.

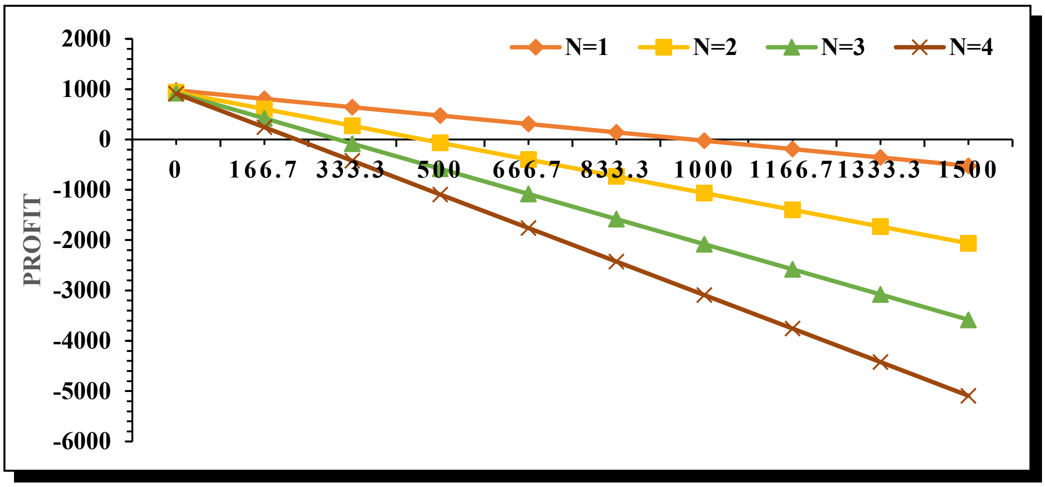

Figure 8. Profit versus IC_0 for $N = 1,2,3$ and 4

Figure 8 shows profit behavior in relation to the installation cost for various numbers of standbys ($N = 1,2,3$, and 4). The profit of the system decreases as the installation cost increases and decreases as the number of standbys rises.

9. Bounds for Profit for Different Values of Parameters

Table 3. Bounds for Activation Cost/ Repairman visit cost/ Revenue/ Installation Cost

Figure number	Parameter value	Number of standby used (N)	Bounds for profitability of the system
	$\beta = 0.3$, $\lambda = 0.1$	$N = 1$	$C_3 > 1895.6$
4	$\gamma = 0.2$, $C_{01} = 1000$	$N = 2$	$C_3 > 1169.63$
	$C_{02} = 600$, $C_1 = 300$,	$N = 3$	$C_3 > 885.7$
	$C_2 = 200$, $IC_0 = 100$	$N = 4$	$C_3 > 572.32$
	$\beta = 0.3$, $\lambda = 0.1$	$N = 1$	$C_2 > 12756.39$
5	$\gamma = 0.2$, $C_{01} = 1800$	$N = 2$	$C_2 > 9565.07$
	$C_{02} = 600$, $C_1 = 300$	$N = 3$	$C_2 > 8771.61$
	$C_3 = 300$, $IC_0 = 150$	$N = 4$	$C_2 > 6121.63$
	$\beta = 0.3$, $\lambda = 0.1$	$N = 1$	$C_{01} < 4758.73$
6	$\gamma = 0.2$, $C_{02} = 600$	$N = 2$	$C_{01} < 5332.21$
	$C_1 = 300$, $C_2 = 200$	$N = 3$	$C_{01} < 8455.35$
	$C_3 = 300$, $IC_0 = 100$	$N = 4$	$C_{01} < 10852.33$
	$\beta = 0.3$, $\lambda = 0.1$	$N = 1$	$C_{02} < 132.31$
7	$\gamma = 0.2$, $C_{01} = 800$	$N = 2$	$C_{02} < 541.45$
	$C_1 = 300$, $C_2 = 200$	$N = 3$	$C_{02} < 720.62$
	$C_3 = 300$, $IC_0 = 100$	$N = 4$	$C_{02} < 952.75$
	$\beta = 0.3$, $\lambda = 0.1$	$N = 1$	$IC_0 > 911.35$
8	$\gamma = 0.2$, $C_{01} = 2000$	$N = 2$	$IC_0 > 477.63$
	$C_{02} = 600$, $C_1 = 300$	$N = 3$	$IC_0 > 305.89$
	$C_2 = 200$, $C_3 = 300$	$N = 4$	$IC_0 > 261.34$

10. Conclusion

Optimizing cold standby redundancy is essential to achieving cost-effectiveness while maintaining reliability and takes into factors like figuring out the optimum number of standby units, creating maintenance plans to guarantee readiness, and creating effective failover procedures. So, this study evaluates a crucial repairable system with two operative unit and N cold standby units with activation time which includes the processes of powering up the standby unit, initializing its configurations, and ensuring it is ready to take over the functions of the failed operative unit. Exponential distributions have been used to get findings for reliability metrics. The model is validated taking specific values of parameters involved. It has been noted that using a number of standby units depends on the situation and may involve using 1, 2, 3, or 4 standby units, depending on the cut-off levels determined in various circumstances.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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