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Research Article

Examining Fuzzy Partial Metric Space and Associated Outcomes

Renu P. Pathak and Rohini R. Gore*

Department of Mathematics, Sandip University, Nashik 422213, Maharashtra, India
***Corresponding author:** gorerohini2612@gmail.com

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Abstract. Using the concept of a PM space with fuzzy, it gives idea of a FPMS in this paper. A point's self-distance in partial metric space does not always equal to zero. Ordinary metric is a subset of partial metric. Additionally, also define continuous t-norms. Partial fuzzy contraction mapping is defined here. It also demonstrates that, in certain circumstances, the complete partial metric space has a common fixed point through use of contraction mapping. Relevant examples are used as provide results.

Keywords. Fixed point theorem, Fuzzy sets, Fuzzy metric space, Partial metric space

Mathematics Subject Classification (2020). 47H10, 37C25, 30L15

1. Introduction

In 1965, Zadeh developed fuzzy set theory [22]. Fuzzy metric concepts have been presented in a variety of ways by numerous writers. George and Veeramani [9] defines concept of FMS and extended Probabilistic MS (see also, Vasuki and Veeramani [19]), and the fuzzy partial metric space concept was expanded upon from Amer [3]. Following that, under certain circumstances, Vasuki and Veeramani [14] and Gregori *et al*. [18] proved a few fixed-point theorems on FMS. Numerous varieties of generalized metric spaces have been introduced in literature by altering the metric condition by Mustafa and Sims [16].

The idea of PMS, which Matthews [15] introduced as an extension of metric space where any point self-distance is not equal to zero, is one of the metric spaces' generalizations. Computer science applications serve as the inspiration for this idea. Bukatin *et al*. [7] demonstrated how metric space's nonzero self-distance mathematics is established. They also discussed a few potential applications for partial metric spaces. By taking into account the continuous t-norm, Yue and Gu [21] defined the concept of fuzzy partial metric space. Additionally, they extended the concept of fixed-point theorems which was explained Gregori and Sapena [11]. Following, Sedghi's acquisition [18] of the FPM space through the generalization of the non-Archimedean fuzzy metric structure, they were able to derive certain fixed-point outcomes within these spaces.

FPMS is a concept that Gregori *et al*. [12] approached by combining PMS and FMS with continuous t-norm. From a fuzzy partial metric, Aygun *et al*. [5] constructs an FMS. This fuzzy metric's topology, Cauchy sequences, and completeness are examined, along with how they relate to the same ideas that underpin the FMS. For generalized contractive type mappings on partial metric space, Altun *et al*. [2] provides a few fixed-point theorems. Under certain limitations, Güner and Aygün [8] found several helpful inequalities in fuzzy partial metric spaces. The outcome of Grabiec's [10] common fixed-point establishment is a fuzzy metric space. They establish a common fixed-point theorem for property in fuzzy metric space in this study.

On fuzzy metric spaces, Beg *et al*. [6] obtained a fixed point of mapping that satisfied an implicit connection. By following the evolution of fuzzy metric space, Amer [4] defined the product space on fuzzy partial metric space. Gregori *et al*. [13] investigated a few characteristics of a fuzzy metric space class. Fixed point generalizations to PMS can be derived from the equivalent metric spaces results, as demonstrated by Haghi *et al*. [14]. O'Neill [17] establishes the inherent duality of partial metrics and suggests that considering a partial metric space as a bitopological space is a natural perspective. In contrast to earlier definitions of fuzzy metric spaces, Xia and Guo's [20] revised fuzzy metric spaces use fuzzy scalars rather than fuzzy numbers or real numbers. They establish the links between FMS and PMS in this work, using FPMS in the sense of Sedghi *et al*. [18]. Next, they demonstrate that, in certain scenarios, every fuzzy partial metric yields a fuzzy metric. Using this idea, also demonstrate that, in some cases (Aldemir *et al*. [1]), these mappings have a unique fixed point. Furthermore, it gives demonstrate how the results of fuzzy metric space can be used to build some of the fixed-point generalizations in FPMS.

2. Preliminaries

Definition 2.1 ([15])**.** A PMS on X is a pair (X,P) such that X is a nonempty set and $P : X \times X \to R^+$ is a mapping satisfy following conditions $\forall\ p,q,r \in X$ such that

(i) $P(p,p) \le P(p,q)$,

(ii) $P(p,p) = P(q,q) = P(p,q)$ if and only if $P = q$,

(iii) $P(p,q) = P(q,p)$,

(iv) $P(p,r) \le P(p,q) + P(q,r) - P(q,q)$.

Note that PMS, a point's self-distance does not always equal zero. Partial metric P is an ordinary metric on X, if $P(p,p) = 0\ \forall\ P \in\ X$. Therefore, a PM is an extension of an ordinary metric.

Example 2.1. Let $P : R^- \times R^- \to R^+$ be a mapping such that, $P(p,q) = -\min(p,q)$, for every $p,q \in R^-$. Then, (R^-,P) is a PMS in which the self-distance of each point $P \in R^-$ does not equal to zero.

Definition 2.2 ([9])**.** A binary operation $\odot$ on $[0,1]$ is called a continuous t-norm if it is satisfied following conditions: $\forall\ p,q,r,s \in [0,1]$

(i) $p \odot q = q \odot p$ and $p \odot (q \odot r) = (p \odot q) \odot r$,

(ii) $\odot$ is continuous on $[0,1] \times [0,1]$,

(iii) $p \odot 1 = p$,

(iv) if $p \le q$ and $r \le s$, then $p \odot r \le q \odot s$.

Definition 2.3 ([9])**.** Let X be a nonempty set, $\odot$ be a continuous t-norm and $F : X \times X \times [0,\infty) \to [0,1]$ be a mapping. Let F be Fuzzy set, and the listed conditions are satisfied $\forall\ p,q,r \in X$ and $u,v \ge 0$, then the triplet $(X,F,\odot)$ is said to be a fuzzy metric space. If it satisfies following properties for

(i) $F(p,q,u) \ge 0$,

(ii) $F(p,q,u) = 1$ if and only if $p = q$,

(iii) $F(p,q,u) = F(q,p,u)$,

(iv) $F(p,q,u+v) \ge F(p,q,u) \odot F(p,q,v)$,

(v) $F(p,q,\odot)$ is continuous on $[0,\infty)$.

If $(X,F,\odot)$ is a fuzzy metric space, then F is a fuzzy metric on X.

Example 2.2. Let (X,d) be a metric space and define $p \odot q = \min(p,q)$ and

$$F(p,q,u) = \frac{u}{u + d(p,q)}.$$

Then $(X,F,\odot)$ is a fuzzy metric space and F as the standard metric space about d.

Definition 2.4 ([6])**.** Let X be a nonempty set, $\odot$ be a continuous t-norm and $F_p : X \times X \times [0,\infty) \to [0,1]$ be a mapping. Let P be partial metric space. If the listed conditions are satisfied $\forall\ p,q,r \in X$ and $u,v \ge 0$, then the triplet $(X,F_p,\odot)$ is said to be a fuzzy partial metric space:

(i) $F_p(p,q,0) = 0$,

(ii) $F_p(p,q,u) = F_p(q,p,u)$,

(iii) $F_p(p,r,u+v) \ge F_p(p,q,u) \odot F_p(q,r,v)$,

(iv) $F_p(p,q,u) \le 1$, $u \ge 0$ and $F_p(p,q,u) = 1$ if and only if $P(p,q) = 0$,

(v) $F_p(p,q,\odot) : [0,\infty) \to [0,1]$ is continuous, where $F_p(p,q,u) = \frac{u}{u+p(p,q)}$.

If $(X,F_p,\odot)$ is an FPMS, then F_p is an FPM on X.

Example 2.3. Let (L,d_1) and (M,d_2) be two PMS with $(L \times M,d)$ be their product with $p(l,m) = \max\{d_1(l_1,m_1), d_2(l_2,m_2)\}$, for each $l = d_1(l_1,m_1)$ and $m = d_2(l_2,m_2)$ in $L \times M$. Denote $x\Delta y = \min(x,y)$, $\forall\ x,y \in [0,1]$. Let $F_p(l,m,u) = \dfrac{u}{u + p(l,m)}$.

Let F_p be fuzzy set on $L \times M \times [0,\infty)$. Then, the triple $(L \times M, F_p, \Delta)$ is a FPMS.

Proof. (i) $F_p(l,m,0) = \dfrac{0}{0 + p(l,m)} = 0$,

(ii) $$F_p(l,m,u) = \frac{u}{u + p(l,m)} = \frac{u}{u + \max\{d_1(l_1,m_1), d_2(l_2,m_2)\}} = \frac{u}{u + \max\{d_2(l_2,m_2), d_1(l_1,m_1)\}}$$

$$= \frac{u}{u+p(l,m)} = F_p(m,l,u).$$

(iii) $F_p(l,m,u)\Delta F_p(m,n,v) = \dfrac{u}{u+p(l,m)}\Delta\dfrac{v}{v+p(m,n)}$

$$= \frac{u}{u+\max\{d_1(l_1,m_1),d_2(l_2,m_2)\}}\Delta\frac{v}{u+\max\{d_1(m_1,n_1),d_2(m_2,n_2)\}}$$

$$= \min\left\{\frac{u}{u+\max\{d_1(l_1,m_1),d_2(l_2,m_2)\}}\frac{v}{u+\max\{d_1(m_1,n_1),d_2(m_2,n_2)\}}\right\}$$

$$\le \frac{u+v}{u+v+\max\{d_1(l_1,n_1),d_2(l_2,n_2)\}}$$

$$= \frac{u+v}{u+v+p(l,n)} \le F_p(l,n,u+v)$$

(iv) $F_p(l,m,u)=1$ if and only if $p(l,m)=0$, $0\le F_p(l,m,u)=\dfrac{u}{u+p(l,m)}\le 1$.

(v) $F_p(l,m,\Delta):[0,\infty)\to[0,1]$ is continuous, where $F_p(l,m,u)=\dfrac{u}{u+p(l,m)}$.

Thus, the triple $(L\times M,F_p,\Delta)$ is a fuzzy partial metric space. □

Definition 2.5. Let (M,P) be a partial metric space and (y_n) be a sequence in M,

(i) (y_n) is converged to $y\in M$ if $P(y_n,y)=P(y,y)$.

(ii) (y_n) is Cauchy sequence if $P(y_n,y_m)$ exist.

(iii) (M,P) is complete, if there is a point $y\in M$ such that $P(y_n,y_m)=P(y_n,y)=P(y,y)$.

Definition 2.6. Let $(M,F,\odot)$ be a fuzzy metric space and (y_n) be a sequence in M,

(i) (y_n) is converged to $y\in M$ if $F(y_n,y,u)=1$, $\forall\ u>0$.

(ii) (y_n) is Cauchy sequence if $F(y_n,y_m,u)=1$, $\forall\ u>0$.

(iii) $(M,F,\odot)$ is complete, if every Cauchy sequence (y_n) converges to a point $y\in M$ such that $F(y_n,y_m,u)=F(y,y,u)$.

Definition 2.7. Let $(M,F_p,\odot)$ be a fuzzy partial metric space and (y_n) be a sequence in M,

(i) (y_n) is converged to $y\in M$ if $F_p(y_n,y,u)=F_p(y,y,u)$, $\forall\ u>0$.

(ii) (y_n) is Cauchy sequence if $F_p(y_n,y_m,u)$ exists, $\forall\ u>0$.

(iii) $(M,F_p,\odot)$ is complete, if every Cauchy sequence (y_n) converges to a point $y\in M$ such that $F_p(y_n,y_m,u)=F_p(y,y,u)$.

Theorem 2.1. *Let $\{y_n\}$, $n\in N$ be a sequence in Y. Then, $\{y_n\}$, $n\in N$ converges to $y\in Y$ if and only if $F_p(y_n,y,u)=F_p(y,y,u)$, $\forall\ u>0$.*

Proof. Part I: Since $\{y_n\}$, $n\in N$ converges to $y\in Y$. Then, for each neighborhood U of y, there exists $n_0\in N$ such that $y_n\in U$, for each $n\ge n_0$.

Let $u>0$ and let $\varepsilon\in(0,1)$. Since $(Y,F_p,\odot)$ is a fuzzy partial metric space.

Then there exists $n_0\in N$ such that

$$y_n\in B_F(y,\varepsilon,u),\quad \text{for each } n\ge n_0$$

i.e., $(y,y,u) \to F_p(y_n,y,u) > 1-\varepsilon$, for each $n \geq n_0$.
Since $p \odot r \leq q \Leftrightarrow p \to q \geq r$, $\forall\ p,q,r \in [0,1]$.
Thus, $u > 0$ for each $n \geq n_0$. $F_p(y,y,u) \geq F_p(y_n,y,u)$, for each $n \in N$.
Therefore, it gives that $F_p(y,y,u) \geq F_p(y_n,y,u) \geq (1-\varepsilon) \odot F_p(y,y,u)$, for each $n \geq n_0$.
Therefore, $F_p(y_n,y,u) = F_p(y,y,u)$, $\forall\ u > 0$. Since u is arbitrary.
Suppose $F_p(y_n,y,u) = F_p(y,y,u)$, $\forall\ u > 0$. Therefore,

$$F_p(y,y,u) \geq F_p(y_n,y,u), \quad \text{for each } n \in N,\ \forall\ u > 0.$$

Part II: Let $u > 0$ and $\epsilon \in (0,1)$, there exists $n_0 \in N$ such that $F_p(y,y,u) \geq F_p(y_n,y,u)$, $u > 0$, $\forall\ n \geq n_0$.
Then, $F_p(y,y,u) \to F_p(y_n,y,u) > 1-\varepsilon$. Let $u > 0$ and let $s \in (0,u)$ and $\epsilon \in (0,1)$, there exists $n_0 \in N$ such that $F_p(y,y,s) \to F_p(y_n,y,s) > 1-\varepsilon$, $\forall\ n \geq n_0$.
Thus, $\forall\ s_1 \in (s,u)$, that $F_p(y,y,u) \to F_p(y_n,y,u) > 1-\varepsilon$, $\forall\ n \geq n_0$, since the function $F_p(y,y,\odot) \to F_p(y,z,\odot)$ is increasing.
Therefore, $\sup\{F_p(y,y,s) \to F_p(y_n,y,s) : s \in (0,u)\} > 1-\epsilon\}$, for each $n \geq n_0$, which is equivalent to $y_n \in B_F(y,\varepsilon,u)$, $\forall\ n \geq n_0$.
Let U be a neighborhood of y. Then, there exists $r \in (0,1)$ and $u > 0$ such that $B_F(y,r,u) \subseteq U$.
There exists $n_0 \in N$ such that $y_n \in B_F(y,\varepsilon,u)$, $\forall\ n \geq n_0$. Hence, $\{y_n\}$, $n \in N$ converges to y. □

Example 2.4. Let $(Y,F_p,\odot)$ be a fuzzy partial metric space. Define $Y = (0,\infty)$ and $P(a,b) = \max(a,b)$, $a,b \in Y$. Consider the sequence $\{y_n\} = \{0,1,0,1,\dots,0,1\}$, $n \in N$, $F_P(y_n,1,u) = \frac{u}{u+p(y_n,1)} = \frac{u}{u+1} = F_p(1,1,u)$, $u > 0$ if and only if $\{y_n\}$, $n \in N$ converges to $1 \in Y$.

Theorem 2.2. *Let (Y,P) be a PMS and $(Y,F_p,\odot)$ be a standard FPMS about P.*

(i) *(y_n) converges to $y \in Y$ in $(Y,F_p,\odot)$ if and only if (y_n) converges to $y \in Y$ in (Y,P).*

(ii) *(y_n) is a Cauchy sequence in $(Y,F_p,\odot)$ if and only if (y_n) is a Cauchy sequence in (Y,P).*

(iii) *$(Y,F_p,\odot)$ is complete if and only if (Y,P) is complete.*

Definition 2.8. Let (Y,P) be a complete PMS and f be a self-mapping on Y. The mapping f is said to be partial contractive mapping on Y, if there exists a $k \in [0,1)$ such that

$$P(f(y_1),f(y_2)) \leq kP(y_1,y_2), \quad \text{for all } y_1,y_2 \in Y.$$

Theorem 2.3. *Let (Y,P) be a complete Partial Metric Space and f be a partial contractive mapping on Y. Then, there exists a unique point $y \in Y$ such that $f(y) = y$ and $P(y,y) = 0$.*

Theorem 2.4. *Let $(Y,F,\odot)$ be a fuzzy metric space. If f is a self-mapping on Y satisfying*

$$F(f(a),f(b),u) > F(a,b,u), \quad \textit{for all } a,b \in Y,\ a \neq b \textit{ and } u > 0,$$

and there is a point $a_0 \in Y$ whose sequence of iterates $(f^n(a_0))$ contains a convergent subsequence $(f^{ni}(a_0))$, then f has a common fixed point in Y.

Non-Archimedean Property. If an FMS $(Y,F,\odot)$ provide the following condition $\forall\ p,q,r \in Y$ and $u,v > 0$, then $(Y,F,\odot)$ is said to be a non-Archimedean in FMS $F(p,q,\max\{u,v\}) \geq F(p,q,u) \odot M(q,r,v)$.

Note 2.1. Each non-Archimedean property in FMS is a FPMS, but the converse may not be true.

Example 2.5. Let (Y,P) be a partial metric space and $x \odot y = xy$, $\forall\ x,y \in [0,1]$. Consider the mapping $F_p : Y \times Y \times (0,\infty) \to [0,1]$ defined by $F(p,q,u) = \frac{u}{u+P(p,q)}$.
Then $(Y,F_p,\odot)$ is a FPMS which is called the standard FPMS. Note that $(Y,F_p,\odot)$ is not an FMS.

Remark 2.1. In an FMS $(Y,F,\odot)$, $F(a,b,\odot) : (0,\infty) \to [0,1]$ is increasing function $\forall\ a,b \in Y$, but in a FPMS $(Y,F_p,\odot)$, $F_p(a,b,\odot) : (0,\infty) \to [0,1]$ may not be increasing function $\forall\ a,b \in Y$.

In the following example, It is show that there are FPMS, but FMS may not be increasing function.

Example 2.6. Let $Y = R$ and $x \odot y = \min\{x,y\}$, $\forall\ x,y \in [0,1]$. Consider the mapping F_p : $Y \times Y \times (0,\infty) \to [0,1]$ defined by

$$F_b(a,b,u) = \begin{cases} e^{-u}, & a = b, \\ \frac{1}{2}e^{-u}, & a \neq b. \end{cases}$$

Definition 2.9. Let $(Y,F_p,\odot)$ be a FPMS and $\psi \in \Psi$. A self-mapping f on Y is called fuzzy partial contraction if there exists a $k \in [0,1)$ such that $F_p(f(y),f(z),ku) \geq F_p(y,z,u)$, $\forall\ y,z \in U$ and $u > 0$.

3. Main Result

Theorem 3.1. *Let $(Y,F_p,\odot)$ be a complete FPMS such that $\lim_{u\to\infty} F(a,b,u) = 1$, $\forall\ a,b \in Y$. If a self-mapping f on Y is a fuzzy partial contraction mapping, then f has a common fixed point in Y. Therefore, $\lim_{n\to\infty} F_p(a_n,a,u) = 1$. Therefore, $f(a) = a$. Hence, a is a fixed point of f.*

Proof. Let $a_0 \in Y$ and $a_n = f^n(a_0)$, $\forall\ n \in N$. Since $\lim_{u\to\infty} F(a,b,u) = 1$, $\forall\ a,b \in Y$.
Then

$$\begin{aligned} F_p(a_{n+1},a_n,u) &= F_p(f(a_n),f(a_{n-1}),u) \\ &\geq F_p\left(a_n,a_{n-1},\frac{u}{k}\right) \\ &= F_p\left(f(a_{n-1}),f(a_{n-2}),\frac{u}{k}\right) \\ &\geq F_p\left(a_{n-1},a_{n-2},\frac{u}{k^2}\right) \\ &\ \vdots \\ &\geq F_p\left(a_1,a_0,\frac{u}{k^2}\right) \to 1, \quad n \to \infty \end{aligned}$$

$$F_p(a_{n+1},a_n,u) = 1, \quad \forall\ u > 0.$$

Let $n,m \in N$ and assume that $n < m$,

$$\begin{aligned} F_p(a_n,a_m,u) &\geq F_p(a_n,a_m,u) \odot F_p(a_{n+1},a_{n+1},u) \\ &\geq F_p(a_{n+1},a_{n+1},u) \odot F_p(a_m,a_{n+1},u) \end{aligned}$$

$$\begin{aligned}&\geq F_p(a_n,a_{n+1},u)\odot F_p(a_m,a_{n+1},u)\odot F_p(a_{n+2},a_{n+2},u)\\&\geq F_p(a_n,a_{n+1},u)\odot F_p(a_{n+1},a_{n+2},u)\odot F_p(a_{n+2},a_m,u)\\&\vdots\\&\geq F_p(a_n,a_{n+1},u)\odot F_p(a_{n+1},a_{n+2},u)\odot\cdots\odot F_p(a_{m-1},a_m,u).\end{aligned}$$

Thus, obtain

$$F_P(a_n,a_m,u)=1,\quad \text{for all } u>0.$$

Hence, (a_n) is a Cauchy sequence in $(Y,F_p,\odot)$.

Then,

$$\begin{aligned}F_p(f(a),a,u)&\geq F_p(f(a),a,u)\odot F_p(a_n,a_n,u)\\&\geq F_p(f(a),a_n,u)\odot F_p(a_n,a_{n-1},u)\\&\geq F(f(a),f(a_{n-1}),u)\odot F_p(a,a,u)\\&\geq F_p(a,a_{n-1},u)\odot F_p(a,a,u).\end{aligned}$$

Since $(Y,F_p,\odot)$ is a complete FPMS, there exists a point $a\in Y$ such that (a_n) converges to a,

$$\lim_{n\to\infty}F_p(a_n,a,u)=F_p(a,a,u)=F_P(a_n,a_m,u)=1,\quad u>0.$$

Therefore, $(f(a),f(b),u)=1$. Thus, $f(a)=a$. Hence, a is a fixed point of function f.

Suppose $a\neq b$. Then,

$$F_p(a,b,u)<F_p(f(a),f(b),u)=F_p(a,b,u)$$

which is a contradiction. Hence, $a=b$. Thus, f has a common fixed point a in Y. □

Corollary 3.1. *Let $(Y,F_p,\odot)$ be a standard FPMS, where P is complete partial metric on Y. If f is self-mapping on Y which is a fuzzy partial contraction mapping, then f has a common fixed point in Y.*

Proof. Since $(Y,F_p,\odot)$ is a standard fuzzy partial metric space, where $P$is complete partial metric on Y. Therefore, $(Y,F_p,\odot)$ is a complete FPMS, where $F_p(a,b,u)=\frac{u}{u+p(a,b)}$, $\forall\ a,b\in Y$, $u>0$. Also, $\lim_{u\to\infty}F(a,b,u)=1$. Therefore, all conditions of Theorem 3.1 are satisfied. By using Theorem 3.1, f has a common fixed point in Y. □

Example 3.1. Define $Y=R^+$, $x\odot y=xy$, $\forall\ x,y\in[0,1]$ and the mapping $F_p:Y\times Y\times(0,\infty)\to[0,1]$ be defined by $F_p(a,b,u)=\frac{u}{u+\max(a,b)}$,

$$f(a)=\begin{cases}2x+\frac{1}{2}, & x\in\left[0,\frac{1}{4}\right],\\ \frac{1}{2}, & x\in\left[\frac{1}{4},1\right].\end{cases}$$

It has a common fixed point which is $x=\frac{1}{2}$.

By using Example 3.1, it gives new proposition.

Proposition 3.1. *Let $(Y,F_p,\odot)$ be a complete FPMS such that $\lim_{u\to\infty}F(a,b,u)$ exist, $\forall\ a,b\in Y$. If a self-mapping f on Y is a fuzzy partial contraction mapping, then f has a common fixed point in Y.*

Example 3.2. Define $Y = R^-$, $x \odot y = xy$, $\forall\ x, y \in [0,1]$ and the mapping $F_p : Y \times Y \times (0,\infty) \to [0,1]$ defined by $F_p(a,b,u) = \frac{u}{u - \min(a,b)}$.

It is a complete FPMS and $\lim\limits_{u\to\infty} F_P(a,b,u) = 1$, $\forall\ a, b \in Y$ is hold.

Let f be a self-mapping defined on Y given by $f(a) = a + 1$. But the mapping f does not satisfy the contraction condition of Theorem. Thus, it has no fixed point.

Example 3.3. Define $Y = [1,\infty)$, $x \odot y = xy$, for every $x, y \in [0,1]$ and the mapping $F_p : Y \times Y \times (0,\infty) \to [0,1]$. Defined by $F_p(a,b,u) = \frac{u}{u + \max(a,b)}$.

It is complete FPMS and $\lim\limits_{u\to\infty} F(a,b,u) = 1$, $\forall\ a, b \in Y$ is hold.

Let f be a self-mapping defined on Y given by $f(a) = a + \frac{1}{a}$. But mapping f does not satisfy the contraction condition of Theorem. Thus, f has no fixed point.

4. Conclusion

This study investigates various results in *Fuzzy Partial Metric Spaces* (FPMS), with particular emphasis on their implications in the context of fuzzy set theory. To support the theoretical findings, a representative example is presented, drawing upon established results and methodologies from prior research in the domain of FPMS and fuzzy set theory.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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