Communications in Mathematics and Applications
Vol. 16, No. 2, pp. 661–668, 2025
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v16i2.2956

http://www.rgnpublications.com

Research Article

Intuitionistic Fuzzy Binary Soft Topological Spaces

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Received: December 12, 2024 **Revised:** March 15, 2025 **Accepted:** April 8, 2025

Abstract. In this article, intuitionistic fuzzy binary soft topological space over two initial universal sets and a parameter set is introduced. Further, intuitionistic fuzzy binary soft neighborhood, intuitionistic fuzzy binary soft interior, intuitionistic fuzzy binary soft closure in an intuitionistic fuzzy binary soft topological space is defined, and their properties are discussed.

Keywords. Intuitionistic fuzzy binary soft topological space, Intuitionistic fuzzy binary soft neighborhood, Intuitionistic fuzzy binary soft interior, Intuitionistic fuzzy binary soft closure

Mathematics Subject Classification (2020). 03B20, 11B05, 03E72, 94D05

1. Introduction

While dealing with the real-world problems, the data or the information associated with the problem is vague. To deal with such uncertainty, Zadeh [17] proposed the concept of fuzzy sets. Using the concept of fuzzy sets, Chang [4] introduced the idea of fuzzy topological space. *intuitionistic fuzzy sets* (IFS) are the extension of fuzzy sets proposed by Atanassov [2]. Çoker [5] introduced the idea of *intuitionistic fuzzy topological spaces* (IFTS) using the idea of IFS.

Parametrization of the sets were done by Moldtsov [9] in 1999 and defined it as soft sets. More results and properties on soft sets were discussed by Maji *et al*. [7]. Soft topological spaces were initiated by Shabir and Naz [13] in 2011. Tanay and Kandemir [15] studied the *fuzzy soft topological spaces* (FSTS). The idea of *intuitionistic fuzzy soft topological spaces* (IFSTS) was studied by Li and Cui [6].

Açkigoz and Tas [1] introduced the idea of *binary soft sets* (BSS). In 2016, using the idea of BSS, Benchalli *et al*. [3] studied the *Binary Soft Topological Spaces* (BSTS). Further, Patil and Bhat [10] contributed towards the theory of BSTS. Metlida and Subhashini [8] defined the *fuzzy binary soft sets* (FBSS) over two universal sets and a fixed parameter set. Patil *et al*. [11] gives the application of FBSS. Combining the IFS with FBSS is done by Sivasankari and Subhashini [14] and defined it as *intuitionistic fuzzy binary soft sets* (IFBST) in 2023.

In this article, *intuitionistic fuzzy binary soft topological spaces* (IFBSTS) using IFBSSs is defined. Further, IFBS neighborhood, IFBS interior and IFBS closure in a IFBSTS is defined, and their properties are discussed.

2. Preliminaries

Definition 2.1 ([17])**.** Let X be a *universal set* and A be a function defined by

$A : X \to [0,1]$ or $\mu_X : X \to [0,1]$.

Then, the set $A = \{(x, A(x)) \mid x \in X\}$ is called a *fuzzy subset* of X.

Definition 2.2 ([2])**.** An *intuitionistic fuzzy set* (IFS) A on universe X can be defined as follows:

$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$,

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ with the property: $0 \le \mu_A(x) + \nu_A(x) \le 1$ ($\forall\ x \in X$). The values $\mu_A(x)$ and $\nu_A(x)$ denotes the degree of membership and non-membership of x to A, respectively.

Definition 2.3 ([9])**.** A pair $(\mathcal{F}, \mathcal{E})$ is called a *soft set* over a universal set $\mathcal{U}$ if F is a mapping of E, a set of parameters into the set of all subsets of set $\mathcal{U}$.

$\mathcal{F} : \mathcal{E} \to P(\mathcal{U})$.

Definition 2.4 ([12])**.** Let $\tilde{P}(\mathcal{U})$be the set of fuzzy subsets of $\mathcal{U}$, a pair $(\tilde{\mathcal{F}}, \mathcal{A})$ is called a *fuzzy soft set* over $\mathcal{U}$, where $\tilde{P}(\mathcal{U})$ is a mapping given by,

$\tilde{\mathcal{F}} : \mathcal{A} \to \tilde{P}(\mathcal{U})$.

A *fuzzy soft set* (FSS) is a mapping from parameters to $\tilde{P}(\mathcal{U})$.

Definition 2.5 ([16])**.** Let $\mathcal{U}$ be an initial universe, $\mathcal{E}$ be a set of parameters and IFS(U) denotes the intuitionistic fuzzy power set of $\mathcal{U}$ and $\mathcal{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathcal{A})$ is called an *intuitionistic fuzzy soft set* (IFSS) over $\mathcal{U}$, where $\mathcal{F}$ is a mapping given by,

$\mathcal{F} : \mathcal{A} \to \text{IFS}(\mathcal{U})$.

Definition 2.6 ([1])**.** Let $\mathcal{U}_1$ and $\mathcal{U}_2$ be two universal sets. E be a set of parameters, $A \subseteq E$. Let $\mathcal{F}$ be a function defined by

$\mathcal{F} : \mathcal{A} \to P(\mathcal{U}_1) \times P(\mathcal{U}_2)$.

Then, the set $(\mathcal{F}, \mathcal{A})$ is *binary soft set* over $\mathcal{U}_1$ and $\mathcal{U}_2$.

Definition 2.7 ([8])**.** Let $\mathcal{U}_1$ and $\mathcal{U}_2$ be two universal sets, $\mathcal{E}$ be the set of parameters, and $\mathcal{A} \subseteq \mathcal{E}$. Let $\mathcal{F}$ be a function defined by

$$\mathcal{F}: \mathcal{A} \to \tilde{P}(\mathcal{U}_1) \times \tilde{P}(\mathcal{U}_2),$$

where $\tilde{P}(\mathcal{U}_1)$ and $\tilde{P}(\mathcal{U}_2)$ are a set of all fuzzy sets of $\mathcal{U}_1$ and $\mathcal{U}_2$, respectively. Then, $(\mathcal{F},\mathcal{A})$ is *fuzzy binary soft set* over $\mathcal{U}_1$ and $\mathcal{U}_2$.

Definition 2.8 ([14])**.** Let $\mathcal{U}_1$ and $\mathcal{U}_2$ be two universal sets, $\mathcal{E}$ be the set of parameters, and $\mathcal{A} \subseteq \mathcal{E}$. Let $\mathcal{F}$ be a function defined by

$$\mathcal{F}: \mathcal{A} \to \tilde{P}(\mathcal{U}_1) \times \tilde{P}(\mathcal{U}_2),$$

where $\tilde{P}(\mathcal{U}_1)$ and $\tilde{P}(\mathcal{U}_2)$ are a set of all intuitionistic fuzzy sets of $\mathcal{U}_1$ and $\mathcal{U}_2$, respectively. Then, $(\mathcal{F},\mathcal{A})$ is *intuitionistic fuzzy binary soft set* over $\mathcal{U}_1$ and $\mathcal{U}_2$.

3. Intuitionistic Fuzzy Binary Soft Sets

Definition 3.1. Let $\tilde{\tilde{\tau}}$ be the collection of IFBSSs over $\mathcal{U}_1$ and $\mathcal{U}_2$. Then, $\tilde{\tilde{\tau}}$ is said to be *intuitionistic fuzzy binary soft* (IFBS) topology on $\mathcal{U}_1,\mathcal{U}_2$ if the following conditions are hold:

(i) $\tilde{\tilde{O}}, \tilde{\tilde{A}} \in \tilde{\tilde{\tau}}$,

(ii) the union of any members of $\tilde{\tilde{\tau}}$ belongs to $\tilde{\tilde{\tau}}$,

(iii) the intersection of any two members of $\tilde{\tilde{\tau}}$ belongs to $\tilde{\tilde{\tau}}$.

Then, $(\mathcal{U}_1,\mathcal{U}_2,\tilde{\tilde{\tau}},\mathcal{E})$ is called IFBS topological space.

Definition 3.2. Let $(\mathcal{U}_1,\mathcal{U}_2,\tilde{\tilde{\tau}},\mathcal{E})$ be IFBSTS on $\mathcal{U}_1,\mathcal{U}_2$. Then, the members of $\tilde{\tilde{\tau}}$ are called *intuitionistic fuzzy binary soft open sets* (IFBSOS) in $\mathcal{U}_1,\mathcal{U}_2$.

Definition 3.3. Let $(\mathcal{U}_1,\mathcal{U}_2,\tilde{\tilde{\tau}},\mathcal{E})$ be IFBSTS on $\mathcal{U}_1,\mathcal{U}_2$. Then, the member of $\tilde{\tilde{\tau}}$, $(\mathcal{F},\mathcal{E})$ is said to be *intuitionistic fuzzy binary soft closed set* (IFBSCS) in $\mathcal{U}_1,\mathcal{U}_2$, if it is relative complement belongs to $\tilde{\tilde{\tau}}$.

Definition 3.4. Let $(\mathcal{U}_1,\mathcal{U}_2,\tilde{\tilde{\tau}},\mathcal{E})$ be IFBSTS on $\mathcal{U}_1,\mathcal{U}_2$, where $\tilde{\tilde{\tau}} = \{\tilde{\tilde{O}},\tilde{\tilde{A}}\}$. Then, $\tilde{\tilde{\tau}}$ is called indiscrete topology and $(\mathcal{U}_1,\mathcal{U}_2,\tilde{\tilde{\tau}},\mathcal{E})$ is called indiscrete IFbSTS.

Definition 3.5. Let $\mathcal{U}_1,\mathcal{U}_2$ be two initial universal sets and $\mathcal{E}$ be a set of parameters. Let $\tilde{\tilde{\tau}}$ be the collection of all IFBSSs over $\mathcal{U}_1,\mathcal{U}_2$. Then, $\tilde{\tilde{\tau}}$ is called discrete topology and $(\mathcal{U}_1,\mathcal{U}_2,\tilde{\tilde{\tau}},\mathcal{E})$ is called IFBS discrete topological space.

Example 3.6. Let $\mathcal{U}_1 = \{\alpha_1,\alpha_2,\alpha_3\}$, $\mathcal{U}_2 = \{\beta_1,\beta_2,\beta_3\}$, $\mathcal{E} = \{e_1,e_2\}$. Let $(\mathcal{F}_1,\mathcal{E})$, $(\mathcal{F}_2,\mathcal{E})$ and $(\mathcal{F}_3,\mathcal{E})$ be three IFBSSs over $\mathcal{U}_1,\mathcal{U}_2$ defined as

$$\begin{aligned}(\mathcal{F}_1,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.6,0.3\rangle,\langle\alpha_2,0.5,0.4\rangle,\langle\alpha_3,0.8,0.1\rangle\},\{\langle\beta_1,0.5,0.4\rangle,\langle\beta_2,0.6,0.3\rangle,\langle\beta_3,0.6,0.3\rangle\})),\\ &(e_2,(\{\langle\alpha_1,0.8,0.1\rangle,\langle\alpha_2,0.5,0.3\rangle,\langle\alpha_3,0.6,0.3\rangle\},\{\langle\beta_1,0.6,0.4\rangle,\langle\beta_2,0.7,0.2\rangle,\langle\beta_3,0.8,0.2\rangle\}))\},\\ (\mathcal{F}_2,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.7,0.3\rangle,\langle\alpha_2,0.6,0.3\rangle,\langle\alpha_3,0.9,0.1\rangle\},\{\langle\beta_1,0.6,0.3\rangle,\langle\beta_2,0.7,0.2\rangle,\langle\beta_3,0.7,0.2\rangle\})),\\ &(e_2,(\{\langle\alpha_1,0.9,0\rangle,\langle\alpha_2,0.7,0.1\rangle,\langle\alpha_3,0.8,0.1\rangle\},\{\langle\beta_1,0.8,0.2\rangle,\langle\beta_2,0.9,0\rangle,\langle\beta_3,0.9,0.1\rangle\}))\},\\ (\mathcal{F}_3,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.8,0.2\rangle,\langle\alpha_2,0.6,0.2\rangle,\langle\alpha_3,0.9,0.1\rangle\},\{\langle\beta_1,0.7,0.2\rangle,\langle\beta_2,0.7,0.1\rangle,\langle\beta_3,0.9,0.1\rangle\})),\\ &(e_2,(\{\langle\alpha_1,1,0\rangle,\langle\alpha_2,0.9,0\rangle,\langle\alpha_3,0.9,0\rangle\},\{\langle\beta_1,0.8,0.2\rangle,\langle\beta_2,0.9,0\rangle,\langle\beta_3,1,0\rangle\}))\}.\end{aligned}$$

Then, $\tilde{\tilde{\tau}} = \{\tilde{\tilde{O}}, \tilde{\tilde{A}}, (\mathcal{F}_1,\mathcal{E}), (\mathcal{F}_2,\mathcal{E}), (\mathcal{F}_3,\mathcal{E})\}$ is IFBS topology on $\mathcal{U}_1, \mathcal{U}_2$.
$\tilde{\tilde{O}}, \tilde{\tilde{A}}, (\mathcal{F}_1,\mathcal{E}), (\mathcal{F}_2,\mathcal{E}), (\mathcal{F}_3,\mathcal{E})$ are IFBS open sets.
$\tilde{\tilde{O}}, \tilde{\tilde{A}}, (\mathcal{F}_1,\mathcal{E})^C, (\mathcal{F}_2,\mathcal{E})^C, (\mathcal{F}_3,\mathcal{E})^C$ are IFBS closed sets.

Remark 3.7. Let $(\mathcal{U}_1,\mathcal{U}_2,\tilde{\tilde{\tau}},\mathcal{E})$ and $(\mathcal{U}_1,\mathcal{U}_2,\tilde{\tilde{\tau}}',\mathcal{E})$ be two IFBSTSs over $\mathcal{U}_1,\mathcal{U}_2$. Then, $(\mathcal{U}_1,\mathcal{U}_2,\tilde{\tilde{\tau}} \cup \tilde{\tilde{\tau}}', E)$ need not be IFBSTS over $\mathcal{U}_1,\mathcal{U}_2$. Example 3.8 supports this statement.

Example 3.8. Let $\mathcal{U}_1 = \{\alpha_1,\alpha_2\}$, $\mathcal{U}_2 = \{\beta_1,\beta_2\}$, $\mathcal{E} = \{e_1,e_2\}$. Let $\tilde{\tilde{\tau}} = \{\tilde{\tilde{O}}, \tilde{\tilde{A}}, (\mathcal{F}_1,\mathcal{E}), (\mathcal{F}_2,\mathcal{E}), (\mathcal{F}_3,\mathcal{E})\}$ be IFBS topology on $\mathcal{U}_1,\mathcal{U}_2$, where

$$\begin{aligned}
(\mathcal{F}_1,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.6,0.4\rangle,\langle\alpha_2,0.5,0.4\rangle\},\{\langle\beta_1,0.5,0.4\rangle,\langle\beta_2,0.6,0.3\rangle\}))\\
&(e_2,(\{\langle\alpha_1,0.8,0.1\rangle,\langle\alpha_2,0.5,0.3\rangle\},\{\langle\beta_1,0.6,0.4\rangle,\langle\beta_2,0.7,0.2\rangle\}))\},\\
(\mathcal{F}_2,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.7,0.3\rangle,\langle\alpha_2,0.6,0.3\rangle\},\{\langle\beta_1,0.6,0.3\rangle,\langle\beta_2,0.7,0.2\rangle\}))\\
&(e_2,(\{\langle\alpha_1,0.9,0\rangle,\langle\alpha_2,0.7,0.1\rangle\},\{\langle\beta_1,0.8,0.2\rangle,\langle\beta_2,0.9,0\rangle\}))\},\\
(\mathcal{F}_3,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.8,0.2\rangle,\langle\alpha_2,0.6,0.2\rangle\},\{\langle\beta_1,0.7,0.2\rangle,\langle\beta_2,0.7,0.1\rangle\}))\\
&(e_2,(\{\langle\alpha_1,1,0\rangle,\langle\alpha_2,0.9,0\rangle\},\{\langle\beta_1,0.8,0.2\rangle,\langle\beta_2,0.9,0\rangle\}))\}.
\end{aligned}$$

Let $\tilde{\tilde{\tau}} = \{\tilde{\tilde{O}}, \tilde{\tilde{A}}, (\mathcal{F}'_1,\mathcal{E}), (\mathcal{F}'_2,\mathcal{E}), (\mathcal{F}'_3,\mathcal{E})\}$ be another IFBS topology on $\mathcal{U}_1,\mathcal{U}_2$, where

$$\begin{aligned}
(\mathcal{F}'_1,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.5,0.3\rangle,\langle\alpha_2,0.6,0.3\rangle\},\{\langle\beta_1,0.6,0.3\rangle,\langle\beta_2,0.5,0.4\rangle\}))\\
&(e_2,(\{\langle\alpha_1,0.7,0.2\rangle,\langle\alpha_2,0.6,0.2\rangle\},\{\langle\beta_1,0.7,0.2\rangle,\langle\beta_2,0.6,0.3\rangle\}))\},\\
(\mathcal{F}'_2,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.6,0.4\rangle,\langle\alpha_2,0.6,0.3\rangle\},\{\langle\beta_1,0.7,0.2\rangle,\langle\beta_2,0.6,0.3\rangle\}))\\
&(e_2,(\{\langle\alpha_1,0.8,0.1\rangle,\langle\alpha_2,0.7,0.1\rangle\},\{\langle\beta_1,0.8,0.1\rangle,\langle\beta_2,0.7,0.2\rangle\}))\},\\
(\mathcal{F}'_3,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.7,0.3\rangle,\langle\alpha_2,0.6,0.3\rangle\},\{\langle\beta_1,0.8,0.1\rangle,\langle\beta_2,0.7,0.2\rangle\}))\\
&(e_2,(\{\langle\alpha_1,0.7,0.3\rangle,\langle\alpha_2,0.6,0.3\rangle\},\{\langle\beta_1,0.9,0\rangle,\langle\beta_2,0.8,0.2\rangle\}))\}.
\end{aligned}$$

Now $\tilde{\tilde{\tau}} \cup \tilde{\tilde{\tau}}' = \{\tilde{\tilde{O}}, \tilde{\tilde{A}}, (\mathcal{G}_1,\mathcal{E}), (\mathcal{G}_2,\mathcal{E}), (\mathcal{G}_3,\mathcal{E}), (\mathcal{G}_4,\mathcal{E}), (\mathcal{G}_5,\mathcal{E}), (\mathcal{G}_6,\mathcal{E}), (\mathcal{G}_7,\mathcal{E}), (\mathcal{G}_8,\mathcal{E}), (\mathcal{G}_9,\mathcal{E})\}$, where

$$\begin{aligned}
(\mathcal{G}_1,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.6,0.4\rangle,\langle\alpha_2,0.6,0.3\rangle\},\{\langle\beta_1,0.6,0.3\rangle,\langle\beta_2,0.6,0.3\rangle\}))\\
&(e_2,(\{\langle\alpha_1,0.8,0.1\rangle,\langle\alpha_2,0.6,0.2\rangle\},\{\langle\beta_1,0.7,0.2\rangle,\langle\beta_2,0.7,0.2\rangle\}))\},\\
(\mathcal{G}_2,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.6,0.4\rangle,\langle\alpha_2,0.6,0.3\rangle\},\{\langle\beta_1,0.7,0.2\rangle,\langle\beta_2,0.6,0.3\rangle\}))\\
&(e_2,(\{\langle\alpha_1,0.8,0.1\rangle,\langle\alpha_2,0.7,0.1\rangle\},\{\langle\beta_1,0.8,0.1\rangle,\langle\beta_2,0.7,0.2\rangle\}))\},\\
(\mathcal{G}_3,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.7,0.3\rangle,\langle\alpha_2,0.6,0.3\rangle\},\{\langle\beta_1,0.8,0.1\rangle,\langle\beta_2,0.7,0.2\rangle\}))\\
&(e_2,(\{\langle\alpha_1,0.8,0.1\rangle,\langle\alpha_2,0.8,0.1\rangle\},\{\langle\beta_1,0.9,0\rangle,\langle\beta_2,0.8,0.2\rangle\}))\},\\
(\mathcal{G}_4,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.7,0.3\rangle,\langle\alpha_2,0.6,0.3\rangle\},\{\langle\beta_1,0.6,0.3\rangle,\langle\beta_2,0.7,0.2\rangle\}))\\
&(e_2,(\{\langle\alpha_1,0.9,0\rangle,\langle\alpha_2,0.7,0.1\rangle\},\{\langle\beta_1,0.8,0.2\rangle,\langle\beta_2,0.9,0\rangle\}))\},\\
(\mathcal{G}_5,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.7,0.3\rangle,\langle\alpha_2,0.6,0.3\rangle\},\{\langle\beta_1,0.7,0.2\rangle,\langle\beta_2,0.7,0.2\rangle\}))\\
&(e_2,(\{\langle\alpha_1,0.9,0\rangle,\langle\alpha_2,0.7,0.1\rangle\},\{\langle\beta_1,0.8,0.1\rangle,\langle\beta_2,0.9,0\rangle\}))\},\\
(\mathcal{G}_6,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.7,0.3\rangle,\langle\alpha_2,0.6,0.3\rangle\},\{\langle\beta_1,0.8,0.1\rangle,\langle\beta_2,0.7,0.2\rangle\}))\\
&(e_2,(\{\langle\alpha_1,0.9,0\rangle,\langle\alpha_2,0.8,0.1\rangle\},\{\langle\beta_1,0.9,0\rangle,\langle\beta_2,0.9,0\rangle\}))\},\\
(\mathcal{G}_7,\mathcal{E}) = \{&(e_1,(\{\langle\alpha_1,0.8,0.2\rangle,\langle\alpha_2,0.6,0.2\rangle\},\{\langle\beta_1,0.7,0.2\rangle,\langle\beta_2,0.7,0.1\rangle\}))\\
&(e_2,(\{\langle\alpha_1,1,0\rangle,\langle\alpha_2,0.9,0\rangle\},\{\langle\beta_1,0.8,0.2\rangle,\langle\beta_2,0.9,0\rangle\}))\},
\end{aligned}$$

$$(\mathcal{G}_8,\mathcal{E})=\{(e_1,(\{\langle\alpha_1,0.8,0.2\rangle,\langle\alpha_2,0.6,0.2\rangle\},\{\langle\beta_1,0.7,0.2\rangle,\langle\beta_2,0.7,0.1\rangle\}))$$
$$(e_2,(\{\langle\alpha_1,1,0\rangle,\langle\alpha_2,0.9,0\rangle\},\{\langle\beta_1,0.8,0.2\rangle,\langle\beta_2,0.9,0\rangle\}))\},$$
$$(\mathcal{G}_9,\mathcal{E})=\{(e_1,(\{\langle\alpha_1,0.8,0.2\rangle,\langle\alpha_2,0.6,0.2\rangle\},\{\langle\beta_1,0.8,0.1\rangle,\langle\beta_2,0.7,0.1\rangle\}))$$
$$(e_2,(\{\langle\alpha_1,1,0\rangle,\langle\alpha_2,0.9,0\rangle\},\{\langle\beta_1,0.9,0\rangle,\langle\beta_2,0.9,0\rangle\}))\},$$

$$(\mathcal{G}_2,\mathcal{E})\cap(\mathcal{G}_4,\mathcal{E})=\{(e_1,(\{\langle\alpha_1,0.6,0.4\rangle,\langle\alpha_2,0.6,0.3\rangle\},\{\langle\beta_1,0.6,0.3\rangle,\langle\beta_2,0.6,0.3\rangle\}))$$
$$(e_2,(\{\langle\alpha_1,0.8,0.1\rangle,\langle\alpha_2,0.7,0.1\rangle\},\{\langle\beta_1,0.8,0.2\rangle,\langle\beta_2,0.7,0.2\rangle\}))\}.$$

Here, $(\mathcal{G}_2,\mathcal{E}),(\mathcal{G}_4,\mathcal{E})\in\tilde{\tau}_1\cup\tilde{\tau}_2$. But, $(\mathcal{G}_2,\mathcal{E})\cap(\mathcal{G}_4,\mathcal{E})\notin\tilde{\tau}_1\cup\tilde{\tau}_2$.
However, $\tilde{\tau}_1\cap\tilde{\tau}_2$ is IFBS topology.

Theorem 3.9. *Let $\tilde{\tilde{\tau}}_1$ and $\tilde{\tilde{\tau}}_2$ be IFBS topology on $\mathcal{U}_1,\mathcal{U}_2$. Then, $\tilde{\tilde{\tau}}_1\cap\tilde{\tilde{\tau}}_2$ is IFBS topology on $\mathcal{U}_1,\mathcal{U}_2$.*

Proof. Let $\tilde{\tilde{\tau}}_1$ and $\tilde{\tilde{\tau}}_2$ be IFBS topology on $\mathcal{U}_1,\mathcal{U}_2$.

(i) $\tilde{\tilde{O}}$, $\tilde{\tilde{A}}\in\tilde{\tilde{\tau}}_1$ and $\tilde{\tilde{O}},\tilde{\tilde{A}}\in\tilde{\tilde{\tau}}_2$
$\Longrightarrow\quad \tilde{\tilde{O}},\tilde{\tilde{A}}\in\tilde{\tilde{\tau}}_1\cap\tilde{\tilde{\tau}}_2$.

(ii) Let $\{(\mathcal{F}_i,\mathcal{E})/i\in I\}$ be family of IFBSSs in $\tilde{\tilde{\tau}}_1\cap\tilde{\tilde{\tau}}_2$
$\Longrightarrow\quad (\mathcal{F}_i,\mathcal{E})\in\tilde{\tilde{\tau}}_1$ and $(\mathcal{F}_i,\mathcal{E})\in\tilde{\tilde{\tau}}_2$
$\Longrightarrow\quad \bigcup_{i\in I}(\mathcal{F}_i,\mathcal{E})\in\tilde{\tilde{\tau}}_1$ and $\cup_{i\in I}(\mathcal{F}_i,\mathcal{E})\in\tilde{\tilde{\tau}}_2$
$\Longrightarrow\quad \bigcup_{i\in I}(\mathcal{F}_i,\mathcal{E})\in\tilde{\tilde{\tau}}_1\cap\tilde{\tilde{\tau}}_2$

(iii) Let $(\mathcal{F},\mathcal{E}),(\mathcal{G},\mathcal{E})$ be any two IFBSSs in $\tilde{\tilde{\tau}}_1\cap\tilde{\tilde{\tau}}_2$
$\Longrightarrow\quad (\mathcal{F},\mathcal{E}),(\mathcal{G},\mathcal{E})\in\tilde{\tilde{\tau}}_1$ and $(\mathcal{F},\mathcal{E}),(\mathcal{G},\mathcal{E})\in\tilde{\tilde{\tau}}_2$
$\Longrightarrow\quad (\mathcal{F},\mathcal{E})\cap(\mathcal{G},\mathcal{E})\in\tilde{\tilde{\tau}}_1$ and $(\mathcal{F},\mathcal{E})\cap(\mathcal{G},\mathcal{E})\in\tilde{\tilde{\tau}}_2$
$\Longrightarrow\quad (\mathcal{F},\mathcal{E})\cap(\mathcal{G},\mathcal{E})\in\tilde{\tilde{\tau}}_1\cap\tilde{\tilde{\tau}}_2$.

Hence, from Definition 3.1, $\tilde{\tilde{\tau}}_1\cap\tilde{\tilde{\tau}}_2$ is IFBS topology on $\mathcal{U}_1,\mathcal{U}_2$. □

Definition 3.10. Let $(\mathcal{U}_1,\mathcal{U}_2,\tilde{\tilde{\tau}},\mathcal{E})$ be IFBSTS over U_1,U_2. Then, an IFBSS $(\mathcal{F},\mathcal{E})$ over $\mathcal{U}_1,\mathcal{U}_2$ is called neighborhood of IFBSS $(\mathcal{G},\mathcal{E})$ over $\mathcal{U}_1,\mathcal{U}_2$ if there exists $(\mathcal{H},\mathcal{E})\in\tilde{\tilde{\tau}}$ such that $(\mathcal{G},\mathcal{E})\subseteq(\mathcal{H},\mathcal{E})\subseteq(\mathcal{F},\mathcal{E})$.

Definition 3.11. Let $(\mathcal{U}_1,\mathcal{U}_2,\tilde{\tilde{\tau}},\mathcal{E})$ be IFBSTS over $\mathcal{U}_1,\mathcal{U}_2$. Let $(\mathcal{F},\mathcal{E})$ and $(\mathcal{G},\mathcal{E})$ be IFBSSs over $\mathcal{U}_1,\mathcal{U}_2$ such that $(\mathcal{G},\mathcal{E})\subseteq(\mathcal{F},\mathcal{E})$. Then, $(\mathcal{G},\mathcal{E})$ is called IFBS interior set of $(\mathcal{F},\mathcal{E})$ if and only if $(\mathcal{F},\mathcal{E})$ is neighborhood of $(\mathcal{G},\mathcal{E})$.
The union of all IFBS interior sets of $(\mathcal{F},\mathcal{E})$ is called interior of $(\mathcal{F},\mathcal{E})$ and denoted by $(\mathcal{F},\mathcal{E})^o$.

Theorem 3.12. *Let $(\mathcal{U}_1,\mathcal{U}_2,\tilde{\tilde{\tau}},\mathcal{E})$ be IFBSTS over $\mathcal{U}_1,\mathcal{U}_2$ and $(\mathcal{F},\mathcal{E})$ be IFBSS over $\mathcal{U}_1,\mathcal{U}_2$. Then,*

(a) *$(\mathcal{F},\mathcal{E})^o$ is an IFBS open set and $(\mathcal{F},\mathcal{E})^o$ is the largest IFBS subset contained in $(\mathcal{F},\mathcal{E})$.*
(b) *The IFBSS $(\mathcal{F},\mathcal{E})$ is IFBS open if and only if $(\mathcal{F},\mathcal{E})=(\mathcal{F},\mathcal{E})^o$.*
(c) *If $(\mathcal{F},\mathcal{E})\subseteq(\mathcal{G},\mathcal{E})$. Then, $(\mathcal{F},\mathcal{E})^o\subseteq(\mathcal{G},\mathcal{E})^o$.*

Proof. (a) $(\mathcal{F},\mathcal{E})^o = \cup\{(\mathcal{G},\mathcal{E})/(\mathcal{F},\mathcal{E})$ is neighborhood of $(\mathcal{G},\mathcal{E})\}$.

Clearly, $(\mathcal{F},\mathcal{E})^o$ is the IFBS interior set of $(\mathcal{F},\mathcal{E})$. Hence, there exists an IFBSS $(\mathcal{H},\mathcal{E}) \in \tilde{\tilde{\tau}}$ such that $(\mathcal{F},\mathcal{E})^o \subseteq (\mathcal{H},\mathcal{E}) \subseteq (\mathcal{F},\mathcal{E})$.

But $(\mathcal{H},\mathcal{E})$ is IFBS interior set of $(\mathcal{F},\mathcal{E})$.

Therefore, $(\mathcal{H},\mathcal{E}) \subseteq (\mathcal{F},\mathcal{E})^o$

$\implies \quad (\mathcal{F},\mathcal{E})^o = (\mathcal{H},\mathcal{E})$.

Hence, $(\mathcal{F},\mathcal{E})^o$ is open and largest open set contained in $(\mathcal{F},\mathcal{E})$.

(b) Let $(\mathcal{F},\mathcal{E})$ be IFBS open set.

$$(\mathcal{F},\mathcal{E}) \subseteq (\mathcal{F},\mathcal{E}). \tag{3.1}$$

But $(\mathcal{F},\mathcal{E})^o$ is the largest subset contained in $(\mathcal{F},\mathcal{E})$.

From eq. (3.1), $(\mathcal{F},\mathcal{E}) \subseteq (\mathcal{F},\mathcal{E})^o \subseteq (\mathcal{F},\mathcal{E})$

$\implies \quad (\mathcal{F},\mathcal{E}) = (\mathcal{F},\mathcal{E})^o$.

Conversely, suppose $(\mathcal{F},\mathcal{E}) = (F,E^o)$. Since, $(\mathcal{F},\mathcal{E})^o$ is an IFBS open set, $(\mathcal{F},\mathcal{E})$ is an IFBS open set.

(c) Let $(\mathcal{F},\mathcal{E}) \subseteq (\mathcal{G},\mathcal{E})$. Then, $(\mathcal{F},\mathcal{E})^o \subseteq (\mathcal{G},\mathcal{E})$. But $(\mathcal{G},\mathcal{E})^o$ is the largest open set contained in $(\mathcal{G},\mathcal{E})$. It follows that, $(\mathcal{F},\mathcal{E})^o \subseteq (\mathcal{G},\mathcal{E})^o$. □

Definition 3.13. Let $(\mathcal{U}_1,\mathcal{U}_2,\tilde{\tilde{\tau}},\mathcal{E})$ be IFBSTS over $\mathcal{U}_1,\mathcal{U}_2$. Let $(\mathcal{F},\mathcal{E})$ be an IFBSSs over $\mathcal{U}_1,\mathcal{U}_2$. Then, IFBS closure of $(\mathcal{F},\mathcal{E})$ is denoted by $\overline{(\mathcal{F},\mathcal{E})}$ and is defined as intersection of all IFBS closed sets containing $(\mathcal{F},\mathcal{E})$. Thus, $\overline{(\mathcal{F},\mathcal{E})}$ is smallest subset containing $(\mathcal{F},\mathcal{E})$.

Theorem 3.14. *Let $(\mathcal{U}_1,\mathcal{U}_2,\tilde{\tilde{\tau}},\mathcal{E})$ be IFBSTS over $\mathcal{U}_1,\mathcal{U}_2$. Let $(\mathcal{F},\mathcal{E})$ and $(\mathcal{G},\mathcal{E})$ be IFBSSs over $\mathcal{U}_1,\mathcal{U}_2$. Then,*

(a) *$\overline{(\mathcal{F},\mathcal{E})}$ is IFBS closed set containing $(\mathcal{F},\mathcal{E})$,*

(b) *$(\mathcal{F},\mathcal{E})$ is IFBS closed set if and only if $(\mathcal{F},\mathcal{E}) = \overline{(\mathcal{F},\mathcal{E})}$,*

(c) *if $(\mathcal{F},\mathcal{E}) \subseteq (\mathcal{G},\mathcal{E})$. Then, $\overline{(\mathcal{F},\mathcal{E})} \subseteq \overline{(\mathcal{G},\mathcal{E})}$.*

Proof. (a) Let $\{(\mathcal{F}_i,\mathcal{E})/i \in I\}$ be the collection of IFBS closed sets containing $(\mathcal{F},\mathcal{E})$.

$\overline{(\mathcal{F},\mathcal{E})} = \cap_{i\in I}(\mathcal{F}_i,\mathcal{E}) \quad \implies \quad \overline{(\mathcal{F},\mathcal{E})}$ is IFBS close set.

Clearly $(\mathcal{F},\mathcal{E}) \subseteq (\mathcal{F}_i,\mathcal{E}), \quad \forall\ i \in I$

$\implies \quad (\mathcal{F},\mathcal{E}) \subseteq \overline{(\mathcal{F},\mathcal{E})}$.

Thus, $\overline{(\mathcal{F},\mathcal{E})}$ IFBS closed set containing $(\mathcal{F},\mathcal{E})$.

(b) Let $(\mathcal{F},\mathcal{E})$ be IFBS closed set

$$(\mathcal{F},\mathcal{E}) \subseteq (\mathcal{F},\mathcal{E}). \tag{3.2}$$

But $\overline{(\mathcal{F},\mathcal{E})}$ is the smallest subset containing $(\mathcal{F},\mathcal{E})$.

From eq. (3.2), $(\mathcal{F},\mathcal{E}) \subseteq \overline{(\mathcal{F},\mathcal{E})} \subseteq (\mathcal{F},\mathcal{E})$. Hence, $(\mathcal{F},\mathcal{E}) = \overline{(\mathcal{F},\mathcal{E})}$.

(c) Let $(\mathcal{F},\mathcal{E}) \subseteq (\mathcal{G},\mathcal{E})$. Then, $(\mathcal{F},\mathcal{E}) \subseteq \overline{(\mathcal{G},\mathcal{E})}$. But $\overline{(\mathcal{F},\mathcal{E})}$ is the smallest IFBS closed set containing $(\mathcal{F},\mathcal{E})$. It follows that, $\overline{(\mathcal{F},\mathcal{E})} \subseteq \overline{(\mathcal{G},\mathcal{E})}$. □

Acknowledgement

The first author is thankful for the financial support provided by Karnatak University Dharwad for the research work under the URS scheme.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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