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Research Article

H-*E*-Super Magic Graceful Labelings of Graphs

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Abstract. This article examines *H-E-Super Magic Graceful Labelings* (*H-E-SMGL*) of graphs. These labelings provide a systematic way to analyze how graphs can be decomposed into subgraphs with specific properties, particularly in the context of *H-coverings*. We investigate several families of graphs, including fans, books, and grids admits *H-E-SMGL*.

Keywords. H-covering, H-magic labeling, H-E-super magic graceful labeling

Mathematics Subject Classification (2020). 05C78

1. Introduction

In this article, we take finite simple undirected graphs. In a graph $G(p,q)$, the set of vertices and edges is represented by $V(G)$ and $E(G)$ respectively, where $p = |V(G)|$ and $q = |E(G)|$. Different types of labeling have been studied and investigated by Gallian [1] provides a comprehensive summary of graph labeling (also see, Hafez *et al.* [4], Ichishima and Muntaner-Batle [5], Iyappan *et al.* [6], Kavitha *et al.* [7], Kumar *et al.* [9], and Mutharasu *et al.* [12]).

A group of subgraphs H_i, $1 \le i \le h$ that ensures every edge of $E(G)$ is a member of one or more of the subgraphs H_i, $1 \le i \le h$ constitutes a covering of G. It is then claimed that G accepts a covering of $(H_1, H_2, \dots, H_h)$. G permits a H-covering if each H_i is isomorphic to a given graph H. Assume that G accepts a covering of H. The term H-magic labeling of G refers to a total labeling, which is a one-one μ from $V(G)\cup E(G)$ onto $\{1,2,\dots,p+q\}$, if $\sum_{v\in V(H')}\mu(v)+\sum_{e\in E(H')}\mu(e)=M$ for any subgraph H' of G that isomorphic to H. A graph that permits for this type of labeling is called H-magic. If $\mu(E(G))=\{1,2,\dots,q\}$, then the function μ is considered as H-E-super magic labeling (H-E-SML).

The H-magic labeling was defined by Gutiérrez and Lladó [3]. Lladó and Moragas [10] examined a few C_n-supermagic graphs. β-valuation is a labeling that was defined by Rosa [13]. A graceful labeling of G is an injection f from the vertices of G to $\{0,1,\dots,q\}$, provided the labels that follow from assigning the label $|\mu(u)-\mu(v)|$ to each edge uv are distinct (see Kumar and Marimuthu [8] for additional details on H-E-SML).

An H-E-SMGL is an one one onto μ from $V(G)\cup E(G)$ onto $\{1,2,\dots,p+q\}$ with the condition that $\mu(E(G))=\{1,2,\dots,q\}$ such that, $\sum_{v\in V(H')}\mu(v)-\sum_{e\in E(H')}\mu(e)=M$ (Murugan and Kumar [11]). We investigate H-E-SMGL of fans, graphs obtained by joining a star $K_{1,n}$ with one isolated vertex, books and grids.

2. C_3-E-Super Magic Graceful Graphs (C_3-E-SMGL)

The C_3-E-SMGL of some connected graph, such as fans, which are graphs formed by combining a star $K_{1,m}$ with a single isolated vertex, is addressed in this section.

Theorem 2.1. *Let $n(\geq 2)$ be a positive integer. Then F_n is C_3-E-SMG.*

Proof. Let $V(F_n)=\{a_i,c: i=1,\dots,n\}$ and $E(F_n)=\{a_ia_{i+1}: i=1,\dots,n-1\}\cup\{a_i,c: i=1,\dots,n\}$. Describe a function μ from $V(F_n)\cup E(F_n)$ onto $\{1,\dots,3n\}$ as follows:

$$\mu(t)=\begin{cases}3n, & \text{when } t=c,\\ 2n+\frac{1}{2}(i-1), & \text{when } t=a_i \text{ for } i\equiv 1(\text{mod } 2),\\ \left\lfloor\frac{1}{2}(5n+i-1)\right\rfloor, & \text{when } t=a_i \text{ for } i\equiv 0(\text{mod } 2),\end{cases}$$

$$\mu(e)=\begin{cases}n-i, & \text{when } e=a_ia_{i+1} \text{ for } i=1,\dots,n-1,\\ n-1+i, & \text{when } e=ca_i \text{ for } i=1,\dots,n.\end{cases}$$

For $i=1,\dots,n-1$, let $C_3^{(i)}$ be the subcycle of F_n. The associated subcycles are $V(C_3^{(i)})=\{c,a_i,a_{i+1}\}$ and $E(C_3^{(i)})=\{ca_i,a_ia_{i+1},c_ia_{i+1}\}$. Now we prove that $f\mu$ is C_3-E-SMG.

For $1\leq i\leq n-1$,

$$\begin{aligned}\sum_{v\in V(C_3^{(i)})}\mu(v)-\sum_{e\in E(C_3^{(i)})}\mu(e)&=\mu(c)+\mu(a_i)+\mu(a_{i+1})-\mu(a_ia_{i+1})-\mu(ca_i)-\mu(c_ia_{i+1})\\ &=3n+2n+\frac{1}{2}(i-1)+\left\lfloor\frac{1}{2}(5n+i)\right\rfloor-(n-i)-(n-1+i)-(n+i)\\ &=\left\lfloor\frac{1}{2}(9n+1)\right\rfloor.\end{aligned}$$

Hence F_n is C_3-E-SMG. □

Theorem 2.2. *The graph $G\cong K_{1,n}+K_1$, $n\ (\geq 1)$ is C_3-E-SMG.*

Proof. Let $V(G)=\{a_1,a_2,b_j: i=1,\dots,n\}$ and $E(G)=\{a_1a_2\}\cup\{a_ib_j: i=1,2; 1\leq j\leq n\}$. Describe a function μ from $V(G)\cup E(G)$ onto $\{1,2,\dots,3n+3\}$ as follows:

Suppose n is odd:

$$\mu(s)=\begin{cases}2n+1+i, & \text{when } s=a_i \text{ for } i=1,2,\\ 3n+3-(i-1), & \text{when } s=b_i \text{ for } 1\le i\le n,\\ 1, & \text{when } s=a_1a_2,\\ \frac{1}{2}(n+3+2i), & \text{when } s=a_1b_i \text{ for } i=1,2,\dots,\frac{n-1}{2},\\ \frac{1}{2}(-n+3+2i), & \text{when } s=a_1b_i \text{ for } i=\frac{n+1}{2},\frac{n+3}{2},\dots,n,\\ 2n+2-2i, & \text{when } s=a_2b_i \text{ for } i=1,2,\dots,\frac{n-1}{2},\\ 3n+2-2i, & \text{when } s=a_2b_i \text{ for } i=\frac{n+1}{2},\frac{n+3}{2},\dots,n.\end{cases}$$

Suppose n is even:

$$\mu(t)=\begin{cases}2n+1+i, & \text{when } t=a_i \text{ for } i=1,2,\\ 3n+3-(i-1), & \text{when } t=b_i \text{ for } 1\le i\le n,\\ \frac{n+2}{2}, & \text{when } t=a_1a_2\\ \frac{2n+3-i}{2}, & \text{when } t=a_1b_i \text{ for } i=1,3,\dots,n-,1\\ \frac{n+2-i}{2}, & \text{when } t=a_1b_i \text{ for } i=2,4,\dots,n,\\ \frac{3n+3-i}{2}, & \text{when } t=a_2b_i \text{ for } i=1,3,\dots,n-1,\\ \frac{4n+4-i}{2}, & \text{when } t=a_2b_i \text{ for } i=2,4,\dots,n.\end{cases}$$

Let $C_3^{(i)}$ be the subcycle of G for $1\le i\le n$, and let $E(C_3^{(i)})=\{a_1a_2,a_1b_i,a_2b_i\}$ and $V(C_3^{(i)})=\{a_1,a_2,b_i\}$.

Case 1: Suppose n is odd.

Subcase 1: For $i=1,2,\dots,\frac{n-1}{2}$,

$$\begin{aligned}\sum_{v\in V(C_3^{(i)})}\mu(v)-\sum_{e\in E(C_3^{(i)})}\mu(e) &= \mu(a_1)+\mu(a_2)+\mu(b_i)-\mu(a_1a_2)-\mu(a_1b_i)-\mu(a_2b_i)\\ &=(2n+2)+(2n+3)+(3n+3-i+1)-(1)\\ &\quad -\left(\frac{1}{2}(n+3+2i)\right)-(2n+2-2i)\\ &=\frac{9n+9}{2}.\end{aligned}$$

Subcase 2: For $i=\frac{n+1}{2},\frac{n+3}{2},\dots,n$,

$$\begin{aligned}\sum_{v\in V(C_3^{(i)})}\mu(v)-\sum_{e\in E(C_3^{(i)})}\mu(e) &= \mu(a_1)+\mu(a_2)+\mu(b_i)-\mu(a_1a_2)-\mu(a_1b_i)-\mu(a_2b_i)\\ &=(2n+2)+(2n+3)+(3n+3-i+1)-(1)\\ &\quad -\left(\frac{1}{2}(-n+3+2i)\right)-(3n+2-2i)\\ &=\frac{9n+9}{2}.\end{aligned}$$

Case 2: Suppose n is even.

Subcase 3: For $i = 1,3,\dots,n-1$,

$$\begin{aligned}\sum_{v\in V(C_3^{(i)})}\mu(v)-\sum_{e\in E(C_3^{(i)})}\mu(e) &= \mu(a_1)+\mu(a_2)+\mu(b_i)-\mu(a_1a_2)-\mu(a_1b_i)-\mu(a_2b_i)\\ &= (2n+2)+(2n+3)+(3n+3-i+1)\\ &\quad -\left(\frac{n+2}{2}\right)-\left(\frac{2n+3-i}{2}\right)-\left(\frac{3n+3-i}{2}\right)\\ &= 4n+5.\end{aligned}$$

Subcase 4: For $i = 2,4,\dots,n$,

$$\begin{aligned}\sum_{v\in V(C_3^{(i)})}\mu(v)-\sum_{e\in E(C_3^{(i)})}\mu(e) &= \mu(a_1)+\mu(a_2)+\mu(b_i)-\mu(a_1a_2)-\mu(a_1b_i)-\mu(a_2b_i)\\ &= (2n+2)+(2n+3)+(3n+3-i+1)\\ &\quad -\left(\frac{n+2}{2}\right)-\left(\frac{n+2-i}{2}\right)-\left(\frac{4n+4-i}{2}\right)\\ &= 4n+5.\end{aligned}$$

Hence G is C_3-E-SMG. □

3. C_4-E-Super Magic Graceful Graphs

In this section, we investigates C_4-E-SMGL of grids and books.

Theorem 3.1. *The graph $B_m = K_{1,m}\times K_2$, $m\ge 2$ is C_4-E-SMG.*

Proof. Let $V(B_m)=\{a_1,a_2\}\cup\{b_i,c_i : 1\le i\le m\}$ and $E(B_m)=\{a_1a_2\}\cup\{a_1c_i,a_2b_i,b_ic_i : 1\le i\le m\}$. Describe a function $\mu : V(B_m)\cup E(B_m)\to\{1,2,\dots,5m+3\}$ as follows:

Suppose m is odd:

$$\mu(s)=\begin{cases}3m+1+i, & \text{when } s=a_i \text{ for } i=1,2,\\ 3m+3+i, & \text{when } s=b_i \text{ for } 1\le i\le m,\\ 5m+4-i, & \text{when } s=c_i \text{ for } 1\le i\le m,\\ 1, & \text{when } s=a_1a_2,\\ 1+i, & \text{when } s=a_2b_i \text{ for } 1\le i\le m,\\ 3m+3-2i, & \text{when } s=a_1c_i \text{ for } 1\le i\le \lceil\frac{m}{2}\rceil,\\ 4m+3-2i, & \text{when } s=a_1c_i \text{ for } \lceil\frac{m}{2}\rceil+1\le i\le m,\\ \frac{1}{2}(3m+1+2i), & \text{when } s=b_ic_i \text{ for } 1\le i\le \lceil\frac{m}{2}\rceil,\\ \frac{1}{2}(m+1+2i), & \text{when } s=b_ic_i \text{ for } \lceil\frac{m}{2}\rceil+1\le i\le m.\end{cases}$$

Suppose m is even:

$$\mu(t)=\begin{cases}3m+1+i, & \text{when } t=a_i \text{ for } i=1,2,\\ 3m+3+i, & \text{when } t=b_i \text{ for } 1\le i\le m,\\ 5m+4-i, & \text{when } t=c_i \text{ for } 1\le i\le m,\\ \frac{m}{2}+1, & \text{when } t=a_1a_2,\\ i, & \text{when } t=a_2b_i \text{ for } 1\le i\le \frac{m}{2},\\ 1+i, & \text{when } t=a_2b_i \text{ for } \frac{m}{2}\le i\le m,\\ 3m+3-2i, & \text{when } t=a_1c_i \text{ for } 1\le i\le \frac{m}{2},\\ 4m+2-2i, & \text{when } t=a_1c_i \text{ for } \frac{m}{2}\le i\le m,\\ \frac{1}{2}(3m+2+2i), & \text{when } t=b_ic_i \text{ for } 1\le i\le \frac{m}{2},\\ \frac{1}{2}(m+2+2i), & \text{when } t=b_ic_i \text{ for } \frac{m}{2}+1\le i\le m.\end{cases}$$

Let $C_4^{(i)}$ be the subcycle of B_m for $1\le i\le m$, and let $E(C_4^{(i)})=\{a_1a_2,a_2b_i,a_1c_i,b_ic_i\}$ and $V(C_4^{(i)})=\{a_1,a_2,b_i,c_i\}$.

Case 1: Suppose m is odd.

Subcase 1: For $i=1,2,\dots,\lceil\frac{m}{2}\rceil$.
Then

$$\begin{aligned}\sum_{v\in V(C_4^{(i)})}\mu(v)-\sum_{e\in E(C_4^{(i)})}\mu(e)&=\mu(a_1)+\mu(a_2)+\mu(b_i)+\mu(c_i)-\mu(a_1a_2)-\mu(a_2b_i)-\mu(a_1c_i)-\mu(b_ic_i)\\&=(3m+2)+(3m+3)+(3m+3+i)+(5m+4-i)-(1)\\&\quad-(1+i)-(3m+3-2i)-\left(\frac{1}{2}(3m+1+2i)\right)\\&=\frac{19m+13}{2}.\end{aligned}$$

Subcase 2: For $i=\lceil\frac{m}{2}\rceil+1,\lceil\frac{m}{2}\rceil+2,\dots,m$.
Then

$$\begin{aligned}\sum_{v\in V(C_4^{(i)})}f(v)-\sum_{e\in E(C_4^{(i)})}\mu(e)&=\mu(a_1)+\mu(a_2)+\mu(b_i)+\mu(c_i)-\mu(a_1a_2)-\mu(a_2b_i)-\mu(b_ic_i)-\mu(a_ic_i)\\&=3m+2+3m+3+3m+3+i+5m+4-i-1-(1+i)\\&\quad-(4m+3-2i)-\left(\frac{1}{2}(m+1+2i)\right)\\&=\frac{19m+13}{2}.\end{aligned}$$

Case 2: Suppose m is even.

Subcase 3: For $i=1,2,\dots,\frac{m}{2}$.
Then

$$\sum_{v\in V(C_4^{(i)})}\mu(v)-\sum_{e\in E(C_4^{(i)})}\mu(e)=\mu(a_1)+\mu(a_2)+\mu(b_i)+\mu(c_i)-\mu(a_1a_2)-\mu(a_2b_i)-\mu(a_1c_i)-\mu(b_ic_i)$$

$$= (3m+2)+(3m+3)+(3m+3+i)+(5m+4-i) - \left(\frac{m}{2}+1\right)-(1+i)-(3m+3-2i)-\left(\frac{1}{2}(3m+2+2i)\right) = 9m+7.$$

Subcase 4: For $i = \frac{m}{2}+1, \frac{m}{2}+2, \dots, m$,

$$\sum_{v\in V(C_4^{(i)})}\mu(v)-\sum_{e\in E(C_4^{(i)})}\mu(e) = \mu(a_1)+\mu(a_2)+\mu(b_i)+\mu(c_i)-\mu(a_1a_2)-\mu(a_2b_i)-\mu(a_1c_i)-\mu(b_ic_i)$$

$$= (3m+2)+(3m+3)+(3m+3+i)+(5m+4-i) - \left(\frac{m}{2}+1\right)-(1+i)-(4m+2-2i)-\left(\frac{1}{2}(m+2+2i)\right) = 9m+7.$$

Hence B_n is C_4-E-SMG. □

Theorem 3.2. *The graph $G = P_t \times P_2$ is C_4-E-SMG.*

Proof. Let $V(G) = \{a_i, b_i : 1 \le i \le t\}$ and $E(G) = \{a_ib_i : 1 \le i \le t\} \cup \{a_ia_{i+1}, b_ib_{i+1} : 1 \le i \le t-1\}$.

Describe a function $f : V(G)\cup E(G) \to \{1,2,\dots,5t-2\}$ as follows:

$$f(s) = \begin{cases} 3t+i-2, & \text{when } s = a_i \text{ for } 1 \le i \le t,\\ 5t-i-1, & \text{when } s = b_i \text{ for } 1 \le i \le t,\\ 3i, & \text{when } s = a_ia_{i+1} \text{ for } 1 \le i \le t-1,\\ 3i-1, & \text{when } s = b_ib_{i+1} \text{ for } 1 \le i \le t-1,\\ 3t+1-3i, & \text{when } s = a_ib_i \text{ for } 1 \le i \le t.\end{cases}$$

For $1 \le i \le t-1$, let $C_4^{(i)}$ be the subcycle of G with $V(C_4^{(i)}) = \{a_i, a_{i+1}, b_i, b_{i+1}\}$ and $E(C_4^{(i)}) = \{a_ia_{i+1}, b_ib_{i+1}, a_ib_i, a_{i+1}b_{i+1}\}$.

For $1 \le i \le t-1$,

$$\sum_{v\in V(C_4^{(i)})} f(v) - \sum_{e\in E(C_4^{(i)})}\mu(e)$$
$$= \mu(a_i)+\mu(a_{i+1})+\mu(b_i)+\mu(b_{i+1})-\mu(a_ia_{i+1})-\mu(b_ib_{i+1})-\mu(a_ib_i)-\mu(a_{i+1}b_{i+1})$$
$$= (3t+i-2)+(3t+i-1)+(5t-i-1)+(5t-i-2)-(3i)-(3i-1)-(3t+1-3i)-(3t+1-3i-3)$$
$$= 10t-4.$$

Hence G is C_4-E-SMG. □

Theorem 3.3. *The graph $G = P_t \times P_3$, $t \ge 2$ is C_4-E-SMG.*

Proof. Let $V(G) = \{a_{i,j} : 1 \le i \le 3, 1 \le j \le t\}$ and $E(G) = \{a_{i,j}a_{i,j+1} : 1 \le i \le 3, 1 \le j \le t-1\} \cup \{a_{i,j}a_{i+1,j} : 1 \le i \le 2, 1 \le j \le t\}$.

Describe a function $\mu : V(G)\cup E(G) \to \{1,2,\dots,8t-3\}$ as follows:

For $1 \le j \le t$,

$$\mu(a_{i,j}) = \begin{cases} (\frac{i-1}{2})t+5t-3+j, & \text{when } i = 1,3,\\ 7t-3+j, & \text{when } i = 2,\end{cases}$$

$$\mu(a_{i,j}a_{i+1,j}) = \begin{cases} 3t+2(j-1)-2, & \text{when } i=1, \\ 3t+2(j-1)-1, & \text{when } i=2. \end{cases}$$

For $1 \le j \le t-1$,

$$\mu(a_{i,j}a_{i,j+1}) = \begin{cases} t-j, & \text{when } i=1, \\ t+j-1, & \text{when } i=2, \\ 3t-j-2, & \text{when } i=3. \end{cases}$$

For $1 \le i \le 2$ and $1 \le j \le t-1$, let $C_4^{(i,j)}$ be the subcycle of G with

$$V(C_4^{(i,j)}) = \{a_{i,j}, a_{i,j+1}, a_{i+1,j}, a_{i+1,j+1}\}$$

and

$$E(C_4^{(i,j)}) = \{a_{i,j}a_{i,j+1}, a_{i+1,j}a_{i+1,j+1}, a_{i,j}a_{i+1,j}, a_{i,j+1}a_{i+1,j+1}\}.$$

For $1 \le i \le 2$ and $1 \le j \le t-1$, then

$$\begin{aligned}\sum_{v\in V(C_4^{(i,j)})} f(v) - \sum_{e\in E(C_4^{(i,j)})} \mu(e) &= \mu(a_{i,j}) + \mu(a_{i,j+1}) + \mu(a_{i+1,j}) + \mu(a_{i+1,j+1}) - \mu(a_{i,j}a_{i,j+1}) \\ &\quad - \mu(a_{i+1,j}a_{i+1,j+1}) - \mu(a_{i,j}a_{i+1,j}) - \mu(a_{i,j+1}a_{i+1,j+1}).\end{aligned}$$

Case 1: Suppose $i=1$, then

$$\begin{aligned}\sum_{v\in V(C_4^{(1,j)})} \mu(v) - \sum_{e\in E(C_4^{(1,j)})} \mu(e) &= (5t-3+j)+(5t-3+j+1)+(7t-3+j)+(7t-3+j+1) \\ &\quad -(t-j)+(t+j-1)+(3t+2(j-1)-2)-(3t+2(j)-2) \\ &= 16t-3.\end{aligned}$$

Case 2: Suppose $i=2$, then

$$\begin{aligned}\sum_{v\in V(C_4^{(2,j)})} \mu(v) - \sum_{e\in E(C_4^{(2,j)})} \mu(e) &= (7t-3+j)+(7t-3+j+1)+(6t-3+j)+(6t-3+j+1) \\ &\quad -(t+j-1)+(3t-j-2)+(3t+2(j-1)-1)-(3t+2j-1) \\ &= 16t-3.\end{aligned}$$

□

4. Conclusion

We studied H-E-SMGL of fans, graphs obtained by joining a star $K_{1,n}$ with one isolated vertex, books and grids in this paper.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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