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Research Article

Integral Transforms of Pragathi-Satyanarayana's *I*-function

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Abstract. Many of the transformations like Euler, Hankel, Sumudu and K-transforms play a vital role in the field of engineering mathematics and have many applications. This paper refers to the study of Pragathi-Satyanarayana I-function of one variable. As a part of this study, we obtain different integral transforms of Pragathi-Satyanarayana I-function of one variable. Also, some of the generalized transforms have been obtained as special cases. The integral transformations developed here are useful in real-world applications of mathematical science.

Keywords. Pragathi-Satyanarayana I-function, Hankel transform, Sumudu transform, K-transform, Euler-beta transforms

Mathematics Subject Classification (2020). 33C60, 33C99

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1. Introduction

Fox [6] created the well-known one-variable H-function and showed that it is a symmetric Fourier kernel to Meijer's G-function (Shah *et al.* [18]). Later, Rathie [15] introduced the I-function to the literature in 1997, which plays an important role in physics, applied mathematics, and in other fields. A novel Pragathi-Satyanarayana I-function (Psi-function), which proved to be the generalized I-function (Ayant [2]) and generalized H-function (Srivastava *et al*. [19]) was recently introduced by Kumar and Satyanarayana [11]. In this paper, we establish different types of integral transforms of Pragathi-Satyanarayana I-function of one variable function.

Psi-function of one variable defined by Kumar and Satyanarayana [11] is given by

$$\Psi^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n};(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\\(b_j,\beta_j;B_j)_{1,m};(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i}\end{matrix}\right.\right]=\frac{1}{2\pi i}\int_L \phi(s)z^s ds, \tag{1.1}$$

where

$$\phi(s)=\frac{\prod\limits_{j=1}^{m}\Gamma^{B_j}(b_j-\beta_j s)\prod\limits_{j=1}^{n}\Gamma^{A_j}(1-a_j+\alpha_j s)}{\sum\limits_{i=1}^{r}\left[\prod\limits_{j=m+1}^{q_i}\Gamma^{B_{ji}}(1-b_{ji}+\beta_{ji}s)\prod\limits_{j=n+1}^{p_i}\Gamma^{A_{ji}}(a_{ji}-\alpha_{ji}s)\right]}. \tag{1.2}$$

Also,

- $z\neq 0$.
- $0\le n\le p_i$, where p_i $(i=1,\dots,r)$, $0\le m\le q_i$, where q_i $(i=1,\dots,r)$ and r is finite.
- α_j, β_j, α_{ji} and β_{ji} are positive integers and A_j, B_j, A_{ji} and B_{ji} are non-negative integers.
- a_j, b_j, a_{ji} and b_{ji} are complex numbers such that no singularities of $\Gamma^{B_j}(b_j-\beta_j s)$, $j=1,\dots,m$ coincides with any singularities of $\Gamma^{A_j}(1-a_j+\alpha_j s)$, $j=1,\dots,n$.
- Contour L is defined over $\sigma-i\infty$ to $\sigma+i\infty$. Using the intricate s-plane in such a manner that all the singularities of $\Gamma^{B_j}(b_j-\beta_j s)$, $j=1,\dots,m$ lies to the right side of L, and all the singularities of $\Gamma^{A_j}(1-a_j+\alpha_j s)$, $j=1,\dots,n$ lie to the left side of L.

Following different transforms (Gradshteyin and Ryzhik [7], and Jasim *et al*. [9]) have been applied on Pragathi-Satyanarayana I-function and the results are shown in Section 2:

(a) *Hankel transform*: The Hankel transform (Ali and Kalla [1]) of order γ of a function $f(r)$ is given by

$$H_\gamma(k)=\int_0^\infty f(r)J_\gamma(kr)r\,dr, \tag{1.3}$$

where J_γ is the Bessel function of the first kind of order γ with $\gamma\ge -\frac{1}{2}$.

(b) *Sumudu transform*: Over the set of functions

$$A=\{f(t)\mid \exists\, M,\tau_1,\tau_2>0, |f(t)|<Me^{|t|/\tau_i},\ \text{if } t\in(-1)^j\times[0,\infty)\},$$

Sumudu transform (Belgacem and Karaballi [3]) is defined by

$$G(u)=S[f(t)]=\int_0^\infty f(ut)e^{-t}dt,\quad u\in(-\tau_1,\tau_2). \tag{1.4}$$

(c) *K-transform*: The K-transform (Nasim [14]) of order γ of a function $f(x)$ is given by

$$R_\gamma\{f(x):\rho\}=\int_0^\infty(\rho x)^{\frac{1}{2}}K_\gamma(\rho x)f(x)dx,\quad \rho>0, \tag{1.5}$$

where K_γ is the known as Bessel modified function of third kind.

(d) *Euler-beta transform*: The Euler-beta transform (Satyanarayana *et al.* [17]) of the function $f(z)$ is defined as

$$B\{f(z);a,b\} = \int_0^1 z^{a-1}(1-z)^{b-1}f(z)dz\,, \tag{1.6}$$

where $\Re(a) > 0$ and $\Re(b) > 0$.

Also, we considered the following Laplace transform (Ricci *et al.* [16]) of Pragathi-Satyanarayana I-function given by Kumar and Satyanarayana [11] as

$$F(\mu) = L[\Psi[ax^{\sigma}];s]$$

$$= \sum_{s=0}^{\infty} \frac{(-\mu)^s}{s!} \frac{a^{-\frac{s+1}{\sigma}}}{\sigma} \frac{\prod_{j=1}^{m} \Gamma^{B_j}\left(b_j + \beta_j\left(\frac{s+1}{\sigma}\right)\right) \prod_{j=1}^{n} \Gamma^{A_j}\left(1 - a_j - \alpha_j\left(\frac{s+1}{\sigma}\right)\right)}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{q_i} \Gamma^{B_{ji}}\left(1 - b_{ji} - \beta_{ji}\left(\frac{s+1}{\sigma}\right)\right) \prod_{j=n+1}^{p_i} \Gamma^{A_{ji}}\left(a_{ji} + \alpha_{ji}\left(\frac{s+1}{\sigma}\right)\right)\right]}. \tag{1.7}$$

In the following section, we are going to derive different integral transforms (1.3), (1.4), (1.5) and (1.6) of Pragathi-Satyanarayana's I-function of single variable.

2. Main Results

This section provides detailed proofs of the integral transforms such as Hankel transform (Ali and Kalla [1]), Sumudu transform (Belgacem *et al.* [4]), K-transform (Nasim [14]) and the Euler-beta transforms (Choudhry *et al.* [5]) applied on Pragathi-Satyanarayana's I-function of a single variable.

Theorem 2.1. *Prove that*

$$H_{\gamma}[\psi(ax^{\sigma})] = \frac{\sqrt{2}}{\rho} \psi^{m,n+1}_{p_i+1,q_i+1;r}\left[a\left(\frac{2}{\rho}\right)^{\sigma} \middle| \begin{matrix} \left(\frac{1}{4} - \frac{\gamma}{2}, \frac{\sigma}{2}; 1\right), (a_j, \alpha_j; A_j)_{1,n}, \\ (a_{ji}, \alpha_{ji}; A_{ji})_{n+1,p_i}, (b_j, \beta_j; B_j)_{1,m}, \\ (b_{ji}, \beta_{ji}, B_{ji})_{m+1,q_i}, \left(\frac{3}{4} - \frac{\gamma}{2}, -\frac{\sigma}{2}; 1\right) \end{matrix}\right] \tag{2.1}$$

provided

(a_1) $Re(\rho) > 0$, $Re(\sigma) > 0$,

(a_2) $Re(b_j + \beta_j s) > 0$,

(a_3) $Re(1 - a_j - \alpha_j s) > 0$,

(a_4) $|\arg a| < \frac{\pi}{2}\Delta$, $\Delta > 0$,

(a_5) $|\arg a| \geq \frac{\pi}{2}\Delta$, $(\nabla + \mu) > 1$, $\Delta \geq 0$.

Proof. Substituting (1.1) in the definition of Hankel transform (1.3), we get

$$H_{\gamma}\{\psi(ax^{\sigma})\} = \int_0^{\infty} (\rho x)^{\frac{1}{2}} J_{\gamma}(\rho x)\left[\frac{1}{2\pi i}\int_L \phi(s) a^s x^{\sigma s} ds\right] dx$$

$$= \frac{1}{2\pi i}\int_L \phi(s) a^s \left[\int_0^{\infty} (\rho x)^{\frac{1}{2}} J_{\gamma}(\rho x) x^{\sigma s} dx\right] ds.$$

With the help of gamma representation to the above integral with the limits from 0 to ∞ gives us

$$H_{\gamma}\{\psi(ax^{\sigma})\} = \frac{1}{2\pi i}\int_L \phi(s) a^s 2^{\left(\sigma s + \frac{1}{2}\right)} \rho^{-\sigma s - 1} \frac{\Gamma\left(\frac{\sigma s}{2} + \frac{\gamma}{2} + \frac{3}{4}\right)}{\Gamma\left(\frac{\gamma}{2} - \frac{\sigma s}{2} + \frac{1}{4}\right)} ds$$

$$=\frac{\sqrt{2}}{\rho}\frac{1}{2\pi i}\int_L \phi(s)\frac{\Gamma\left(\frac{\sigma s}{2}+\frac{\gamma}{2}+\frac{3}{4}\right)}{\Gamma\left(\frac{\gamma}{2}-\frac{\sigma s}{2}+\frac{1}{4}\right)}\left(\frac{a2^{\sigma}}{\rho}\right)^s ds.$$

From the representation of (1.2), we have

$$H_\gamma\{\psi(ax^\sigma)\}=\frac{\sqrt{2}}{\rho}\psi^{m,n+1}_{p_i+1,q_i+1,r}\left[a\left(\frac{2}{\rho}\right)^{\sigma}\left|\begin{matrix}\left(\frac{1}{4}-\frac{\gamma}{2},\frac{\sigma}{2};1\right),(a_j,\alpha_j;A_j)_{1,n},\\ (a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i},(b_j,\beta_j;B_j)_{1,m},\\ (b_{ji},\beta_{ji};B_{ji})_{m+1,q_i},\left(\frac{3}{4}-\frac{\gamma}{2},-\frac{\sigma}{2},1\right)\end{matrix}\right.\right]. \qquad \square$$

Theorem 2.2. *Prove that*

$$S\{\psi(ax^\sigma)\}=\frac{1}{\sigma}\sum_{s=0}^{\infty}\frac{(-1)^s}{\mu^{s+1}s!}a^{-\frac{s+1}{\sigma}}\frac{\prod_{j=1}^{m}\Gamma^{B_j}\left(b_j+\beta_j\left(\frac{s+1}{\sigma}\right)\right)\prod_{j=1}^{n}\Gamma^{A_j}\left(1-a_j-\alpha_j\left(\frac{s+1}{\sigma}\right)\right)}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{q_i}\Gamma^{B_{ji}}\left(1-b_{ji}-\beta_{ji}\left(\frac{s+1}{\sigma}\right)\right)\prod_{j=n+1}^{p_i}\Gamma^{A_{ji}}\left(a_{ji}+\alpha_{ji}\left(\frac{s+1}{\sigma}\right)\right)\right]} \tag{2.2}$$

provided

(a$_1$) $Re(\rho)>0$, $Re(\sigma)>0$,

(a$_2$) $Re\left(b_j+\beta_j\left(\frac{s+1}{\sigma}\right)\right)>0$,

(a$_3$) $Re\left(1-a_j-\alpha_j\left(\frac{s+1}{\sigma}\right)\right)>0$,

(a$_4$) $\Delta>0$, $|\arg a|<\frac{\pi}{2}\Delta$,

(a$_5$) $\Delta\geq 0$, $|\arg z|\geq\frac{\pi}{2}\Delta$, $(\nabla+\mu)>1$.

Proof. Using the dual property of Sumudu transform (Belgacem and Karaballi [3]), and Laplace transform (Ricci *et al.* [16]), we have

$$S\{\psi(ax^\sigma)\}=\frac{1}{\mu}F\left(\frac{1}{\mu}\right).$$

Using (1.7), Sumudu transform of Pragathi-Satyanarayan's I-function becomes

$$S\{\psi(ax^\sigma)\}=\frac{1}{\sigma}\sum_{s=0}^{\infty}\frac{(-1)^s}{\mu^{s+1}s!}a^{-\frac{s+1}{\sigma}}\frac{\prod_{j=1}^{m}\Gamma^{B_j}\left(b_j+\beta_j\left(\frac{s+1}{\sigma}\right)\right)\prod_{j=1}^{n}\Gamma^{A_j}\left(1-a_j-\alpha_j\left(\frac{s+1}{\sigma}\right)\right)}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{q_i}\Gamma^{B_{ji}}\left(1-b_{ji}-\beta_{ji}\left(\frac{s+1}{\sigma}\right)\right)\prod_{j=n+1}^{p_i}\Gamma^{A_{ji}}\left(a_{ji}+\alpha_{ji}\left(\frac{s+1}{\sigma}\right)\right)\right]}. \qquad \square$$

Theorem 2.3. *Prove that*

$$\int_0^\infty x^{\rho-1}K_\eta(ax)\psi(bx^\sigma)dx=2^{\rho-2}a^{-\rho}\psi^{m,n+2}_{p_i+2,q_i;r}\left[b\left(\frac{2}{a}\right)^{\sigma}\left|\begin{matrix}\left(1-\frac{\rho}{2}-\frac{\eta}{2},\frac{\sigma}{2};1\right),\left(1-\frac{\rho}{2}+\frac{\eta}{2},\frac{\sigma}{2};1\right),\\ (b_j,\beta_j;B_j)_{1,m},(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i},\\ (a_j,\alpha_j;A_j)_{1,n},(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}\end{matrix}\right.\right] \tag{2.3}$$

provided

(a$_1$) $\sigma>0$, $Re(a)>0$, $\mu>0$, $Re(\rho)>0$,

(a$_2$) $Re(b_j+\beta_j s)>0$,

(a$_3$) $Re(1-a_j-\alpha_j s)>0$,

(a$_4$) $|\arg a|<\frac{\pi}{2}\Delta$, $\Delta>0$,

(a$_5$) $|\arg z|\geq\frac{\pi}{2}\Delta$, $(\nabla+\mu)>1$ *and* $\Delta\geq 0$,

(a$_6$) $\eta\in C$.

Proof. Using the definition of K-transform (1.5) and substituting (1.1) in the left-hand side of the theorem statement, we get

$$\int_0^\infty x^{\rho-1}K_\eta(ax)\psi(bx^\sigma)dx = \int_0^\infty x^{\rho-1}K_\gamma(ax)\left(\frac{1}{2\pi i}\int_L \phi(s)b^s x^{\sigma s}ds\right)dx$$

$$= \frac{1}{2\pi i}\int_L \phi(s)\left(\int_0^\infty x^{\rho+\sigma s-1}K_\eta(ax)dx\right)b^s ds.$$

Using the formula

$$\int_0^\infty x^{\rho-1}K_\eta(ax)dx = 2^{\rho-2}a^{-\rho}\Gamma\left(\frac{\rho\pm\eta}{2}\right) \qquad \text{(Mathai and Saxena [12, p. 78])}$$

$$= \frac{1}{2\pi i}\int_L \phi(s)2^{\rho+\sigma s-2}a^{-\rho-\sigma s}\Gamma\left(\frac{\rho+\sigma s\pm\eta}{2}\right)b^s ds$$

$$= 2^{\rho-2}a^{-\rho}\frac{1}{2\pi i}\int_L \phi(s)\Gamma\left(\frac{\rho+\sigma s\pm\eta}{2}\right)(ba^{-\sigma}2^\sigma)^s ds.$$

From the equation (1.2), we have

$$\int_0^\infty x^{\rho-1}K_\eta(ax)\psi(bx^\sigma)dx = 2^{\rho-2}a^{-\rho}\psi^{m,n+2}_{p_i+2,q_i;r}\left[b\left(\frac{2}{a}\right)^\sigma \left| \begin{matrix} \left(1-\frac{\rho}{2}-\frac{\eta}{2},\frac{\sigma}{2};1\right),\left(1-\frac{\rho}{2}+\frac{\eta}{2},\frac{\sigma}{2};1\right), \\ (a_j,\alpha_j;A_j)_{1,n},(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i}, \\ (b_j,\beta_j;B_j)_{1,m},(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i} \end{matrix}\right.\right]. \qquad \square$$

Theorem 2.4. *Prove that*

$$\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\psi[ax^\mu(t-x)^\delta]dx$$

$$= t^{\rho+\sigma-1}\psi^{m,n+2}_{p_i+2,q_i;r}\left[at^{\mu+\delta}\left| \begin{matrix} (1-\rho,\mu;1),(1-\sigma,\delta;1),(a_j,\alpha_j;A_j)_{1,n}, \\ (a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i},(b_j,\beta_j;B_j)_{1,m}, \\ (b_{ji},\beta_{ji};B_{ji})_{m+1,q_i},(1-(\rho+\sigma),\mu+\delta;1) \end{matrix}\right.\right] \tag{2.4}$$

provided

(a_1) $\sigma>0$, $Re(a)>0$, $\mu>0$, $Re(\rho)>0$,

(a_2) $Re(b_j+\beta_j s)>0$,

(a_3) $Re(1-a_j-\alpha_j s)>0$,

(a_4) $|\arg a|<\frac{\pi}{2}\Delta$, $\Delta>0$,

(a_5) $|\arg z|\geq\frac{\pi}{2}\Delta$, $(\nabla+\mu)>1$ *and* $\Delta\geq 0$,

(a_6) $\eta\in C$.

Proof. Using the definition of Euler-beta transform (1.6) and substituting (1.1), we get

$$\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\psi[ax^\mu(t-x)^\delta]dx$$

$$= \int_0^t x^{\rho-1}(t-x)^{\sigma-1}\left(\frac{1}{2\pi i}\int_L \phi(s)a^s x^{\mu s}(t-x)^{\delta s}ds\right)dx.$$

Rearranging,

$$\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\psi[ax^\mu(t-x)^\delta]dx = \frac{1}{2\pi i}\int_L \phi(s)\left[\int_0^t x^{\rho+\mu s-1}(t-x)^{\sigma+\delta s-1}dx\right]a^s ds.$$

Applying the formula $\int_0^t x^{m-1}(t-x)^{n-1}dx = t^{m+n-1}\beta(m,n)$, we get

$$\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\psi[ax^{\mu}(t-x)^{\delta}]dx = \frac{1}{2\pi i}\int_L \phi(s)t^{\rho+\sigma+(\mu+\delta)s-1}\beta(\rho+\mu s,\sigma+\delta s)a^s ds.$$

Using the relation between beta and gamma functions (Jafari [8]), we arrive to

$$\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\psi[ax^{\mu}(t-x)^{\delta}]dx = \frac{1}{2\pi i}t^{\rho+\sigma-1}\int_L \phi(s)\frac{\Gamma(\rho+\mu s)\Gamma(\sigma+\delta s)}{\Gamma(\rho+\sigma+(\mu+\delta)s)}(at^{\mu+\delta})^s ds.$$

From the equation (1.2), we have

$$\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\psi[ax^{\mu}(t-x)^{\delta}]dx$$
$$= t^{\rho+\sigma-1}\psi^{m,n+2}_{p_i+2,q_i+1;r}\left[at^{\mu+\delta}\left|\begin{matrix}(1-\rho,\mu;1),(1-\sigma,\delta;1),(a_j,\alpha_j;A_j)_{1,n},\\(a_{ji},\alpha_{ji};A_{ji})_{n+1,p_i},(b_j,\beta_j;B_j)_{1,m},\\(b_{ji},\beta_{ji};B_{ji})_{m+1,q_i},(1-(\rho+\sigma),\mu+\delta;1)\end{matrix}\right.\right]. \qquad \square$$

In the next section, we will investigate the exceptional instances in the form of special cases by assigning various values to different parameters.

3. Special Cases

Corollary 3.1. *In Theorem* 2.1*, if we assume $r=1$, $\alpha_{ji}=\alpha_j$, $\beta_{ji}=\beta_j$, $A_{ji}=A_j$ and $B_{ji}=B_j$ then the Hankel transform of Pragathi-Satyanarayana I-function* (2.1) *reduces to the Hankel transform of the one variable I-function specified by Jayarama and Rathie* [10] *and the result is as follows:*

$$H_{\gamma}\{\psi(ax^{\sigma})\} = \frac{\sqrt{2}}{\rho}I^{m,n+1}_{p_i+1,q_i+1}\left[a\left(\frac{2}{\rho}\right)^{\sigma}\left|\begin{matrix}\left(\frac{1}{4}-\frac{\gamma}{2},\frac{\sigma}{2};1\right),(a_j,\alpha_j,A_j)_{1,p_i},\\(b_j,\beta_j,B_j)_{1,q_i},\left(\frac{3}{4}-\frac{\gamma}{2},-\frac{\sigma}{2};1\right)\end{matrix}\right.\right]. \tag{3.1}$$

Corollary 3.2. *In Theorem* 2.2*, if we assume $r=1$, $\alpha_{ji}=\alpha_j$, $\beta_{ji}=\beta_j$, $A_{ji}=A_j$ and $B_{ji}=B_j$, then the Sumudu transform of Pragathi-Satyanarayana I-function* (2.2) *reduces to the Sumudu transform of the one variable I-function defined by Jayarama and Rathie* [10] *and the result is as follows:*

$$S\{\psi(ax^{\sigma})\} = S\{I(ax^{\sigma})\}$$
$$= \frac{1}{\sigma}\sum_{s=0}^{\infty}\frac{(-1)^s}{\mu^{s+1}s!}a^{-\frac{s+1}{\sigma}}\frac{\prod_{j=1}^{m}\Gamma^{B_j}\left(b_j+\beta_j\left(\frac{s+1}{\sigma}\right)\right)\prod_{j=1}^{n}\Gamma^{A_j}\left(1-a_j-\alpha_j\left(\frac{s+1}{\sigma}\right)\right)}{\prod_{j=m+1}^{q_i}\Gamma^{B_j}\left(1-b_j-\beta_j\left(\frac{s+1}{\sigma}\right)\right)\prod_{j=n+1}^{p_i}\Gamma^{A_j}\left(a_j+\alpha_j\left(\frac{s+1}{\sigma}\right)\right)}. \tag{3.2}$$

Corollary 3.3. *In Theorem* 2.3*, if we assume $r=1$, $\alpha_{ji}=\alpha_j$, $\beta_{ji}=\beta_j$, $A_{ji}=A_j$ and $B_{ji}=B_j$, then the K-transform of Pragathi-Satyanarayana I-function reduces to the I-function of one variable defined by Jayarama and Rathie* [10] *as follows:*

$$\int_0^{\infty}x^{\rho-1}K_{\eta}(ax)\psi(bx^{\sigma})dx$$
$$= 2^{\rho-2}a^{-\rho}I^{m,n+2}_{p_i+2,q_i}\left[b\left(\frac{2}{a}\right)^{\sigma}\left|\begin{matrix}\left(1-\frac{\rho}{2}-\frac{\eta}{2},\frac{\sigma}{2};1\right),\left(1-\frac{\rho}{2}+\frac{\eta}{2},\frac{\sigma}{2};1\right),\\(a_j,\alpha_j;A_j)_{1,p_i},(b_j,\beta_j;B_j)_{1,q_i}\end{matrix}\right.\right]. \tag{3.3}$$

Corollary 3.4. *In Theorem* 2.4*, if we assume $r = 1$, $\alpha_{ji} = \alpha_j$, $\beta_{ji} = \beta_j$, $A_{ji} = A_j$ and $B_{ji} = B_j$, then the Euler-beta transform of Pragathi-Satyanarayana I-function reduces to the one variable I-function defined by Jayarama and Rathie* [10] *as follows:*

$$\begin{aligned}&\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\psi[ax^{\mu}(t-x)^{\delta}]dx\\&= t^{\rho+\sigma-1} I^{m,n+2}_{p_i+2,q_i+1}\left[at^{\mu+\delta}\left|\begin{matrix}(1-\rho,\mu;1),(1-\sigma,\delta;1),(a_j,\alpha_j;A_j)_{1,p_i},\\(b_j,\beta_j;B_j)_{1,q_i},(1-(\rho+\sigma),\mu+\delta;1)\end{matrix}\right.\right].\end{aligned} \tag{3.4}$$

4. Conclusion

We investigated the effects of applying several integral transforms [8] on Pragathi-Satyanarayana's I-function including Hankel, Sumudu, K, and Euler-beta transforms. At the end, by substituting parameters with specific values, we obtained different integral transforms of I-function defined by Jayarama and Rathie [10] which was already proved to be the generalization of H-function. Based on this one may conclude that Pragathi-Satyanarayana's I-function (Kumar and Satyanarayana [11]) is a generalization of the I-function (Mishra and Vandana [13]) and H-function (Srivastava *et al*. [19]). As a result, this function can lead to a wide range of fascinating results in applied mathematics, physics and in engineering. One can extend and can able to find the integral transforms of two or more variables.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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