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Research Article

An M/G/1 Retrial G-queue with Multiple Working Vacation and a Waiting Server

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Abstract. An M/G/1 retrial G-queue with multiple working vacation and a waiting server is taken into consideration in this study. Both the retrial times and service times are assumed to follow general distribution and the waiting server follows an exponential distribution. During the working vacation period customers are served at a lesser rate of service. Before switching over to a vacation the server waits for some arbitrary amount of time and so is called a waiting server. We obtain the PGF for the number of customers and the mean number of customers in the invisible waiting area which is acquired by utilizing the supplementary variable technique. We compute the waiting time distribution. Out of interest a few special cases are conferred. Numerical outcomes are exhibited.

Keywords. Retrial queue, Working vacation, Supplementary variable technique, Waiting server, Negative customers

Mathematics Subject Classification (2020). 60K25, 90B22

1. Introduction

Retrial queues are expressed by the fact that if a customer observed that the server is occupied then they are entered into the invisible waiting area called an orbit. In recent years numerous researchers have examined the retrial queue. For a more in-depth analysis of the retrial queues, one can refer [5, 13, 16].

In Queueing theory queueing models with server vacation has a most impactful application. In addition to the vacation strategy Servi and Finn [11] developed a newest vacation strategy, called as Working Vacation (WV). In the WV period the server provides a lesser rate of service to the customers than during the regular service period. Wu and Takagi [15] examined M/G/1/MWV. Kalyanaraman and Murugan [8] have developed the retrial queue with vacation, Murugan and Santhi [9] studied the M/G/1 retrial queue with MWV. For a comprehensive study on WV one can refer [4].

Whenever the system becomes empty the server leaves from the *regular service* period (RS) and goes on a WV, but in a waiting server model the server wait for a arbitrary amount of time before going to WV. For a detailed study on waiting server model one can refer [3, 7, 12]. Arrival of a negative customers causes the server need to repair as well as the removal of the customer from the system. Once it has been repaired, the server is as good as new. For a comprehensive study on G-queue one can refer [10].

In this article, we consider an M/G/1 retrial G-queue with multiple WV and a waiting server. This article has the following structure. We explain the model in Section 2. In Section 3 model analysis are framed. In Section 4 performance measures are established. Section 5 discusses some special cases. In Section 6 numerical outcomes are exhibited along with the graphical analysis. The conclusion is given in Section 7.

2. Model Description

We examine an M/G/1 retrial G-queue with multiple WV and a waiting server where the primary customers arrival follows a Poison process with arrival rate λ. If an approaching customer discovers that the server is occupied then they exit the service area because we assume that there is no waiting area and they joins the orbit. At a service completion instant, if the number of customer is one at the extreme front end of the orbit, is permitted to approach the server with a distribution function $G(x)$ and the retrial time follows a general distribution. For the normal service period, let $g(x)$ and $G^*(\theta)$ signify the pdf and LST respectively, and for WV period, let $l(x)$, $L^*(\theta)$ signify the pdf and LST respectively. On the service completion epoch of each customer, if there is a contest between primary customer and an orbit customer, then it will be determined with $R_s(x)$, $r_s(x)$, $R_s^*(\theta)$ as its distribution function, pdf, LST with general distribution. The service delivered among the WV period follows general distribution with $W_v(x)$, $w_v(x)$, $W_v^*(\theta)$ as its distribution function, pdf, LST.

The server waits for a arbitrary period of time once the orbit turns empty which follows an exponential distribution with rate α. After completion of waiting time the server goes for WV which follows an exponential distribution with rate β and Inter-arrival times, retrial periods, RS periods, and WV periods are all presumed to be independent of one another.

The negative customers arrival follows a Poison process with arrival rate δ. Arrival of a negative customers causes the server need to repair as well as the removal of the customer from the system. The negative customer will disappear and have no further effect on the system if the server is inactive, undergoing repair, or taking a vacation.The repair time follows general distribution with $N_r(x)$, $n_r(x)$, $N_r^*(\theta)$ as its distribution function, pdf, LST.

Let's use the subsequent random variables.

$O(t)$ — the orbit size at time t;

$L^0(t)$ — the remaining retrial time in Working Vacation period;

$W_v^0(t)$ — the remaining service time in Working Vacation period;

$G^0(t)$ — the remaining retrial time in regular service period;

$R_s^0(t)$ — the remaining service time in regular service period;

$N_r^0(t)$ — the remaining repair time in RS period.

Further variables are introduced, to generate bivariate Markov Process,

$$\{(E(t), B(t));\, t \geq 0\},$$

where

$$B(t) = \begin{cases} L^0(t), & \text{if } E(t) = 0, \\ G^0(t), & \text{if } E(t) = 1, \\ W_v^0(t), & \text{if } E(t) = 2, \\ R_s^0(t), & \text{if } E(t) = 3, \\ N_r^0(t), & \text{if } E(t) = 4. \end{cases}$$

At time "t" the four distinct states of the server are

$$E(t) = \begin{cases} 0 & \text{if the server is not occupied in WV}, \\ 1 & \text{if the server is not occupied in RS period}, \\ 2 & \text{if the server is occupied in WV}, \\ 3 & \text{if the server is occupied in RS period}, \\ 4 & \text{if the server is under repair in RS period}, \end{cases}$$

$$W_{0,0} = \lim_{t\to\infty} P[O(t) = 0, E(t) = 0],$$

$$R_{0,0} = \lim_{t\to\infty} P[O(t) = 0, E(t) = 1],$$

$$W_{0,h}(x) = \lim_{t\to\infty} P[O(t) = h, E(t) = 0, x < L^0(t) \leq x + dx]; \quad h \geq 1,$$

$$R_{0,h}(x) = \lim_{t\to\infty} P[O(t) = h, E(t) = 1, x < G^0(t) \leq x + dx]; \quad h \geq 1,$$

$$W_{1,h}(x) = \lim_{t\to\infty} P[O(t) = h, E(t) = 2, x < W_v^0(t) \leq x + dx]; \quad h \geq 0,$$

$$R_{1,h}(x) = \lim_{t\to\infty} P[O(t) = h, E(t) = 3, x < R_s^0(t) \leq x + dx]; \quad h \geq 0,$$

$$N_h(x) = \lim_{t\to\infty} P[O(t) = h, E(t) = 4, x < N_r^0(t) \leq x + dx]; \quad h \geq 1.$$

Following are the limiting probabilities:

$$R_s^*(\theta) = \int_0^\infty e^{-\theta x} r_s(x)dx, \qquad W_v^*(\theta) = \int_0^\infty e^{-\theta x} w_v(x)dx,$$

$$L^*(\theta) = \int_0^\infty e^{-\theta x} l(x)dx, \qquad G^*(\theta) = \int_0^\infty e^{-\theta x} g(x)dx,$$

$$W_{0,h}^*(\theta) = \int_0^\infty e^{-\theta x} W_{0,h}(x)dx, \quad W_{0,h}^*(0) = \int_0^\infty W_{0,h}(x)dx,$$

$$W_{1,h}^*(\theta) = \int_0^\infty e^{-\theta x} W_{1,h}(x)dx, \quad W_{1,h}^*(0) = \int_0^\infty W_{1,h}(x)dx,$$

$$R_{0,h}^*(\theta) = \int_0^\infty e^{-\theta x} R_{0,h}(x)dx, \quad R_{0,h}^*(0) = \int_0^\infty R_{0,h}(x)dx,$$

$$N_h^*(\theta) = \int_0^\infty e^{-\theta x} N_h(x)dx, \quad N_h^*(0) = \int_0^\infty N_h(x)dx,$$
$$N_r^*(\theta) = \int_0^\infty e^{-\theta x} n_r(x)dx, \quad W_0^*(z,\theta) = \sum_{h=1}^\infty W_{0,h}^*(\theta)z^h,$$
$$W_0^*(z,0) = \sum_{h=1}^\infty W_{0,h}^*(0)z^h, \quad W_0(z,0) = \sum_{h=1}^\infty W_{0,h}(0)z^h,$$
$$W_1^*(z,\theta) = \sum_{h=0}^\infty W_{1,h}^*(\theta)z^h, \quad W_1^*(z,0) = \sum_{h=0}^\infty W_{1,h}^*(0)z^h,$$
$$W_1(z,0) = \sum_{h=0}^\infty W_{1,h}(0)z^h, \quad R_0^*(z,\theta) = \sum_{h=1}^\infty R_{0,h}^*(\theta)z^h,$$
$$R_0^*(z,0) = \sum_{h=1}^\infty R_{0,h}^*(0)z^h, \quad R_0(z,0) = \sum_{h=1}^\infty R_{0,h}(0)z^h,$$
$$R_1^*(z,\theta) = \sum_{h=0}^\infty R_{1,h}^*(\theta)z^h, \quad R_1^*(z,0) = \sum_{h=0}^\infty R_{1,h}^*(0)z^h,$$
$$R_1(z,0) = \sum_{h=0}^\infty R_{1,h}(0)z^h, \quad N^*(z,\theta) = \sum_{h=0}^\infty N_h^*(\theta)z^h,$$
$$N^*(z,0) = \sum_{h=0}^\infty N_h^*(0)z^h, \quad N(z,0) = \sum_{h=0}^\infty N_h(0)z^h.$$

The above mentioned are the LST and PGF which we have defined.

3. The Orbit Size Distribution

In steady state the system was illustrated by the subsequent differential difference equations:

$$\lambda W_{0,0} = W_{1,0}(0) + \alpha R_{0,0} + N_0(0), \tag{3.1}$$
$$-\frac{d}{dx}W_{0,h}(x) = -(\beta+\lambda)W_{0,h}(x) + W_{1,h}(0)l(x); \quad h \geq 1, \tag{3.2}$$
$$-\frac{d}{dx}W_{1,0}(x) = -(\beta+\lambda)W_{1,0}(x) + W_{0,1}(0)w_v(x) + \lambda W_{0,0}w_v(x), \tag{3.3}$$
$$-\frac{d}{dx}W_{1,h}(x) = -(\beta+\lambda)W_{1,h}(x) + \lambda W_{1,h-1}(x) + W_{0,h+1}(0)w_v(x) + \lambda\int_0^\infty W_{0,h}(x)dx\, w_v(x); h \geq 1, \tag{3.4}$$
$$(\lambda+\alpha)R_{0,0} = R_{1,0}(0), \tag{3.5}$$
$$-\frac{d}{dx}R_{0,h}(x) = -\lambda R_{0,h}(x) + R_{1,h}(0)g(x) + N_h(0)g(x) + \beta\int_0^\infty W_{0,h}(x)dx\, g(x); \quad h \geq 1, \tag{3.6}$$
$$-\frac{d}{dx}R_{1,0}(x) = -(\lambda+\delta)R_{1,0}(x) + R_{0,1}(0)r_s(x) + \beta r_s(x)\int_0^\infty W_{1,0}(x)r_s(x)dx, \tag{3.7}$$
$$-\frac{d}{dx}R_{1,h}(x) = -(\lambda+\delta)R_{1,h}(x) + \lambda R_{1,h-1}(x) + \beta r_s(x)\int_0^\infty W_{1,h}(x)dx + R_{0,h+1}(0)r_s(x)$$

$$+\lambda r_s(x)\int_0^\infty R_{0,h}(x)dx; \quad h \ge 1, \tag{3.8}$$

$$-\frac{d}{dx}N_0(x) = -\lambda N_0(x) + \delta n_r(x)\int_0^\infty R_{1,0}(x)dx, \tag{3.9}$$

$$-\frac{d}{dx}N_h(x) = -\lambda N_h(x) + \lambda N_{h-1}(x) + \delta n_r(x)\int_0^\infty R_{1,h}(x)dx. \tag{3.10}$$

Taking the LST from (3.2) to (3.10) on both sides results

$$\theta W^*_{0,h}(\theta) - W_{0,h}(0) = (\lambda+\beta)W^*_{0,h}(\theta) - W_{1,h}(0)L^*(\theta); \quad h \ge 1, \tag{3.11}$$

$$\theta W^*_{1,0}(\theta) - W_{1,0}(0) = (\lambda+\beta)W^*_{1,0}(\theta) - W_{0,1}(0)W^*_v(\theta) - \lambda W_{0,0}W^*_v(\theta), \tag{3.12}$$

$$\theta W^*_{1,h}(\theta) - W_{1,h}(0) = (\lambda+\beta)W^*_{1,h}(\theta) - W_{0,h+1}(0)W^*_v(\theta) - \lambda W^*_{1,h-1}(\theta) - \lambda W^*_{0,h}(0)W^*_v(\theta); \ \ h \ge 1, \tag{3.13}$$

$$\theta R^*_{0,h}(\theta) - R_{0,h}(0) = \lambda R^*_{0,h}(\theta) - R_{1,h}(0)G^*(\theta) - N_h(0)G^*(\theta) - \beta G^*(\theta)W^*_{0,h}(0); \ \ h \ge 1, \tag{3.14}$$

$$\theta R^*_{1,0}(\theta) - R_{1,0}(0) = (\lambda+\delta)R^*_{1,0}(\theta) - R_{0,1}(0)R^*_s(\theta) - \beta R^*_s(\theta)W^*_{1,0}(0) - \lambda R_{0,0}R^*_s(\theta), \tag{3.15}$$

$$\begin{aligned}\theta R^*_{1,h}(\theta) - R_{1,h}(0) &= (\lambda+\delta)R^*_{1,h}(\theta) - \lambda R^*_{1,h-1}(\theta) - R^*_s(\theta)R_{0,h+1}(0) - \beta R^*_s(\theta)W^*_{1,h}(0) \\ &\quad - \lambda R^*_s(\theta)R^*_{0,h}(0); \ \ h \ge 1,\end{aligned} \tag{3.16}$$

$$\theta N^*_0(\theta) - N_0(0) = \lambda N^*_0(\theta) - \delta R^*_{1,0}(0)N^*_r(\theta), \tag{3.17}$$

$$\theta N^*_h(\theta) - N_h(0) = \lambda N^*_h(\theta) - \lambda N^*_{h-1}(\theta) - \delta R^*_{1,n}(0)N^*_r(\theta). \tag{3.18}$$

Summing over h from 1 to infinity × (3.11) with z^h and results,

$$W^*_0(z,\theta)[\theta - (\beta+\lambda)] = W_0(z,0) - L^*(\theta)[W_1(z,0) - W_{1,0}(0)]. \tag{3.19}$$

Summing over h from 1 to infinity ×(3.13) with z^h and comprise with (3.12) results,

$$W^*_1(z,\theta)[\theta - (\beta - \lambda z + \lambda)] = W_1(z,0) - \frac{W^*_v(\theta)}{z}W_0(z,0) - \lambda W_{0,0}W^*_v(\theta) - \lambda W^*_v(\theta)W^*_0(z,0). \tag{3.20}$$

Placing $\theta = \beta + \lambda$ in (3.19), results

$$W_0(z,0) = L^*(\beta+\lambda)[W_1(z,0) - W_{1,0}(0)]. \tag{3.21}$$

Placing $\theta = 0$ and (Sub.) (3.21) in (3.19), results

$$W^*_0(z,0) = \frac{(1 - L^*(\lambda+\beta))[W_1(z,0) - W_{1,0}(0)]}{\lambda+\beta}. \tag{3.22}$$

Placing $\theta = \beta - \lambda z + \lambda$ and (Sub.) (3.21) and (3.22) in (3.20), results

$$W_1(z,0) = \frac{W^*_v(\lambda - \lambda z + \beta)[\lambda z(\lambda+\beta)W_{0,0} - (L^*(\lambda+\beta)(\lambda - \lambda z + \beta) + \lambda z)W_{1,0}(0)]}{z(\lambda+\beta) - W^*_v(\lambda - \lambda z + \beta)(L^*(\lambda+\beta)(\lambda - \lambda z + \beta) + \lambda z)}. \tag{3.23}$$

(Sub.) (3.23) in (3.21), results

$$W_0(z,0) = \frac{zL^*(\lambda+\beta)(\lambda+\beta)[\lambda W^*_v(\lambda - \lambda z + \beta)W_{0,0} - W_{1,0}(0)]}{z(\lambda+\beta) - W^*_v(\lambda - \lambda z + \beta)(L^*(\lambda+\beta)(\lambda - \lambda z + \beta) + \lambda z)}. \tag{3.24}$$

Let $f(z) = (\beta+\lambda)z - W^*_v(\beta+\lambda-\lambda z)(L^*(\lambda+\beta)(\beta+\lambda-\lambda z) + \lambda z)$, for $f(z) = 0$ we obtain $f(0) < 0$ and $f(1) > 0$ which ⇒ that ∃ a real root $z_1 \in (0,1)$.

At $z = z_1$ (3.24) is converted in to

$$W_{1,0}(0) = \lambda W^*_v(\lambda - \lambda z_1 + \beta)W_{0,0}. \tag{3.25}$$

(Sub.) (3.25) in (3.23), results

$$W_1(z,0)=\frac{\lambda W_v^*(\lambda-\lambda z+\beta)U(z)}{z(\lambda+\beta)-W_v^*(\lambda-\lambda z+\beta)(L^*(\lambda+\beta)(\lambda-\lambda z+\beta)+\lambda z)}W_{0,0}, \tag{3.26}$$

where

$$U(z)=z(\lambda+\beta)-W_v^*(\lambda-\lambda z_1+\beta)[\lambda z+L^*(\beta+\lambda)(\beta-\lambda z+\lambda)].$$

(Sub.) (3.25) in (3.24), results

$$W_0(z,0)=\frac{L^*(\beta+\lambda)\lambda z(\beta+\lambda)[W_v^*(\lambda-\lambda z+\beta)-W_v^*(\lambda-\lambda z_1+\beta)]}{z(\lambda+\beta)-W_v^*(\lambda-\lambda z+\beta)(L^*(\lambda+\beta)(\lambda-\lambda z+\beta)+\lambda z)}W_{0,0}. \tag{3.27}$$

(Sub.) (3.25) and (3.26) in (3.22), results

$$W_0^*(z,0)=\frac{(1-L^*(\lambda+\beta)\lambda z)[W_v^*(\lambda-\lambda z+\beta)-W_v^*(\lambda-\lambda z_1+\beta)]}{z(\lambda+\beta)-W_v^*(\lambda-\lambda z+\beta)(L^*(\lambda+\beta)(\lambda-\lambda z+\beta)+\lambda .z)}W_{0,0}. \tag{3.28}$$

Placing $\theta=0$ and (Sub.) (3.26), (3.27) and (3.28) in (3.20), results

$$W_1^*(z,0)=\frac{\lambda(1-W_v^*(\lambda+\beta-\lambda z))U(z)}{(\lambda+\beta-\lambda z)\{(\beta+\lambda)z-W_v^*(\lambda+\beta-\lambda z)(L^*(\beta+\lambda)(\lambda+\beta-\lambda z)+\lambda z)\}}W_{0,0}. \tag{3.29}$$

Summing over h from 1 to infinity × (3.14) with z^h and results

$$R_0^*(z,\theta)(\theta-\lambda)=R_0(z,0)-G^*(\theta)[R_1(z,0)-R_{1,0}(0)]-G^*(\theta)[N(z,0)-N_0(0)]-W_0^*(z,0)\beta G^*(\theta). \tag{3.30}$$

(Sub.) $W_{1,0}(0)=\lambda W_v^*(\lambda-\lambda z_1+\beta)W_{0,0}$ in (3.1), we get

$$\alpha R_{0,0}+N_0(0)=\lambda(1-W_v^*(\lambda-\lambda z_1+\beta))W_{0,0}$$

Placing $\theta=\lambda$ and (Sub.) $R_{1,0}(0)+N_0(0)=\lambda(1-W_v^*(\lambda-\lambda z_1+\beta))W_{0,0}+\lambda R_{0,0}$ in (3.30), results

$$R_0(z,0)=[R_1(z,0)+N(z,0)-\lambda(1-W_v^*(\lambda-\lambda z_1+\beta))W_{0,0}-\lambda R_{0,0}+\beta W_0^*(z,0)]G^*(\lambda). \tag{3.31}$$

Summing over h from 1 to infinity × (3.16) with z^h and comprise with (3.15) results

$$R_1^*(z,\theta)[\theta-\lambda+\lambda z-\delta)]=R_1(z,0)-\left[\frac{R_0(z,0)}{z}+\beta W_1^*(z,0)+\lambda R_0^*(z,0)+\lambda R_{0,0}\right]R_s^*(\theta). \tag{3.32}$$

Placing $\theta=0$ and (Sub.) (3.31) and $R_{1,0}(0)+N_0(0)=(1-W_v^*(\lambda-\lambda z_1+\beta))\lambda W_{0,0}+\lambda R_{0,0}$ in (3.26), results

$$R_0^*(z,0)=\left[\frac{(1-G^*(\lambda))}{\lambda}\right]\left[R_1(z,0)+N(z,0)-(1-W_v^*(\lambda-\lambda z_1+\beta))\lambda W_{0,0}-\lambda R_{0,0}+\beta W_0^*(z,0)\right]. \tag{3.33}$$

Placing $\theta=\lambda-\lambda z+\delta$ and (Sub.) in (3.32), results

$$R_1(z,0)=\left[\frac{R_0(z,0)}{z}+\beta W_1^*(z,0)+\lambda W_0^*(z,0)+\lambda R_{0,0}\right]R_s^*(\lambda-\lambda z+\delta). \tag{3.34}$$

Placing $\theta=0$ and (Sub.) in (3.32), results

$$R_1^*(z,0)=\left[\frac{(1-R_s^*(\lambda-\lambda z+\delta))}{(\lambda-\lambda z+\delta)}\right]\left[\frac{R_0(z,0)}{z}+\beta W_1^*(z,0)+\lambda W_0^*(z,0)+\lambda R_{0,0}\right]. \tag{3.35}$$

Summing over h from 1 to infinity × (3.18) with z^h and comprise with (3.17) results

$$(\theta-\lambda+\lambda z)N^*(z,\theta)=N(z,0)-\delta N_r^*(\theta)R_1^*(z,0). \tag{3.36}$$

Placing $\theta=(\lambda-\lambda z)$ and (Sub.) in (3.31), results

$$N(z,0)=\delta N_r^*(\lambda-\lambda z)R_1^*(z,0). \tag{3.37}$$

Placing $\theta = 0$ and (Sub.) in (3.31), results

$$N^*(z,0) = \frac{\delta(1-N_r^*(\lambda-\lambda z))R_1^*(z,0)}{(\lambda-\lambda z)}. \tag{3.38}$$

Solving equations (3.31), (3.34) and (3.38) in (3.34), results

$$R_0(z,0) = \frac{\begin{bmatrix} G^*(\lambda)[\beta W_0^*(z,0)-(1-W_v^*(\lambda-\lambda z_1+\beta))\lambda W_{0,0}-\lambda R_{0,0}]z \\ \times(\lambda-\lambda z+\delta)+[\beta z W_1^*(z,0)+\lambda z R_{0,0}][(\lambda-\lambda z+\delta) \\ \times R_s^*(\lambda-\lambda z+\delta))+\delta(1-R_s^*(\lambda-\lambda z+\delta))N_r^*(\lambda-\lambda z)] \end{bmatrix}}{Dr_2(z)}, \tag{3.39}$$

$$R_1(z,0) = \frac{\begin{bmatrix} (\lambda-\lambda z+\delta)R_s^*(\lambda-\lambda z+\delta)\Big\{(G^*(\lambda)(1-z)+z)[\beta W_0^*(z,0) \\ -(1-W_v^*(\lambda-\lambda z_1+\beta))\lambda W_{0,0}-\lambda R_{0,0}]+[\beta z W_1^*(z,0)+\lambda z R_{0,0}]\Big\} \end{bmatrix}}{Dr_2(z)}, \tag{3.40}$$

$$N(z,0) = \frac{\begin{bmatrix} \delta(1-R_s^*(\lambda-\lambda z+\delta))N_r^*(\lambda-\lambda z)\Big\{(G^*(\lambda)(1-z)+z)(\beta W_0^*(z,0) \\ -(1-W_v^*(\lambda-\lambda z_1+\beta))\lambda W_{0,0}-\lambda R_{0,0})+[\beta z W_1^*(z,0)+\lambda z R_{0,0}]\Big\} \end{bmatrix}}{Dr_2(z)}. \tag{3.41}$$

(Sub.) (3.39), (3.40), (3.41) in (3.33), (3.35) and (3.38)

$$R_0^*(z,0) = \frac{\begin{bmatrix} (1-G^*(\lambda))[\beta W_0^*(z,0)-(1-W_v^*(\lambda-\lambda z_1+\beta))\lambda W_{0,0}-\lambda R_{0,0}] \\ z(\lambda-\lambda z+\delta)+[\beta z W_1^*(z,0)+\lambda z R_{0,0}][(\lambda-\lambda z+\delta) \\ \times R_s^*(\lambda-\lambda z+\delta))+\delta(1-R_s^*(\lambda-\lambda z+\delta))N_r^*(\lambda-\lambda z)] \end{bmatrix}}{\lambda Dr_2(z)}, \tag{3.42}$$

$$R_1^*(z,0) = \frac{\begin{bmatrix} (1-R_s^*(\lambda-\lambda z+\delta))\Big\{(G^*(\lambda)(1-z)+z)(\beta W_0^*(z,0) \\ -(1-W_v^*(\lambda-\lambda z_1+\beta))\lambda W_{0,0}-\lambda R_{0,0})+[\beta z W_1^*(z,0)+\lambda z R_{0,0}]\Big\} \end{bmatrix}}{Dr_2(z)}, \tag{3.43}$$

$$N^*(z,0) = \frac{\begin{bmatrix} \delta(1-N_r^*(\lambda-\lambda z))(1-R_s^*(\lambda-\lambda z+\delta))\Big\{(G^*(\lambda)(1-z)+z)[-\lambda R_{0,0} \\ \beta W_0^*(z,0)-(1-W_v^*(\lambda-\lambda z_1+\beta))\lambda W_{0,0}]+[\beta z W_1^*(z,0)+\lambda z R_{0,0}]\Big\} \end{bmatrix}}{(\lambda-\lambda z)Dr_2(z)}. \tag{3.44}$$

(Sub.) (3.28) and (3.29) in (3.42), (3.43), (3.44), results

$$\begin{aligned} Nr_1(z) = \Big\{&\beta z(\lambda-\lambda z+\delta)(\lambda+\beta-\lambda z)(W_v^*(\lambda+\beta-\lambda z)-W_v^*(\lambda+\beta-\lambda z_1)) \\ &\times(1-L^*(\lambda+\beta))-(\lambda-\lambda z+\delta)(1-W_v^*(\lambda+\beta-\lambda z_1))(\lambda+\beta-\lambda z) \\ &\times\{(\beta+\lambda)z-W_v^*(\lambda+\beta-\lambda z)[\lambda z+(\lambda+\beta-\lambda z)L^*(\lambda+\beta)]\}-\frac{\lambda}{\alpha} \\ &\times(\lambda+\beta-\lambda z)(1-W_v^*(\lambda+\beta-\lambda z_1))\{(\beta+\lambda)z-W_v^*(\lambda+\beta-\lambda z) \\ &\times[\lambda z+(\lambda+\beta-\lambda z)L^*(\lambda+\beta)]\}(\lambda-\lambda z+\delta)+(1-W_v^*(\lambda+\beta-\lambda z)) \\ &\times\beta[(\lambda-\lambda z+\delta)R_s^*(\lambda-\lambda z+\delta)+\delta N_r^*(\lambda-\lambda z)(1-R_s^*(\lambda-\lambda z+\delta))] \\ &\times\{(\beta+\lambda)z-W_v^*(\lambda+\beta-\lambda z_1)[\lambda z+(\lambda+\beta-\lambda z)L^*(\lambda+\beta)]\} \\ &+(\lambda+\beta-\lambda z)[(\lambda-\lambda z+\delta)R_s^*(\lambda-\lambda z+\delta)+(1-R_s^*(\lambda-\lambda z+\delta)) \\ &\times\delta N_r^*(\lambda-\lambda z)]\frac{\lambda}{\alpha}\{(\beta+\lambda)z-W_v^*(\lambda+\beta-\lambda z_1)[\lambda z+(\lambda+\beta-\lambda z) \end{aligned}$$

$$\times L^*(\lambda+\beta)]\Big\}(1-W_v^*(\lambda+\beta-\lambda z_1))\Big\}, \tag{3.45}$$

$$\begin{aligned}
Nr_3(z) &= [G^*(\lambda)(1-z)+z](1-L^*(\lambda+\beta))[W_v^*(\lambda+\beta-\lambda z)-W_v^*(\lambda+\beta-\lambda z_1)] \\
&\quad \times(\lambda+\beta-\lambda z)+\{z(\lambda+\beta)-W_v^*(\lambda+\beta-\lambda z_1)(L^*(\lambda+\beta)(\lambda+\beta-\lambda z) \\
&\quad +\lambda z)\}\beta\lambda z(1-W_v^*(\lambda+\beta-\lambda z))-\lambda(1-W_v^*(\lambda+\beta-\lambda z_1))(\lambda+\beta-\lambda z) \\
&\quad \times[G^*(\lambda)(1-z)+z][z(\lambda+\beta)-W_v^*(\lambda+\beta-\lambda z)(L^*(\lambda+\beta)(\lambda+\beta-\lambda z) \\
&\quad +\lambda z]-\frac{\lambda}{\alpha}(1-W_v^*(\lambda+\beta-\lambda z_1))(\lambda+\beta-\lambda z)\{z(\lambda+\beta)-W_v^*(\lambda+\beta-\lambda z) \\
&\quad \times(L^*(\lambda+\beta)(\lambda+\beta-\lambda z)+\lambda z\}[G^*(\lambda)(1-z)+z],
\end{aligned} \tag{3.46}$$

$$R_0^*(z,0)=\frac{z(1-G^*(\lambda))W_{0,0}}{(\lambda+\beta-\lambda z)Dr_1(z)Dr_2(z)}Nr_1(z), \tag{3.47}$$

$$R_1^*(z,0)=\frac{(1-R_s^*(\lambda-\lambda z))W_{0,0}}{Dr_2(z)(\lambda+\beta-\lambda z)Dr_1(z)}Nr_3(z), \tag{3.48}$$

$$N^*(z,0)=\frac{(1-R_s^*(\lambda-\lambda z))\delta(1-N_r^*(\lambda-\lambda z))}{(\lambda-\lambda z)(\lambda+\beta-\lambda z)Dr_1(z)Dr_2(z)}Nr_3(z). \tag{3.49}$$

We define $R_S(z)=R_0^*(z,0)+R_1^*(z,0)+N^*(z,0)+R_{0,0}$,

$$\begin{aligned}
R_S(z) &= \frac{W_{0,0}}{(\lambda+\beta-\lambda z)(Dr_1(z)Dr_2(z))}\Big\{z(1-G^*(\lambda))\Big\{\beta z(\lambda-\lambda z+\delta)(\lambda+\beta-\lambda z) \\
&\quad \times(W_v^*(\lambda+\beta-\lambda z)-W_v^*(\lambda+\beta-\lambda z_1))(1-L^*(\lambda+\beta))-(\lambda-\lambda z+\delta) \\
&\quad \times(1-W_v^*(\lambda+\beta-\lambda z_1))(\lambda+\beta-\lambda z)\Big\{(\beta+\lambda)z-W_v^*(\lambda+\beta-\lambda z)[\lambda z \\
&\quad +(\lambda+\beta-\lambda z)L^*(\lambda+\beta)]\Big\}-\frac{\lambda}{\alpha}\{(\beta+\lambda)z-[\lambda z+(\lambda+\beta-\lambda z)L^*(\lambda+\beta)] \\
&\quad \times W_v^*(\lambda+\beta-\lambda z)\}(\lambda-\lambda z+\delta)(\lambda+\beta-\lambda z)(1-W_v^*(\lambda+\beta-\lambda z_1)) \\
&\quad +\beta[\delta N_r^*(\lambda-\lambda z)(1-R_s^*(\lambda-\lambda z+\delta))+(\lambda-\lambda z+\delta)R_s^*(\lambda-\lambda z+\delta)] \\
&\quad \times(1-W_v^*(\lambda+\beta-\lambda z))\Big\{(\beta+\lambda)z-W_v^*(\lambda+\beta-\lambda z_1)[\lambda z+(\lambda+\beta-\lambda z) \\
&\quad \times L^*(\lambda+\beta)]\Big\}+(\lambda+\beta-\lambda z)\frac{\lambda}{\alpha}[(1-R_s^*(\lambda-\lambda z+\delta))\delta N_r^*(\lambda-\lambda z) \\
&\quad +(\lambda-\lambda z+\delta)R_s^*(\lambda-\lambda z+\delta)](1-W_v^*(\lambda+\beta-\lambda z_1))\Big\{(\beta+\lambda)z \\
&\quad -W_v^*(\lambda+\beta-\lambda z_1)[\lambda z+(\lambda+\beta-\lambda z)L^*(\lambda+\beta)]\Big\}\Big\}+\Big\{(1-L^*(\lambda+\beta)) \\
&\quad \times\beta z[\lambda z+(\lambda+\beta-\lambda z)G^*(\lambda)][W_v^*(\lambda+\beta-\lambda z)-W_v^*(\lambda+\beta-\lambda z_1)] \\
&\quad -(1-W_v^*(\lambda+\beta-\lambda z_1))\{(\beta+\lambda)z-[\lambda z+L^*(\lambda+\beta)(\lambda+\beta-\lambda z)] \\
&\quad \times W_v^*(\lambda+\beta-\lambda z)\}[\lambda z+G^*(\lambda)(\lambda+\beta-\lambda z)]+\beta zL^*(\beta+\lambda)[W_v^*(\lambda+\beta-\lambda z) \\
&\quad -W_v^*(\lambda+\beta-\lambda z_1)](\beta+\lambda)\Big\}(1-R_s^*(\lambda-\lambda z))[\lambda-\lambda z+\delta(1-N_r^*(\lambda-\lambda z))] \\
&\quad -\frac{\lambda}{\alpha}\{(\lambda+\beta)z-W_v^*(\lambda+\beta-\lambda z)[\lambda z+L^*(\lambda+\beta)(\lambda+\beta-\lambda z)\}(\lambda+\beta-\lambda z) \\
&\quad \times(1-W_v^*(\lambda+\beta-\lambda z_1))G^*(\lambda)(1-z)\Big\}.
\end{aligned} \tag{3.50}$$

when the server is on RS period, as the PGF for the no of customers in the orbit.

We define $W_v(z) = W_0^*(z,0) + W_1^*(z,0) + W_{0,0}$

$$
\begin{aligned}
W_v(z) = &\frac{W_{0,0}}{(\lambda+\beta-\lambda z)D_1(z)}\Big\{(\lambda+\beta-\lambda z)\lambda z(1-L^*(\lambda+\beta))(W_v^*(\lambda+\beta-\lambda z)\\
&-W_v^*(\lambda+\beta-\lambda z_1)) + \lambda\big\{z(\lambda+\beta)-(\lambda z+L^*(\lambda+\beta)(\lambda+\beta-\lambda z))\\
&\times W_v^*(\lambda+\beta-\lambda z_1)\big\}(1-W_v^*(\lambda+\beta-\lambda z)) + (\lambda+\beta-\lambda z)[z(\beta+\lambda)\\
&-W_v^*(\lambda+\beta-\lambda z)(\lambda z+L^*(\beta+\lambda)(\lambda+\beta-\lambda z))]\Big\},
\end{aligned}
$$

when the server is on WV period, as the PGF for the no of customers in orbit. Again, we define $R(z) = R_S(z) + W_v(z)$,

$$
\begin{aligned}
R(z) = &\frac{W_{0,0}}{(\lambda+\beta-\lambda z)(D_1(z)D_2(z))}\Big\{z(1-G^*(\lambda))\Big\{\beta z(\lambda-\lambda z+\delta)(\lambda+\beta-\lambda z)\\
&\times(W_v^*(\lambda+\beta-\lambda z)-W_v^*(\lambda+\beta-\lambda z_1))(1-L^*(\lambda+\beta))-(\lambda-\lambda z+\delta)\\
&\times(1-W_v^*(\lambda+\beta-\lambda z_1))(\lambda+\beta-\lambda z)\big\{(\beta+\lambda)z-W_v^*(\lambda+\beta-\lambda z)[\lambda z\\
&+(\lambda+\beta-\lambda z)L^*(\lambda+\beta)]\big\}-\frac{\lambda}{\alpha}\big\{(\beta+\lambda)z-[\lambda z+(\lambda+\beta-\lambda z)L^*(\lambda+\beta)]\\
&\times W_v^*(\lambda+\beta-\lambda z)\big\}(\lambda-\lambda z+\delta)(\lambda+\beta-\lambda z)(1-W_v^*(\lambda+\beta-\lambda z_1))\\
&+\beta[\delta N_r^*(\lambda-\lambda z)(1-R_s^*(\lambda-\lambda z+\delta))+(\lambda-\lambda z+\delta)R_s^*(\lambda-\lambda z+\delta)]\\
&\times(1-W_v^*(\lambda+\beta-\lambda z))\big\{(\beta+\lambda)z-W_v^*(\lambda+\beta-\lambda z_1)[\lambda z+(\lambda+\beta-\lambda z)\\
&\times L^*(\lambda+\beta)]\big\}+(\lambda+\beta-\lambda z)\frac{\lambda}{\alpha}[(1-R_s^*(\lambda-\lambda z+\delta))\delta N_r^*(\lambda-\lambda z)\\
&+(\lambda-\lambda z+\delta)R_s^*(\lambda-\lambda z+\delta)](1-W_v^*(\lambda+\beta-\lambda z_1))\big\{(\beta+\lambda)z\\
&-W_v^*(\lambda+\beta-\lambda z_1)[\lambda z+(\lambda+\beta-\lambda z)L^*(\lambda+\beta)]\big\}\Big\}+\Big\{(1-L^*(\lambda+\beta))\\
&\times\beta z[\lambda z+(\lambda+\beta-\lambda z)G^*(\lambda)][W_v^*(\lambda+\beta-\lambda z)-W_v^*(\lambda+\beta-\lambda z_1)]\\
&-(1-W_v^*(\lambda+\beta-\lambda z_1))\{(\beta+\lambda)z-[\lambda z+L^*(\lambda+\beta)(\lambda+\beta-\lambda z)]\\
&\times W_v^*(\lambda+\beta-\lambda z)\}[\lambda z+G^*(\lambda)(\lambda+\beta-\lambda z)]+\beta z L^*(\beta+\lambda)[W_v^*(\lambda+\beta-\lambda z)\\
&-W_v^*(\lambda+\beta-\lambda z_1)](\beta+\lambda)\Big\}(1-R_s^*(\lambda-\lambda z))[\lambda-\lambda z+\delta(1-N_r^*(\lambda-\lambda z))]\\
&-\frac{\lambda}{\alpha}\{(\lambda+\beta)z-W_v^*(\lambda+\beta-\lambda z)[\lambda z+L^*(\lambda+\beta)(\lambda+\beta-\lambda z)\}(\lambda+\beta-\lambda z)\\
&\times(1-W_v^*(\lambda+\beta-\lambda z_1))G^*(\lambda)(1-z)\Big\}+\Big\{\lambda z(\lambda-\lambda z+\beta)(1-L^*(\lambda+\beta))\\
&\times(W_v^*(\lambda+\beta-\lambda z)-W_v^*(\lambda+\beta-\lambda z_1))+\lambda\{z(\lambda+\beta)-W_v^*(\lambda+\beta-\lambda z_1)\\
&\times(\lambda z+L^*(\beta+\lambda)(\lambda+\beta-\lambda z))\}(1-W_v^*(\lambda+\beta-\lambda z))+\{z(\lambda+\beta)\\
&-W_v^*(\lambda+\beta-\lambda z)(\lambda z+L^*(\lambda+\beta)(\lambda+\beta-\lambda z))\}\Big\}\Big\{z-R_s^*(\lambda-\lambda z)\\
&\times[z+(1-z)R^*(\lambda)]\Big\}\Big\},
\end{aligned}
\tag{3.51}
$$

as the PGF for the no of customers in the orbit, where

$$
Dr_1(z) = z(\lambda+\beta)-W_v^*(\lambda+\beta-\lambda z)[\lambda z+L^*(\lambda+\beta)(\lambda+\beta-\lambda z)], \tag{3.52}
$$

$$
Dr_2(z) = (\lambda-\lambda z+\delta)[z-R_s^*(\lambda-\lambda z+\delta)[G^*(\lambda)(1-z)+z]
$$

$$-\delta N_r^*(\lambda-\lambda z)(1-R_s^*(\lambda-\lambda z+\delta))[G^*(\lambda)(1-z)+z], \tag{3.53}$$

where $Dr_1(z)$ and $Dr_2(z)$ are given in (3.53) and (3.54). Make use of the normalizing condition $R(1)=1$ to find out that $W_{0,0}$. Using L'Hospitals rule and (Sub.) $z=1$ results,

$$W_{0,0}=\frac{1-\rho_s}{\left[\begin{array}{l}\left\{\frac{O(s)}{\beta G^*(\lambda)[\lambda+\beta-W_v^*(\beta)(\lambda+\beta L^*(\beta+\lambda))]}\right\}\\ -\left\{\frac{P(s)}{G^*(\lambda)[\lambda+\beta-W_v^*(\beta)(\lambda+\beta L^*(\lambda+\beta))]}\right\}\\ +\left\{\frac{\beta W_v^*(\lambda-\lambda z_1+\beta)L^*(\beta+\lambda)(1-G^*(\lambda))}{G^*(\lambda)[\lambda+\beta-W_v^*(\beta)(\lambda+\beta L^*(\beta+\lambda))]}+Q\right\}\end{array}\right]}, \tag{3.54}$$

$$R_{0,0}=\frac{\lambda}{\alpha}(1-W_v^*(\lambda-\lambda z_1+\beta))W_{0,0}, \tag{3.55}$$

where

$$O(s)=(\lambda-\lambda W_v^*(\lambda-\lambda z_1+\beta)+\beta)[\lambda+\beta G^*(\lambda)-W_v^*(\beta)(\lambda+\beta L^*(\lambda+\beta))],$$

$$P(s)=\frac{\lambda}{\delta}W_v^*(\beta)[1-R_s^*(\delta)][1+\delta E(N_r)][\lambda+\beta-W_v^*(\lambda-\lambda z_1+\beta)(\lambda+\beta L^*(\beta+\lambda))],$$

$$Q=\frac{\lambda}{\alpha}(1-W_v^*(\lambda+\beta-\lambda z_1)),$$

$$\rho_s<1,$$

$$\rho_s=\frac{\frac{\lambda}{\delta}[1-R_s^*(\delta)][1+\delta E(N_r)]}{G^*(\lambda)}.$$

4. Performance Measures

Mean Orbit Length

We assume that:

W_v, R_s — mean orbit size in WV and RS period.

W_{vw}, R_{sw} — mean waiting time of the customer in the orbit during WV period and RS period.

Then

$$\begin{aligned}W_v&=\frac{d}{dz}W_v(z)\Big|_{z=1}\\&=\frac{d}{dz}[W_1^*(z,0)+W_0^*(z,0)]\Big|_{z=1}\\&=\frac{d}{dz}\left[\frac{S(z)}{(\beta-\lambda z+\lambda)Dr_1(z)}+\frac{K(z)}{Dr_1(z)}\right]W_{0,0}\Big|_{z=1}\\&=\left[\begin{array}{l}\frac{[-Dr_1'(z)S(z)[(\beta-\lambda z+\lambda)-Dr_1(z)\lambda]+Dr_1(z)(\lambda-\lambda z+\beta)S'(z)]}{Dr_1(z)^2(\beta-\lambda z+\lambda)}\\+\left[\frac{K'(z)Dr_1(z)-Dr_1'(z)K(z)}{(Dr_1(z))^2}\right]\end{array}\right]\times W_{0,0}\Big|_{z=1},\end{aligned}$$

where

$$S(z)=\lambda(1-W_v^*(\lambda+\beta-\lambda z))[z(\lambda+\beta)-W_v^*(\lambda+\beta-\lambda z_1)(L^*(\lambda+\beta)(\lambda+\beta-\lambda z)+\lambda z)],$$

$$K(z)=\lambda z(1-L^*(\lambda+\beta))(W_v^*(\lambda+\beta-\lambda z)-W_v^*(\lambda+\beta-\lambda z_1)),$$

$$Dr_1(z) = z(\lambda+\beta) - W_v^*(\lambda+\beta-\lambda z)(L^*(\lambda+\beta)(\lambda+\beta-\lambda z)+\lambda z).$$

Differentiating $S(z)$, $K(z)$ and $Dr_1(z)$ with respect to z, we get

$$\begin{aligned}
S'(z) &= \lambda^2 W_v^{*'}(\lambda+\beta-\lambda z)[z(\lambda+\beta)-[(\beta-\lambda z+\lambda)L^*(\lambda+\beta)+\lambda z] \\
&\quad \times W_v^*(\lambda+\beta-\lambda z_1)+\lambda[\lambda+\beta-(\lambda-\lambda L^*(\lambda+\beta)) \\
&\quad \times W_v^*(\lambda+\beta-\lambda z_1)](1-W_v^*(\lambda+\beta-\lambda z)), \\
K'(z) &= (1-L^*(\beta+\lambda))\lambda(W_v^*(\lambda+\beta-\lambda z)-W_v^*(\lambda+\beta-\lambda z_1)) \\
&\quad +\lambda z(1-L^*(\beta+\lambda))(-\lambda W_v^{*'}(\lambda+\beta-\lambda z)), \\
Dr_1'(z) &= (\beta+\lambda)+\lambda W_v^{*'}(\lambda+\beta-\lambda z)(\lambda z+L^*(\beta+\lambda)(\lambda+\beta-\lambda z)) \\
&\quad -W_v^*(\lambda+\beta-\lambda z)(\lambda-\lambda L^*(\lambda+\beta)).
\end{aligned}$$

At $z=1$ W_v turns,

$$W_v = \left[\frac{\beta Dr_1(1)S'(1)-S(1)[\beta Dr_1'(1)-\lambda Dr_1(1)]}{(\beta Dr_1(1))^2}+\frac{Dr_1(1)K'(1)-K(1)Dr_1'(1)}{(Dr_1(1))^2}\right]W_{0,0}.$$

By Little's formula,

$$W_{vw} = \frac{W_v}{\lambda},$$

where

$$\begin{aligned}
S(1) &= \lambda(1-W_v^*(\beta))[\beta+\lambda-W_v^*(\lambda-\lambda z_1+\beta)(\lambda+\beta L^*(\beta+\lambda))], \\
S'(1) &= \lambda^2 W_v^{*'}(\beta)[\lambda+\beta-W_v^*(\lambda+\beta-\lambda z_1)(\lambda+\beta L^*(\lambda+\beta))] \\
&\quad +\lambda(1-W_v^*(\beta))[\lambda+\beta-W_v^*(\lambda+\beta-\lambda z_1)(\lambda-\lambda L^*(\lambda+\beta))], \\
K(1) &= \lambda(1-L^*(\beta+\lambda))(W_v^*(\beta)-W_v^*(\beta-\lambda z_1+\lambda)), \\
K'(1) &= \lambda(1-L^*(\beta+\lambda))[W_v^*(\beta)-W_v^*(\beta+\lambda-\lambda z_1)-W_v^{*'}(\beta)\lambda], \\
Dr_1(1) &= \beta-(\lambda+\beta L^*(\beta+\lambda))W_v^*(\beta)+\lambda, \\
Dr_1'(1) &= \beta+\lambda W_v^{*'}(\beta)(\lambda+L^*(\lambda+\beta)\beta)+\lambda-(\lambda-\lambda L^*(\lambda+\beta))W_v^*(\beta), \\
R_s &= \frac{d}{dz}R_S(z)\Big|_{z=1} \\
&= \frac{d}{dz}[R_1^*(z,0)+R_0^*(z,0)]\Big|_{z=1} \\
&= \frac{d}{dz}\left[\frac{Nr_1(z)(1-G^*(\lambda))+Nr_2(z)Nr_3(z)}{Dr_1(z)(\lambda-\lambda z+\beta)Dr_2(z)}\right]W_{0,0}\Big|_{z=1} \\
&= \frac{\left[\begin{array}{l}
[Dr_2'(z)2Nr_1'(z)(\lambda Dr_1(z)-(\lambda+\beta-\lambda z)Dr_1'(z)) \\
+(\lambda-\lambda z+\beta)Dr_1(z)Nr_1''(z)(Dr_2'(z)-Dr_2''(z)Nr_1'(z))] \\
\times(1-G^*(\lambda))+2(\beta-\lambda z+\lambda)Nr_2'(z)Dr_2'(z)(Nr_3'(z)Dr_1(z) \\
-Nr_3(z)Dr_1'(z))+Nr_3(z)[2\lambda Nr_2'(z)Dr_2'(z)+(\lambda+\beta-\lambda z) \\
\times Dr_2'(z)Nr_2''(z)-(\lambda+\beta-\lambda z)Dr_2''(z)Nr_2'(z)]Dr_1(z)
\end{array}\right]}{2(Dr_1(z)(\lambda+\beta-\lambda z)Dr_2'(z))^2}W_{0,0}\Big|_{z=1},
\end{aligned}$$

where

$$Nr_1(z) = \Big\{\beta z(\lambda-\lambda z+\delta)(\lambda+\beta-\lambda z)(W_v^*(\lambda+\beta-\lambda z)-W_v^*(\lambda+\beta-\lambda z_1))$$

$$\times(1-L^*(\lambda+\beta))-(\lambda-\lambda z+\delta)(1-W_v^*(\lambda+\beta-\lambda z_1))(\lambda+\beta-\lambda z)$$
$$\times\left\{(\beta+\lambda)z-W_v^*(\lambda+\beta-\lambda z)[\lambda z+(\lambda+\beta-\lambda z)L^*(\lambda+\beta)]\right\}-\frac{\lambda}{\alpha}$$
$$\times(\lambda+\beta-\lambda z)(1-W_v^*(\lambda+\beta-\lambda z_1))\{(\beta+\lambda)z-W_v^*(\lambda+\beta-\lambda z)$$
$$\times[\lambda z+(\lambda+\beta-\lambda z)L^*(\lambda+\beta)]\}(\lambda-\lambda z+\delta)+(1-W_v^*(\lambda+\beta-\lambda z))$$
$$\times\beta[(\lambda-\lambda z+\delta)R_s^*(\lambda-\lambda z+\delta)+\delta N_r^*(\lambda-\lambda z)(1-R_s^*(\lambda-\lambda z+\delta))]$$
$$\times\left\{(\beta+\lambda)z-W_v^*(\lambda+\beta-\lambda z_1)[\lambda z+(\lambda+\beta-\lambda z)L^*(\lambda+\beta)]\right\}$$
$$+(\lambda+\beta-\lambda z)[(\lambda-\lambda z+\delta)R_s^*(\lambda-\lambda z+\delta)+(1-R_s^*(\lambda-\lambda z+\delta))$$
$$\times\delta N_r^*(\lambda-\lambda z)]\frac{\lambda}{\alpha}\{(\beta+\lambda)z-W_v^*(\lambda+\beta-\lambda z_1)[\lambda z+(\lambda+\beta-\lambda z)$$
$$\times L^*(\lambda+\beta)]\}(1-W_v^*(\lambda+\beta-\lambda z_1))\Big\},$$

$$Dr_1(z)=(\lambda+\beta)z-W_v^*(\lambda+\beta-\lambda z)[L^*(\lambda+\beta)(\lambda+\beta-\lambda z)+\lambda z],$$

$$Dr_2(z)=(\lambda-\lambda z+\delta)[z-R_s^*(\lambda-\lambda z+\delta)[G^*(\lambda)(1-z)+z]$$
$$-\delta N_r^*(\lambda-\lambda z)(1-R_s^*(\lambda-\lambda z+\delta))[G^*(\lambda)(1-z)+z],$$

$$Nr_2(z)=(1-R_s^*(\lambda-\lambda z+\delta))[\lambda-\lambda z+\delta(1-N_r^*(\lambda-\lambda z))],$$

$$Nr_3(z)=[G^*(\lambda)(1-z)+z](1-L^*(\lambda+\beta))[W_v^*(\lambda+\beta-\lambda z)-W_v^*(\lambda+\beta-\lambda z_1)]$$
$$\times(\lambda+\beta-\lambda z)+\{z(\lambda+\beta)-W_v^*(\lambda+\beta-\lambda z_1)(L^*(\lambda+\beta)(\lambda+\beta-\lambda z)$$
$$+\lambda z)\}\beta\lambda z(1-W_v^*(\lambda+\beta-\lambda z))-\lambda(1-W_v^*(\lambda+\beta-\lambda z_1))(\lambda+\beta-\lambda z)$$
$$\times[G^*(\lambda)(1-z)+z][z(\lambda+\beta)-W_v^*(\lambda+\beta-\lambda z)(L^*(\lambda+\beta)(\lambda+\beta-\lambda z)+\lambda z]$$
$$-\frac{\lambda}{\alpha}(1-W_v^*(\lambda+\beta-\lambda z_1))(\lambda+\beta-\lambda z)\{z(\lambda+\beta)-W_v^*(\lambda+\beta-\lambda z)$$
$$\times(L^*(\lambda+\beta)(\lambda+\beta-\lambda z)+\lambda z\}[G^*(\lambda)(1-z)+z].$$

At $z=1$ R_s turns,

$$R_s=\frac{\left[\begin{array}{l}(1-G^*(\lambda))\big[2Nr_1'(1)Dr_2'(1)(\lambda Dr_1(1)-\beta Dr_1'(1))+\beta Dr_1(1)\\(Dr_2'(1)Nr_1''(1)-Nr_1'(1)Dr_2''(1))\big]+2\beta Nr_2'(1)Dr_2'(1)\\(Dr_1(1)Nr_3'(1)-Nr_3(1)Dr_1'(1))+Nr_3(1)Dr_1(1)[2\lambda\\Nr_2'(1)Dr_2'(1)+\beta Dr_2'(1)Nr_2''(1)-\beta Nr_2'(1)Dr_2''(1)]\end{array}\right]}{2(\beta Dr_1(1)Dr_2'(1))^2}W_{0,0}\,.$$

By Little's formula,

$$R_{sw}=\frac{R_s}{\lambda},$$

where

$$Nr_3(1)=(1-W_v^*(\beta))\{\beta G^*(\lambda)W_v^*(\lambda+\beta-\lambda z_1)(\lambda+\beta L^*(\beta+\lambda))-\beta\lambda-\beta\lambda G^*(\lambda)$$
$$-\beta^2G^*(\lambda)\}+(1-W_v^*(\lambda+\beta-\lambda z_1))\{-\lambda^2(1-W_v^*(\beta))+\beta\lambda W_v^*(\beta)$$
$$\times L^*(\lambda+\beta)\}+\beta^2L^*(\lambda+\beta)(W_v^*(\beta)-W_v^*(\lambda-\lambda z_1+\beta))-\frac{\lambda}{\alpha}\beta G^*(\lambda)$$
$$\times\{\lambda+\beta-W_v^*(\beta)(L^*(\beta+\lambda)\beta+\lambda)\}(1-W_v^*(\lambda-\lambda z_1+\beta)),$$

$$
\begin{aligned}
Nr_3'(1) &= (W_v^*(\beta) - W_v^*(\lambda - \lambda z_1 + \beta))[(1 - L^*(\lambda + \beta))(\beta\lambda(1 - G^*(\lambda)) + \beta^2 G^*(\lambda)) \\
&\quad + \beta^2 L^*(\lambda + \beta)] + G^*(\lambda)(\lambda + \beta + \lambda W_v^*(\beta))(\beta W_v^*(\lambda - \lambda z_1 + \beta) - \lambda) \\
&\quad - \beta\lambda L^*(\lambda + \beta)(1 - W_v^*(\lambda - \lambda z_1 + \beta))(W_v^*(\beta) + \beta W_v^{*'}(\beta)) + \lambda G^*(\lambda) \\
&\quad \times W_v^*(\lambda - \lambda z_1 + \beta)[\lambda + \beta - \beta W_v^*(\beta) L^*(\beta + \lambda) + \lambda W_v^*(\beta)] - \beta G^*(\lambda) \\
&\quad \times [\lambda + \beta - \lambda^2 W_v^{*'}(\beta) + \lambda W_v^*(\beta)] + [\lambda W_v^{*'}(\beta)(\beta L^*(\beta + \lambda) - \lambda) + \lambda W_v^*(\beta) \\
&\quad \times W^*(\lambda + \beta)][\beta G^*(\lambda) W_v^*(\lambda - \lambda z_1 + \beta) - \lambda(1 - W_v^*(\lambda + \beta - \lambda z_1))] \\
&\quad + \frac{\lambda}{\alpha} G^*(\lambda)(1 - W_v^*(\lambda - \lambda z_1 + \beta))\{\lambda(\beta + \lambda) - 2\lambda\beta W_v^*(\beta) L^*(\lambda + \beta) \\
&\quad - \lambda^2 W_v^*(\beta) - \lambda^2 \beta W_v^{*'}(\beta) - \lambda\beta^2 W_v^{*'}(\beta) L^*(\lambda + \beta) + \lambda\beta W_v^*(\beta) - \beta(\beta + \lambda)\}, \\
Nr_1'(1) &= -\beta\lambda\delta W_v^*(\beta) + \beta[-\lambda R_s^*(\delta) + \lambda\delta R_s^*(\delta) E(N_r) - \lambda\delta R_s^*(\delta) E(N_r)](1 - W_v^*(\beta)) \\
&\quad \times [\lambda + \beta - \beta W_v^*(\lambda - \lambda z_1 + \beta) L^*(\beta + \lambda) - \lambda W_v^*(\lambda + \beta - \lambda z_1)] - \beta^2 \delta W_v^*(\beta) \\
&\quad \times L^*(\beta + \lambda) + \beta^2 \delta W_v^*(\lambda - \lambda z_1 + \beta) L^*(\beta + \lambda) + \lambda\delta[\beta + \lambda - \beta L^*(\lambda + \beta) \\
&\quad \times W_v^*(\beta) - \lambda W_v^*(\beta)] + \lambda^2 \delta W_v^*(\lambda - \lambda z_1 + \beta) W_v^*(\beta) + \lambda\beta\delta W_v^*(\lambda - \lambda z_1 + \beta) \\
&\quad \times W_v^*(\beta) L^*(\lambda + \beta) - \lambda\beta^2 W_v^*(\beta) - \lambda\beta^2 L^*(\lambda + \beta) W_v^*(\lambda - \lambda z_1 + \beta) + \lambda^2 \beta \\
&\quad + \lambda\beta^2 - \lambda^2 \beta W_v^*(\beta) - \lambda^2 \beta W_v^*(\lambda + \beta - \lambda z_1) + \lambda\beta^2 W_v^*(\beta) W_v^*(\lambda - \lambda z_1 + \beta) \\
&\quad \times L^*(\lambda + \beta) + \lambda^2 \beta W_v^*(\beta) W_v^*(\lambda - \lambda z_1 + \beta) + \frac{\lambda}{\alpha} \beta(1 - W_v^*(\lambda + \beta - \lambda z_1)) \\
&\quad \times [\lambda - \lambda\delta R_s^{*'}(\delta) - \lambda R_s^*(\delta) + \lambda\delta R_s^{*'}(\delta) + \lambda\delta E(N_r) - \lambda\delta E(N_r) R_s^*(\delta)] \\
&\quad \times \{(\lambda + \beta) + W_v^*(\beta)[\lambda + \beta L^*(\lambda + \beta)]\}, \\
Nr_1''(1) &= [\lambda + \beta - \lambda W_v^*(\lambda + \beta - \lambda z_1)][(1 - W_v^*(\beta))[-4\lambda\beta\delta R_s^*(\delta) E(N_r) - 4\lambda\beta R_s^*(\delta) \\
&\quad + 4\lambda\beta\delta E(N_r)] + 2\lambda\beta(1 + \delta W_v^{*'}(\beta)) + [R_s^*(\delta) - \delta E(N_r) + \delta R_s^*(\delta) E(N_r)] \\
&\quad \times (1 - W_v^*(\beta))[2\lambda\beta^2 W_v^*(\lambda + \beta - \lambda z_1) L^*(\lambda + \beta) - 2\lambda^2 \beta W_v^*(\lambda + \beta - \lambda z_1) \\
&\quad \times L^*(\lambda + \beta)] + 4\lambda\delta(1 - W_v^*(\beta))(\lambda + \beta) - 2\beta^2 \delta[W_v^*(\beta) - W_v^*(\lambda + \beta - \lambda z_1)] \\
&\quad - \lambda\beta^2 \delta W_v^{*'}(\beta) W_v^*(\lambda + \beta - \lambda z_1) L^*(\lambda + \beta) + \lambda^2 \beta\delta W_v^{*'}(\beta) W_v^*(\lambda + \beta - \lambda z_1) \\
&\quad \times L^*(\lambda + \beta) - \lambda^2 \beta\delta W_v^{*''}(\beta)[2\lambda + \beta - \beta L^*(\lambda + \beta) - 2\lambda W_v^*(\lambda + \beta - \lambda z_1)] \\
&\quad + \lambda^3 \beta W_v^{*'}(\beta)[\delta E(N_r) - R_s^*(\delta) - \delta R_s^*(\delta) E(N_r)] - \lambda^2 \beta W_v^{*'}(\beta)[\beta\delta R_s^{*'}(\delta) \\
&\quad + R_s^*(\delta) - \beta R_s^*(\delta) W_v^*(\lambda + \beta - \lambda z_1) L^*(\lambda + \beta) + \beta\delta E(N_r) W_v^*(\lambda + \beta - \lambda z_1) \\
&\quad \times L^*(\lambda + \beta)(1 - R_s^*(\delta))] - 3\lambda^2 \beta W_v^*(\lambda + \beta - \lambda z_1) + 3\lambda\beta^2 W_v^*(\lambda + \beta - \lambda z_1) \\
&\quad - 4\lambda\beta^2 W_v^*(\beta) + 2\lambda\beta L^*(\lambda + \beta) W_v^*(\beta)(\lambda + \beta) + 2\lambda\beta L^*(\lambda + \beta)(\lambda - 2\beta) \\
&\quad \times W_v^*(\lambda + \beta - \lambda z_1) - \lambda\beta\delta L^*(\lambda + \beta) W_v^*(\lambda + \beta - \lambda z_1) + 2\lambda^2 \beta^2 W_v^{*'}(\beta) \\
&\quad - 2\lambda^2 \beta W_v^*(\beta)[1 + L^*(\lambda + \beta) W_v^*(\lambda + \beta - \lambda z_1)] + 2\lambda\beta W_v^*(\lambda + \beta - \lambda z_1) \\
&\quad \times W_v^*(\beta)[2\lambda + \beta L^*(\lambda + \beta)] - 2W_v^*(\lambda + \beta - \lambda z_1)(1 - 2W_v^*(\lambda + \beta - \lambda z_1)) \\
&\quad \times \lambda^2 \delta + \beta(1 - W_v^*(\beta))[2\lambda^2 R_s^{*'}(\delta) + \lambda^2 \delta(1 - R_s^*(\delta)) E(N_r^2) + 2\lambda^2 \delta R_s^{*'}(\delta) \\
&\quad \times E(N_r)][\lambda + \beta - W_v^*(\lambda + \beta - \lambda z_1)(\lambda + \beta L^*(\lambda + \beta))] + \frac{\lambda}{\alpha}\Big\{2\lambda\beta[\lambda + \beta
\end{aligned}
$$

$$+\lambda W_v^{*'}(\beta)(\lambda+\beta L^*(\lambda+\beta))-W_v^*(\beta)(\lambda-\lambda L^*(\lambda+\beta))][\delta E(N_r)-R_s^*(\delta)$$
$$-\delta E(N_r)R_s^*(\delta)+1]+[\lambda+\beta-W_v^*(\beta)(\lambda+\beta L^*(\lambda+\beta))]\{2\lambda^2(R_s^*(\delta)-1$$
$$+\beta R_s^{*'}(\delta))+\lambda^2\delta(1-R_s^*(\delta))[\beta E(N_r^2)-2E(N_r)]\}\Big\}(1-W_v^*(\lambda+\beta-\lambda z_1)),$$

$$Nr_2'(1)=-\lambda[1-R_s^*(\delta)][1+\delta E(N_r)],$$

$$Nr_2''(1)=-2\lambda R_s^{*'}(\delta)[1+\delta E(N_r)]-\lambda^2\delta E(N_r^2)(1-R_s^*(\delta)),$$

$$Dr_1(1)=\beta+\lambda-(\lambda+\beta L^*(\beta+\lambda))W_v^*(\beta),$$

$$Dr_1'(1)=\beta+\lambda+\lambda(\lambda+\beta L^*(\lambda+\beta))W_v^{*'}(\beta)-\lambda(1-L^*(\lambda+\beta))W_v^*(\beta),$$

$$Dr_2'(1)=-\lambda+\lambda R_s^*(\delta)-\lambda\delta E(N_r)+\lambda\delta E(N_r)R_s^*(\delta)+\delta G^*(\lambda),$$

$$Dr_2''(1)=-2\lambda[1+\lambda R_s^{*'}(\delta)+\lambda R_s^{*'}(\delta)(1-G^*(\lambda))]+\delta[R_s^{*'}(\delta)(1-G^*(\lambda))-\lambda^2R_s^{*''}(\delta)$$
$$+\lambda(1-G^*(\lambda))R_s^{*'}(\delta)]-\lambda^2\delta E(N_r^2)(1-R_s^*(\delta))-\lambda^2\delta E(N_r)R_s^{*'}(\delta)$$
$$-\lambda\delta E(N_r)(1-R_s^*(\delta))(1-G^*(\lambda)),$$

where $E(N_r)$ is the mean repair times for regular service time.

5. Special Cases

(i) If the service time distribution follows an exponential distribution, no service among the vacation period and there is no negative arrival then the present model will be remodeled as "Time dependent analysis of M/M/1 queue with server vacations and a waiting server".

(ii) If the server does not wait after the completion of the RS period and there is no negative arrival then the present model will be remodeled as "An M/G/1 retrial queue with multiple working vacation".

(iii) If the server does not wait after the completion of the RS period, there is no negative arrival and there is no retrial time in the system then the present model will be remodeled as "An M/G/1 queue with multiple working vacation".

(iv) If the server does not wait after the completion of the RS period, there is no negative arrival and the server never takes the vacation then the present model will be remodeled as "An M/G/1 retrial queue".

(v) If the server does not wait after the completion of the RS period, there is no negative arrival, the server never takes the vacation and there is no retrial time in the system then the present model will be remodeled as "An M/G/1 queue".

6. Numerical Result and Graphical Analysis

The values tabulated in Table 1 and the curved graph constructed in Figure 1 are obtained by setting the fixed values $\mu_v=7.6$, $\mu_s=9.8$, $\mu_{vr}=3.5$, $\mu_{sr}=4.3$, $\alpha=0.9$, $\delta=0.5$ and varying the value of λ from 1 to 2 incremented with 0.2 and extending the values of β from 1.5 to 1.9 in steps of 0.2, we observed that as λ rises W_v also rises and hence the stability of the model is verified.

λ	$\beta = 1.5$	$\beta = 1.7$	$\beta = 1.9$
1.0	0.0775	0.0660	0.0571
1.2	0.1122	0.0952	0.0821
1.4	0.1521	0.1285	0.1104
1.6	0.1960	0.1650	0.1414
1.8	0.2426	0.2036	0.1739
2.0	0.2904	0.2428	0.2069

Table 1. W_v with turn over of λ

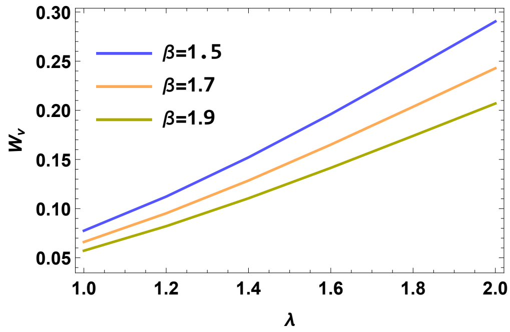

Figure 1. W_v with turn over of λ

The values tabulated in Table 2 and the curved graph constructed in Figure 2 are obtained by setting the fixed values $\mu_v = 7.6$, $\mu_s = 9.8$, $\mu_{vr} = 3.5$, $\mu_{sr} = 4.3$, $\alpha = 2.5$, $\delta = 0.5$ and varying the value of λ from 1 to 2 incremented with 0.2 and extending the values of β from 1 to 1.8 in steps of 0.4. We observed that as λ rises W_{vw} also rises which is expected.

λ	$\beta = 1$	$\beta = 1.4$	$\beta = 1.8$
1.0	0.1275	0.0845	0.0613
1.2	0.1565	0.1022	0.0735
1.4	0.1849	0.1191	0.0849
1.6	0.2120	0.1346	0.0953
1.8	0.2370	0.1484	0.1043
2.0	0.2594	0.1602	0.1118

Table 2. W_{vw} with turn over of λ

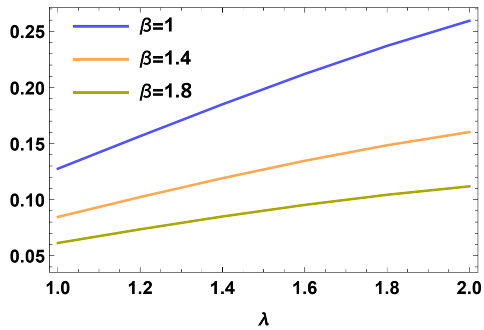

Figure 2. W_{vw} with turn over of λ

The values tabulated in Table 3 and the curved graph constructed in Figure 3 and are obtained by setting the fixed values $\mu_v = 0.1$, $\mu_s = 9$, $\mu_{vr} = 1.5$, $\mu_{sr} = 4.5$, $\alpha = 0.6$, $\delta = 0.3$ and varying the values of λ from 1 to 2 incremented with 0.2 and extending the values of β from 0.4 to 0.6 in steps of 0.1. We observed that as λ rises R_s also rises which shows the stability of the model.

λ	$\beta = 0.4$	$\beta = 0.5$	$\beta = 0.6$
1.0	0.7058	0.3831	0.1552
1.2	1.0603	0.6005	0.2738
1.4	1.4994	0.8749	0.4273
1.6	2.0277	1.2055	0.6092
1.8	2.6492	1.5880	0.8069
2.0	3.3682	2.0155	1.0019

Table 3. R_s with turn over of λ

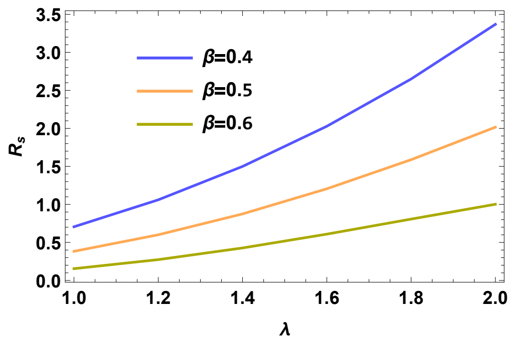

Figure 3. R_s with turn over of λ

The values tabulated in Table 4 and the curved graph constructed in Figure 4 are obtained by setting the fixed values $\mu_v = 0.8$, $\mu_s = 5$, $\mu_{vr} = 1.5$, $\mu_{sr} = 4.5$, $\alpha = 0.6$, $\delta = 0.3$ and altering the value of λ from 1 to 2 incremented with 0.1 and extending the values of β from 0.4 to 0.6 in steps of 0.1. From the graph, we studied that as λ rises R_{sw} also rises which shows the stability of the model.

λ	$\beta=0.4$	$\beta=0.5$	$\beta=0.6$
1.0	1.0479	0.4801	0.1400
1.2	1.4111	0.7081	0.2872
1.4	1.8176	0.9835	0.4842
1.6	2.2680	1.3103	0.7370
1.8	2.7594	1.6910	1.0513
2.0	3.2772	2.1207	1.4284

Table 4. R_{sw} with turn over of λ

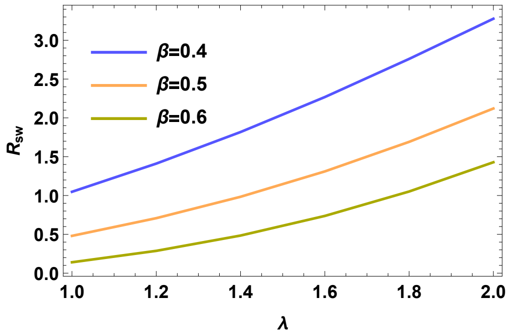

Figure 4. R_{sw} with turn over of λ

7. Conclusion

In this paper, an $M/G/1$ retrial G-queue with multiple working vacation and a waiting server is evaluated. We obtained the PGF for the number of customers and the mean number of customers in the orbit. We worked out the waiting time distribution. We also derived the performance measures. We performed some particular cases. We illustrate some numerical results.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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