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Research Article

Bayesian Estimators of Dynamic Cumulative Residual Entropy for Pareto Type II Distribution

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Abstract. In this paper, Pareto type II distribution is used to propose Bayes estimator of dynamic cumulative residual entropy. To calculate posterior risks, various informative and non-informative priors are used. Using different loss functions, Bayes estimators and associated posterior risks for the distribution have been calculated. Numerical computation is carried out with the help of a real data set. In the last, Monte Carlo Simulation study and Graphical analysis are also given along with the conclusion drawn.

Keywords. Bayesian estimation, Pareto type II distribution, Loss functions, Priors, Fisher information matrix

Mathematics Subject Classification (2020). 60E05, 62F15

1. Introduction

In literature, a number of statistical models are applicable for analysing the life time data and business failure data. The models are preferred so as to express the life times as close as practicable. Recently, for the analysis of data stemming in the field of economics, actuarial statistics, queuing problems (Johnson *et al*. [10]), Pareto type II distribution has become very attractive. It is intended by Lomax [11] and also familiar as Lomax distribution or Pearson Type VI distribution and is a special case of the *generalised Pareto distribution* (GPD). It is broadly applicable in reliability and life testing (Hassan and Al-Ghamdi [9]), problems in engineering as well as in survival analysis as an alternative distribution. Harris [8] also used this distribution to income and wealth data, queuing problems, computer files on servers and biological sciences. When the data are heavy-tailed, Bryson [5] have used this distribution in place of the exponential distribution.

The parameters of Pareto type II distribution are based on Generalized probability weighted moments and they were estimated by Abd-Elfattah and Alharbey [1]. Nasiri and Hosseini [12] compared the *maximum likelihood estimation* (MLE) and the Bayes estimation obtained using a proper prior distribution. Pareto type II distribution has the *probability density function* (pdf) as

$$f(x) = \frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}, \tag{1.1}$$

where $\lambda > 0$, $\alpha > 0$ are, respectively the scale and shape parameter of this distribution. Shannon [17] firstly introduced the entropy for the measurement of uncertainty. Let X is a random variable having c.d.f (F) with p.d.f (f) and then entropy of the random variable X is defined as

$$H(f) = -\int_0^\infty f(x)\log f(x)dx. \tag{1.2}$$

The information about the current age of the system has been used in reliability and survival analysis. The concept of residual entropy was introduced by Ebrahimi and Pellerey [6], and Ebrahimi [7] in form of condition measure. The residual entropy function for a non-negative random variable X given $X > t$ is defined as

$$H(t) = -\int_t^\infty \frac{f(x)}{\overline{F}(t)}\log\frac{f(x)}{\overline{F}(t)}dx. \tag{1.3}$$

$H(t)$ resolves the distribution uniquely if $H(t)$ is increasing in t (Belzunce *et al*. [4]). To measure the randomness of system, *cumulative residual entropy* (CRE) was intended by Rao *et al*. [14] and is defined as

$$E(x) = -\int_0^\infty \overline{F}(x)\log\overline{F}(x)dx. \tag{1.4}$$

Recently, *dynamic cumulative residual entropy* (DCRE) is proposed by Asadi and Zohrevand [3] and is defined as

$$E(t) = -\int_0^\infty \frac{\overline{F}(x)}{\overline{F}(t)}\log\frac{\overline{F}(x)}{\overline{F}(t)}dx, \tag{1.5}$$

where $\overline{F}(t) = 1-F(t)$, is survival function of t. If an item has been sustained up to time t then DCRE measures the uncertainty in its remaining life. The application and properties of DCRE are discussed by Asadi and Zohrevand [3], Navarro *et al*. [13], and Renjini *et al*. [16]. For classical Pareto distribution, the Bayesian estimation of dynamic cumulative residual entropy was intended by Renjini *et al*. [16].

The aim of this paper is to study estimators of *dynamic cumulative residual entropy* (DCRE) of Pareto type II distribution using Bayesian techniques as discussed in Renjini *et al*. [16]. The DCRE for (1.1) is simplified as

$$E(t) = \frac{\alpha(t+\lambda)}{(\alpha-1)^2}. \tag{1.6}$$

2. Prior and Loss Functions

Recently, most researchers have shown a keen interest in Bayesian estimation. In Bayesian analysis prior information is used and it is an important approach to statistics. In Bayesian estimation, prior probability distribution represents uncertainty about the latent variable. We require appropriate choice of priors for the parameters in the Bayesian deduction. Arnold and Press [2] have discussed that there is no method to check the superiority of one prior over other prior, i.e. one is better than other. If we have enough information about the parameter then informative priors are mostly used and for non-informative priors see Upadhyay *et al*. [19].

Harold Jeffrey gives a convenient non-informative (objective) prior distribution known as Jeffrey's prior in Bayesian probability. The likelihood function is given by

$$L(x/\alpha) = \left(\frac{\alpha}{\lambda}\right)^n \prod_{i=1}^{n} \left(1 + \frac{x}{\lambda}\right)^{-(\alpha+1)}. \tag{2.1}$$

The posterior distributions for Jeffrey's, Hartigan, Uniform and Conjugate Gamma priors are, respectively given by $\pi_1(\alpha/\underline{x})$, $\pi_2(\alpha/\underline{x})$, $\pi_3(\alpha/\underline{x})$ and $\pi_4(\alpha/\underline{x})$ and are as follows:

$$\pi_1(\alpha/\underline{x}) = \frac{W^n}{\Gamma(n)} \alpha^{n-1} \exp(-\alpha W), \tag{2.2}$$

$$\pi_2(\alpha/\underline{x}) = \frac{W^{n-2}}{\Gamma(n-2)} \alpha^{n-3} \exp(-\alpha W), \tag{2.3}$$

$$\pi_3(\alpha/\underline{x}) = \frac{W^{n+1}}{\Gamma(n+1)} \alpha^{n} \exp(-\alpha W), \tag{2.4}$$

$$\pi_4(\alpha/\underline{x}) = \frac{Q^N}{\Gamma(N)} \alpha^{N-1} \exp(-\alpha Q), \tag{2.5}$$

where $W = \sum_{i=1}^{n} \log\left(1 + \frac{x_i}{\lambda}\right)$, $Q = a + \sum_{i=1}^{n} \log\left(1 + \frac{x_i}{\lambda}\right)$, $N = n + b$.

A simple process is provided by Bayesian statistics for updating uncertainty. For the selection of best estimator, a loss function must be identified and also used to show the associated penalty. The statement that Bayesian methods always performs good regardless of the situations, is not always true, for reference see Ren *et al*. [15]. In this paper, we have used different loss function such as SELF (*Squared Error Loss Function*), WSELF (*Weighted Squared Error Loss Function*) and M/QSELF (*Modified / Quadratic Squared Error Loss Function*).

The SELF is a *symmetrical loss function* and defined as

$$L(\widehat{\alpha}, \alpha) = (\widehat{\alpha} - \alpha)^2.$$

The *Weighted form of Squared Error Loss Function* is known as WSELF and is defined as

$$L(\widehat{\alpha}, \alpha) = \frac{(\widehat{\alpha} - \alpha)^2}{\alpha}.$$

The *Modified form of Squared Error Loss Function* is known as M/Q SELF and is defined as

$$L(\widehat{\alpha}, \alpha) = \left(1 - \frac{\widehat{\alpha}}{\alpha}\right)^2.$$

The Bayes estimators and posterior risks of different loss functions are given in the following table:

Table 1. Bayes estimators and posterior risks of different loss functions

Loss Function (LF)	*Bayes Estimator* (BE)	*Posterior Risk* (PR)
SELF	$E(\alpha/\underline{x})$	$V(\alpha/\underline{x})$
WSELF	$[E(\alpha^{-1}/\underline{x})]^{-1}$	$E(\alpha/\underline{x})-[E(\alpha^{-1}/\underline{x})]^{-1}$
M/Q SELF	$\frac{E(\alpha^{-1}/\underline{x})}{E(\alpha^{-2}/\underline{x})}$	$1-\frac{[E(\alpha^{-1}/\underline{x})]^2}{E(\alpha^{-2}/\underline{x})}$

3. Bayesian Estimators

Recently, for analyzing statistical data, many researchers have shown a great interest towards Bayesian approach. In the inferential procedure, the Bayesian approach permits prior subjective knowledge on parameters to be included. To get the same quality of inferences Bayesian methods require less sample data than methods based on sampling theory. In the following theorem, we derive the Bayes estimators of α and DCRE by use of different loss functions and priors as discussed in Section 2.

Theorem 3.1. *For Pareto type II distribution* $f(x)=\frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}$ *if prior density function is given by*

$$\pi_1(\alpha/\underline{x})=\frac{W^n}{\Gamma(n)}\alpha^{n-1}\exp(-\alpha W),$$

where $W=\sum_{i=1}^{n}\log\left(1+\frac{x_i}{\lambda}\right)$.

Then Bayes estimators of α *under the loss functions defined in Table* 1 *are given by*

$$\hat{\alpha}_{JS}=\frac{n}{W},\quad \hat{\alpha}_{JW}=\frac{n-1}{W},\quad \hat{\alpha}_{JM}=\frac{n-2}{W}.$$

Proof. We have

$$E\left[\frac{\alpha}{\underline{x}}\right]=\int_0^\infty\frac{W^n}{\Gamma(n)}\alpha^n\exp(-\alpha W)\,d\alpha=\frac{n}{W}.$$

Therefore,

$$\hat{\alpha}_{JS}=E_\pi\left[\frac{\alpha}{\underline{x}}\right]=\frac{n}{W},$$

$$E\left[\frac{1}{\alpha}/\underline{x}\right]=\int_0^\infty\frac{W^n}{\Gamma(n)}\alpha^{n-2}\exp(-\alpha W)\,d\alpha=\frac{\Gamma(n-1)W}{\Gamma(n)}=\frac{W}{n-1}.$$

Therefore,

$$\hat{\alpha}_{JW}=\left[E_\pi\left[\frac{1}{\alpha}/\underline{x}\right]\right]^{-1}=\left[\frac{W}{n-1}\right]^{-1}=\frac{n-1}{W},$$

$$E\left[\frac{1}{\alpha^2}/\underline{x}\right]=\frac{W^2\Gamma(n-2)}{\Gamma(n)}=\frac{W^2}{(n-1)(n-2)}.$$

Therefore

$$\hat{\alpha}_{JM} = \frac{E\left[\frac{1}{\alpha}/\underline{x}\right]}{E\left[\frac{1}{\alpha^2}/\underline{x}\right]} = \frac{(n-2)}{W}.$$ □

Theorem 3.2. *For Pareto type II distribution* $f(x) = \frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}$ *if prior density function is given by*

$$\pi_1(\alpha/\underline{x}) = \frac{W^n}{\Gamma(n)}\alpha^{n-1}\exp(-\alpha W),$$

where $W = \sum_{i=1}^{n}\log(1+\frac{x_i}{\lambda})$.
Then Bayes estimators of DCRE under the loss functions defined in Table 1 *are given by*

$$\hat{E}(t)_{JS} = \frac{W^n(t+\lambda)}{\Gamma(n)}\sum_{k=1}^{\infty}k\frac{\Gamma(n-k)}{W^{n-k}},$$

$$\hat{E}(t)_{JW} = \frac{(t+\lambda)W(n-1)}{(n-2W)(n-1)+W^2},$$

$$\hat{E}(t)_{JM} = \frac{(t+\lambda)W(n-2)[(n-2W)(n-1)+W^2]}{(n-1)(n-2)[n(n+1-4W)+6W^2]-4W^3(n-2)+W^4}.$$

Proof.

$$E[E(t)/\underline{x}] = \int_0^\infty \frac{W^n}{\Gamma(n)}\frac{\alpha(t+\lambda)}{(\alpha-1)^2}\alpha^{n-1}e^{-\alpha W}d\alpha = \frac{(t+\lambda)W^n}{\Gamma(n)}\left[\sum_{k=1}^{\infty}k\frac{\Gamma(n-k)}{W^{n-k}}\right].$$

Therefore,

$$\hat{E}(t)_{JS} = E[E(t)/\underline{x}] = \frac{W^n(t+\lambda)}{\Gamma(n)}\sum_{k=1}^{\infty}k\frac{\Gamma(n-k)}{W^{n-k}},$$

$$E\left[\frac{1}{E(t)}/\underline{x}\right] = \frac{W^n}{(t+\lambda)\Gamma(n)}\int_0^\infty \frac{(\alpha-1)^2}{\alpha t}\alpha^{n-1}e^{-\alpha W}d\alpha = \frac{(n-2W)(n-1)+W^2}{(t+\lambda)W(n-1)}.$$

Therefore,

$$\hat{E}(t)_{JW} = E\left[\frac{\frac{1}{E(t)}}{\underline{x}}\right]^{-1} = \frac{(t+\lambda)W(n-1)}{(n-2W)(n-1)+W^2},$$

$$E\left[\frac{1}{E(t)^2}/\underline{x}\right] = \frac{W^n}{\Gamma(n)}\int_0^\infty \frac{(\alpha-1)^4}{(\alpha(t+\lambda))^2}\alpha^{n-1}e^{-\alpha W}d\alpha$$
$$= \frac{(n-1)(n-2)[n(n+1-4W)+6W^2]-4W^3(n-2)+W^4}{W^2(t+\lambda)^2(n-1)(n-2)}.$$

Therefore,

$$\hat{E}(t)_{JM} = \frac{E\left[\frac{1}{E(t)}/\underline{x}\right]}{E\left[\frac{1}{E(t)^2}/\underline{x}\right]} = \frac{(t+\lambda)W(n-2)[(n-2W)(n-1)+W^2]}{(n-1)(n-2)[n(n+1-4W)+6W^2]-4W^3(n-2)+W^4}.$$ □

Theorem 3.3. *For Pareto type II distribution* $f(x)=\frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}$ *if prior density function is given by*

$$\pi_1(\alpha/\underline{x})=\frac{W^n}{\Gamma(n)}\alpha^{n-1}\exp(-\alpha W),$$

where $W=\sum_{i=1}^{n}\log\left(1+\frac{x_i}{\lambda}\right)$.

Then Posterior risk of α *under the loss functions defined in Table* 1 *are given by*

$$\widehat{\alpha}_{JPS}=\frac{n}{W^2},\quad \widehat{\alpha}_{JPW}=\frac{1}{W},\quad \widehat{\alpha}_{JPM}=\frac{1}{n-1}.$$

Theorem 3.4. *For Pareto type II distribution* $f(x)=\frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}$ *if prior density function is given by*

$$\pi_1(\alpha/\underline{x})=\frac{W^n}{\Gamma(n)}\alpha^{n-1}\exp(-\alpha W),$$

where $W=\sum_{i=1}^{n}\log\left(1+\frac{x_i}{\lambda}\right)$.

Then Posterior Risk of DCRE under the loss functions defined in Table 1 *are given by*

$$\widehat{E}(t)_{JPS}=\frac{W^n(t+\lambda)^2}{\Gamma(n)}\sum_{k=1}^{\infty}\sum_{s=1}^{\infty}ks\frac{\Gamma(n-k-s)}{W^{n-k-s}}-\left(\frac{W^n(t+\lambda)}{\Gamma(n)}\sum_{k=1}^{\infty}k\frac{\Gamma(n-k)}{W^{n-k}}\right)^2,$$

$$\widehat{E}(t)_{JPW}=\left(\frac{W^n(t+\lambda)}{\Gamma(n)}\sum_{k=1}^{\infty}k\frac{\Gamma(n-k)}{W^{n-k}}\right)-\frac{(t+\lambda)W(n-1)}{(n-2W)(n-1)+W^2},$$

$$\widehat{E}(t)_{JPM}=1-\frac{(n-2)[(n-2W)(n-1)+W^2]^2}{(n-1)[(n-1)(n-2)[n(n+1-4W)+6W^2]-4W^3(n-2)+W^4]}$$

4. Numerical Computation and Graphical Interpretation

To explain the suggested estimators in Table 1, a real data set as used in Singh *et al*. [18] has been taken. The ordered real data set is given as:

0.2, 0.3, 0.5, 0.5, 0.5, 0.5, 0.6, 0.6, 0.7, 0.7, 0.7, 0.8, 0.8, 1.0, 1.0, 1.0, 1.0,1.1, 1.3, 1.5, 1.5, 1.5,1.5, 2.0, 2.0, 2.2, 2.5, 2.7, 3.0, 3.0, 3.3, 3.3, 4.0, 4.0,4.5, 4.7, 5.0, 5.4, 5.4, 7.0, 7.5, 8.8, 9.0, 10.3, 22.0, 24.5.

The following tables gives Bayes estimators and posterior risks (in the parenthesis) of α and DCRE for real data:

Table 2. Bayes estimators and posterior risk (in the parenthesis) of (α)

Loss function	Jeffrey	Hartigan	Uniform	Conjugate Gamma
SELF	3.549235(0.2738493)	3.39492(0.2619428)	3.626392(0.2798025)	3.491688(0.2594017)
WSELF	3.472078(0.077157)	3.317763(0.077157)	3.549235(0.077157)	3.417396(0.074291)
M/Q SELF	3.39492(0.022222)	3.240606(0.023256)	3.472078(0.021739)	3.343105(0.021739)

Table 3. Bayes estimators and posterior risks (in the parenthesis) of DCRE

t	Loss function	Jeffrey	Hartigan	Uniform	Conjugate Gamma
0.5	SELF WSELF M/QSELF	5.70948(2.96475) 5.29928(0.410204) 4.95945(0.064127)	6.242(4.52642) 5.7395(0.502496) 5.10709(0.070323)	5.47656(2.53312) 5.10239(0.374171) 4.78922(0.061377)	5.87858(3.16945) 5.4565(0.422071) 5.336(0.064037)
1.0	SELF WSELF M/QSELF	6.0027(3.27708) 5.57143(0.43127) 5.21415(0.064127)	6.56256(5.00327) 6.03426(0.528301) 5.61004(0.070323)	5.75781(2.79998) 5.36442(0.393386) 5.03517(0.061377)	6.18047(3.50334) 5.73672(0.443747) 5.36936(0.064037)
1.5	SELF WSELF M/QSELF	6.29591(3.60505) 5.84357(0.452336) 5.46884(0.064127)	6.88312(5.504) 6.32901(0.554107) 5.88407(0.070323)	6.03906(3.0802) 5.62646(0.412602) 5.28112(0.061377)	6.48237(3.85396) 6.01695(0.465423) 5.63164(0.064037)
2.0	SELF WSELF M/QSELF	6.58912(3.94866) 6.11572((0.473402) 5.72354(0.064127)	7.20368(6.0286) 6.62377(0.579913) 6.1581(0.070323)	6.32031(3.37378) 5.88849(0.431818) 5.52707(0.061377)	6.78426(4.22129) 6.29717(0.487098) 5.89392(0.064037)
2.5	SELF WSELF M/QSELF	6.88233(4.3079) 6.38786(0.494469) 5.97823(0.064127)	7.52424(6.57707) 6.91852(0.605719) 6.43213(0.070323)	6.60156(3.68073) 6.15053(0.451033) 5.77302(0.061377)	7.08616(4.60534) 6.57739(0.508774) 6.15619(0.064037)
3.0	SELF WSELF M/QSELF	7.17555(4.68278) 6.66001(0.515535) 6.23293(0.064127)	7.8448(7.14943) 7.21328(0.631525) 6.70617(0.070323)	6.88281(4.00103) 6.41256(0.470249) 6.01898(0.061377)	7.38806(5.0061) 6.85761(0.530449) 6.41847(0.064037)
3.5	SELF WSELF M/QSELF	7.46876(5.07331) 6.93216(0.536601) 6.48762(0.064127)	8.16536(7.74566) 7.50803(0.657331) 6.9802(0.070323)	7.16406(4.3347) 6.6746(0.489465) 6.26493(0.061377)	7.68995(5.42359) 7.13783(0.552125) 6.68074(0.064037)
4.0	SELF WSELF M/QSELF	7.76197(5.47947) 7.2043(0.557667) 6.74232(0.064127)	8.48592(8.36576) 7.80278(0.683137) 7.25423(0.070323)	7.44531(4.68173) 6.93663(0.50868) 6.51088(0.061377)	7.99185(5.85779) 7.41805(0.573801) 6.94302(0.064037)

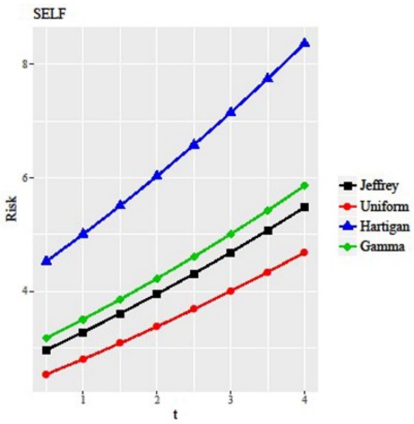

Figure 1

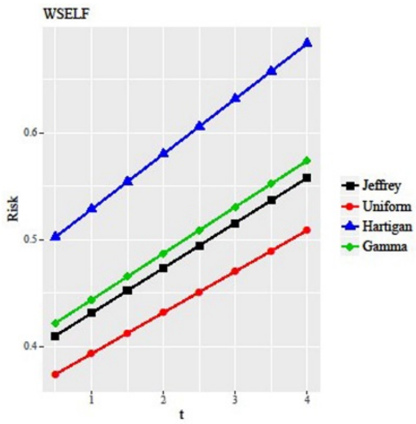

Figure 2

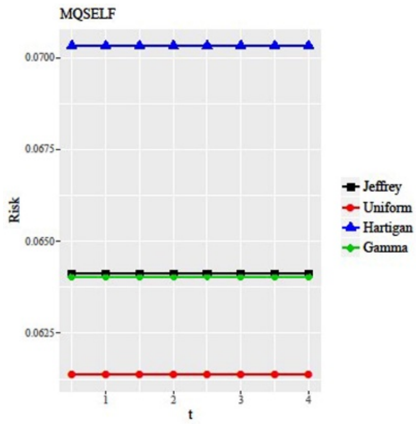

Figure 3

The graphs in Figure 1 to Figure 3 represent posterior risks for different values of t for different loss functions. The posterior risk is higher for Hartigan prior and smaller for Uniform prior. Comparatively to non-informative priors, it is concluded that Uniform prior is better than both the Jeffrey's and Hartigan priors as the posterior risk is smaller for Uniform prior. In Figure 1 and Figure 2, posterior risk increases as t increases for all priors whereas in Figure 3, posterior risk is invariant under t for all priors.

Simulation Study

A simulation study has been done to check the capability of the estimators. For computation, MATHEMATICA software is used whereas graphs are plotted using R software. For the purpose, the sample size is taken as 5, 10, 15, 20, 25, 50, 100 and $\alpha = 5, 7.5, 10, 12.5, 15$, $c = 0.9$ and $\lambda = 20$. The Bayes estimators and posterior risks (in the parenthesis) of DCRE for $n = 5$ and $n = 50$ are given in the following tables:

Table 4. Bayes estimators and posterior risks (in the parenthesis) of DCRE for $n = 5$

α	Loss function	Jeffrey	Hartigan	Uniform	Conjugate Gamma
5	SELF WSELF M/QSELF	18.4398(-101.346) 7.2638(11.176) 4.95643(0.317654)	36.1548(-1057.37) 16.0877(20.0671) 7.6276(0.525874)	10.8565(15.6002) 5.56748(5.28898) 4.14793(0.254972)	29.7279(-337.317) 9.83887(19.8891) 6.71844(0.317154)
7.5	SELF WSELF M/QSELF	7.6365(8.30078) 4.08396(3.55254) 3.01354(0.262104)	18.5524(-233.167) 8.33642(10.2159) 4.65898(0.441129)	4.90066(9.89511) 3.2247(1.67597) 2.53767(0.213053)	14.1056(-0.58861) 6.51882(7.58678) 4.7571(0.270252)
10	SELF WSELF M/QSELF	4.59589(7.4174) 2.82183(1.77406) 2.15581(0.236022)	11.8327(-77.5621) 5.48221(6.35047) 3.31729(0.3949)	3.14451(4.37413) 2.26008(0.884424) 1.82219(0.19375)	9.61086(17.6423) 5.1566(4.45426) 3.87734(0.248081)
12.5	SELF WSELF M/QSELF	3.25771(4.66636) 2.15132(1.10639) 1.6758(0.221034)	8.46699(-31.7217) 4.05313(4.41386) 2.56931(0.366093)	2.31863(2.29174) 1.73743(0.581196) 1.41996(0.182724)	7.60238(16.7977) 4.42001(3.18237) 3.38017(0.235256)
15	SELF WSELF M/QSELF	2.51865(3.01266) 1.73687(0.781776) 1.3698(0.211339)	6.50074(-14.5039) 3.20545(3.2953) 2.09439(0.346614)	1.83817(1.36809) 1.41037(0.427804) 1.1627(0.175609)	6.48489(14.5258) 3.95965(2.52524) 3.06114(0.226917)

Table 5. Bayes estimators and posterior risk (in the parenthesis) of DCRE for $n = 50$

α	Loss function	Jeffrey	Hartigan	Uniform	Conjugate Gamma
5	SELF WSELF M/QSELF	7.09456(2.76588) 6.75343(0.341123) 6.45131(0.044736)	7.58748(3.47244) 7.19231(0.395172) 6.84606(0.048142)	6.87092(2.48275) 6.5529(0.318024) 6.26988(0.04319)	7.39253(3.01913) 7.03557(0.356966) 6.71969(0.044897)
7.5	SELF WSELF M/QSELF	3.9407(0.607155) 3.80011(0.14059) 3.67111(0.033948)	4.17157(0.732078) 4.01259(0.158979) 3.86763(0.036127)	3.83452(0.555171) 3.70197(0.132546) 3.58(0.032949)	4.2085(0.700126) 4.05692(0.151573) 3.91802(0.034239)
10	SELF WSELF M/QSELF	2.71514(0.24507) 2.63145(0.083684) 2.55357(0.029595)	2.86168(0.290459) 2.76808(0.093606) 2.68138(0.03132)	2.64733(0.225876) 2.56803(0.079297) 2.49407(0.028798)	2.9667(0.296627) 2.87415(0.09255) 2.78815(0.029923)
12.5	SELF WSELF M/QSELF	2.06806(0.129235) 2.00963(0.058424) 1.95485(0.027258)	2.17441(0.151708) 2.10946(0.064944) 2.04881(0.028752)	2.01868(0.119646) 1.96316(0.055522) 1.911(0.026566)	2.30987(0.163717) 2.24372(0.066154) 2.18178(0.027603)
15	SELF WSELF M/QSELF	1.66907(0.079042) 1.62457(0.044502) 1.58265(0.025804)	1.75221(0.092218) 1.70294(0.049269) 1.65669(0.027158)	1.63039(0.073387) 1.58802(0.042373) 1.54804(0.025175)	1.90443(0.10461) 1.8529(0.051531) 1.80443(0.026158)

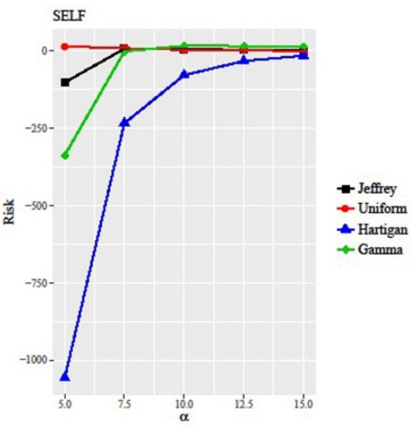

Figure 4

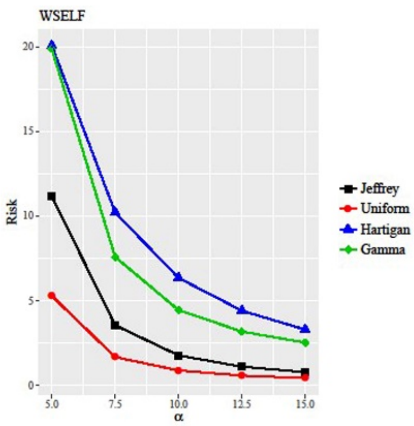

Figure 5

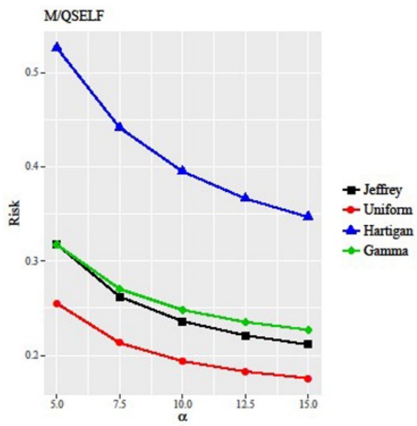

Figure 6

The graphs in Figure 4 to Figure 6 represent posterior risks of DCRE for $n = 5$ with different loss functions for distinct values of α. In Figure 4, the posterior risk increases sharply for small values of α, that is, from $\alpha = 5$ to 7.5 for the Hartigan prior thereafter it increases slowly. The posterior risks increase slowly for all other priors (Jeffrey's and Gamma) but for the Uniform prior the posterior risk decreases from $\alpha = 5$ onwards. In Figure 5 and Figure 6, the posterior risks declines for all the priors as well as for all values of α.

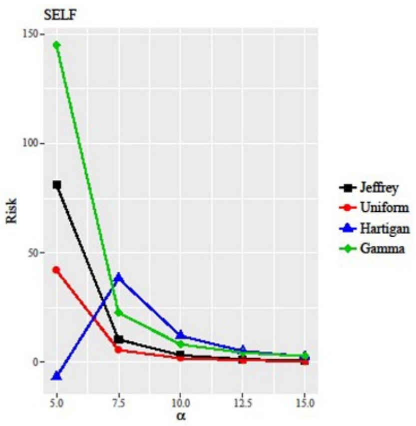

Figure 7

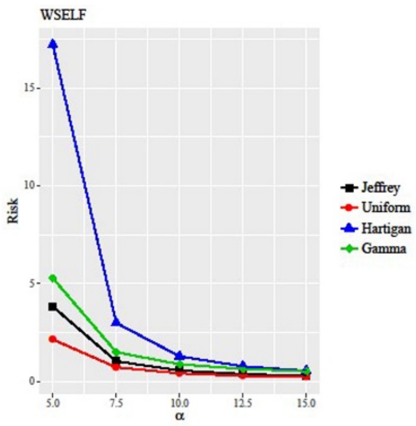

Figure 8

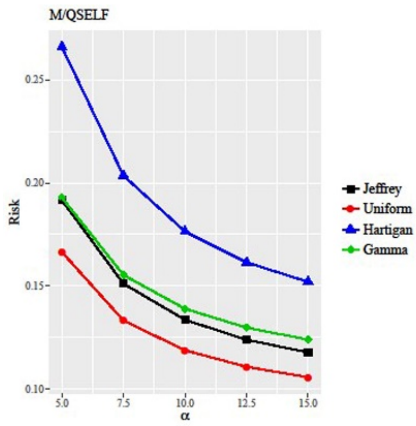

Figure 9

The patterns in Figure 7 to Figure 9 exhibit posterior risks of DCRE for $n = 10$ with different loss functions for distinct values of α. In Figure 7, the posterior risk increases sharply for small values of α, that is from $\alpha = 5$ to 7.5, for the Hartigan prior thereafter it decreases. The posterior risks decline for all other priors (Jeffrey's, Uniform and Gamma) from $\alpha = 5$ onwards. In Figure 8 and Figure 9, the posterior risks decline for all the priors and for all values of α.

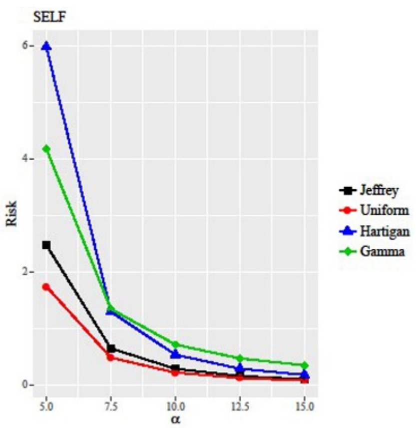

Figure 10

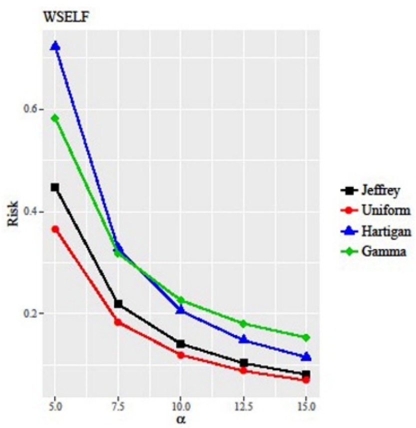

Figure 11

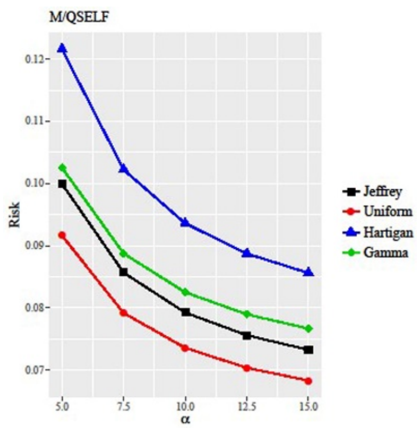

Figure 12

Figure 10 to Figure 12 represent the patterns of posterior risks of DCRE for $n = 15$ with different loss functions for varying values of α. The posterior risk is higher for the Hartigan prior and smaller for the Uniform prior. The posterior risk rapidly decreases from $\alpha = 5.0$ to 7.5 for the Hartigan prior and slowly decreases $\alpha = 7.5$ onwards. Comparatively to non-informative priors, it is concluded that Uniform prior is better than both the Jeffrey's and Hartigan priors as the posterior risk is smaller for the Uniform prior. Further, the posterior risks decline for distinct values of α for all the priors.

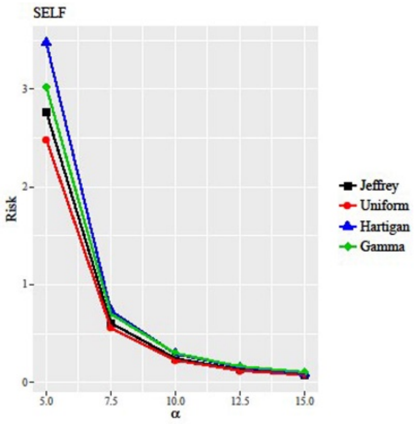

Figure 13

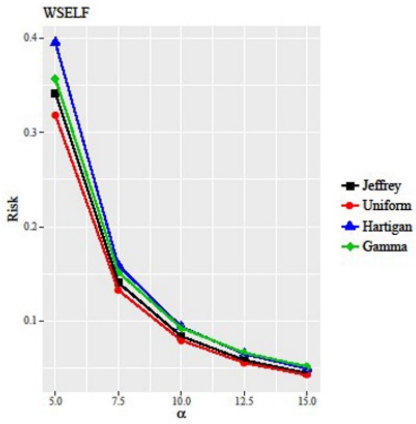

Figure 14

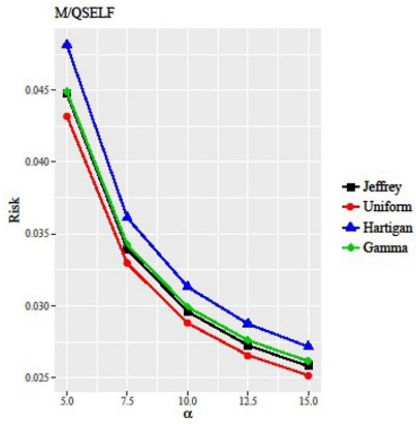

Figure 15

The graphs in Figure 13 to Figure 15 show the posterior risk of DCRE for $n = 50$ with different loss functions for different values of α. The posterior risk is higher for the Hartigan prior and smaller for the Uniform prior. The posterior risks rapidly decrease from $\alpha = 5.0$ to 7.5 for all priors thereafter slowly decrease from $\alpha = 7.5$ to 10.0 and however, during $\alpha = 10.0$ to $\alpha = 15.0$ posterior risks are almost same. Comparatively to non-informative priors, it is concluded that Uniform prior is better than both the Jeffrey's and Hartigan priors as the posterior risk is smaller for the Uniform prior. The posterior risks decline for distinct values of α and for all the priors. Moreover, difference between the posterior risks for all priors are less for $n = 50$ than the difference between the posterior risks for all priors for $n = 25$.

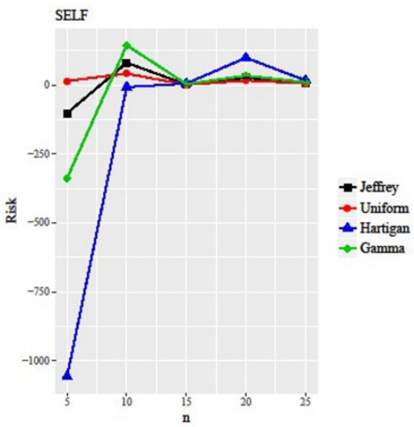

Figure 16

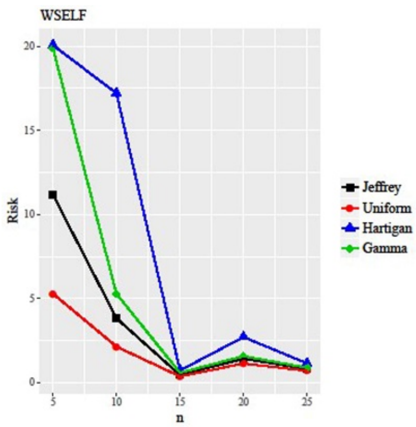

Figure 17

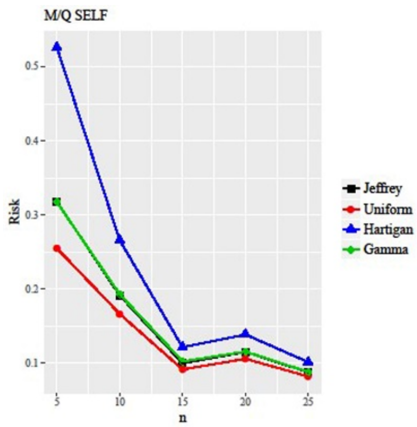

Figure 18

Figure 16 to Figure 18 exhibit the patterns of posterior risks of DCRE for $\alpha = 5$ with different loss functions for varying values of n. In Figure 16, the posterior risk increases sharply for small values of n, that is from $n = 5$ to 10, for the Hartigan prior thereafter it slowly increases from $n = 10$ to 20 and then $n = 20$ onwards it decreases. The posterior risks for all the priors (Jeffrey's, Uniform and Gamma) increase sharply for small values of n, that is from $n = 5$ to 10, then decrease from $n = 10$ to 15, and thereafter, these increase slowly from $n = 15$ to 20 and

$n = 20$ onwards these decrease however, less in comparison to the Hartigan prior. Further, in Figure 17 and Figure 18, the posterior risks decline for all the priors.

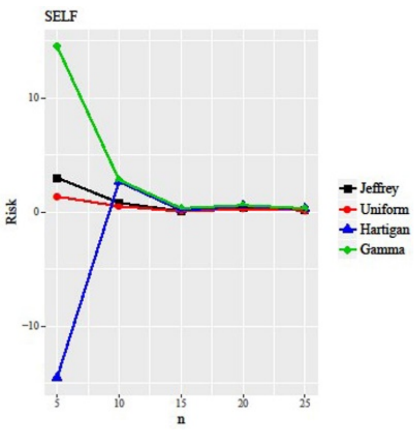

Figure 19

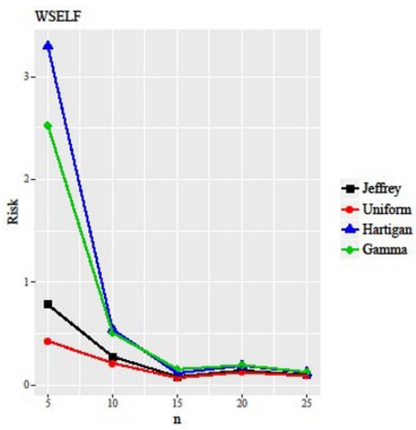

Figure 20

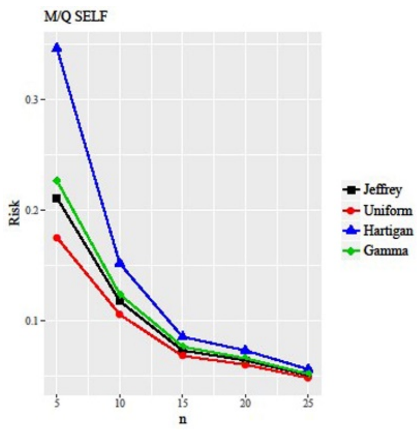

Figure 21

The graphs in Figure 19 to Figure 21 show the posterior risks of DCRE for $\alpha = 15$ with different loss functions for varying values of n. In Figure 19, the posterior risk increases sharply for small values of n, that is from $n = 5$ to 10 and thereafter, it decreases from $n = 10$ onwards for the Hartigan prior. The posterior risks decrease slowly for all the priors (Jeffrey's, Uniform and Gamma). Further, in Figure 20 and Figure 21, the posterior risks declines for all the priors.

5. Conclusion

From the above study, it is concluded that posterior risks of DCRE with different loss functions increase as t increases for all the priors taken here. For small values of n, i.e. $n = 5$ and $n = 10$, the posterior risk increases for SELF, however for all other loss functions, it decreases. For $n = 15$ onwards, the posterior risks with all the loss functions decreases for different values of α. In these cases, the Uniform prior is better than all other priors as it has smaller risk as comparison to other priors and there after Jeffrey and Gamma priors are more preferable. The Hartigan prior has always higher risk as comparison to all the other priors for different loss functions taken in this paper.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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