Research Article

Strongly $T_k^{g^*}$-Spaces

R. Parimelazhagan and V. Jeyalakshmi*

Department of Mathematics, RVS Technical Campus, Coimbatore 641 402, Tamilnadu, India

*Corresponding author: jeya198022@gmail.com

Abstract. In this paper, we introduce the spaces called Strongly-$T_0^{g^*}$-space, Strongly-$T_1^{g^*}$-space and Strongly-$T_2^{g^*}$ in topological spaces. Also, we introduce Strongly $g^*$-symmetric and studied some of their properties.

Keywords. Strongly-$T_0^{g^*}$-space; Strongly-$T_1^{g^*}$-space and Strongly-$T_2^{g^*}$

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1. Introduction

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the variously modified forms of continuity, separation axioms etc. by utilizing generalized open sets. For a subset $A$ of a topological space $(X, \tau)$, $\text{Cl}(A)$ and $\text{Int}(A)$ denote the closure of $A$ and the interior of $A$, respectively. Wilansky [22] has introduced the concept of $US$ spaces. Aull [4] studied some separation axioms between the $T_1$ and $T_2$ spaces, namely, $S_1$ and $S_2$. Next, Arya et al. [3] have introduced and studied the concept of semi-$US$ spaces in the year 1982 and also they made study of $s$-convergence, sequentially semi-closed sets, sequentially $s$-compact notions. Navlagi studied $P$-Normal Almost-$P$-Normal and Mildly-$P$-Normal spaces. Closedness are basic concept for the study and investigation in general topological spaces. This concept has been generalized and studied by many authors from different points of views. Njastad [16] introduced and defined an $\alpha$-open and $\alpha$-closed set.
After the works of Njastad on \( \alpha \)-open sets, various mathematicians turned their attention to the generalizations of various concepts in topology by considering semi-open, \( \alpha \)-open sets. The concept of \( g \)-closed \([9]\), \( s \)-open \([10]\) and \( \alpha \)-open sets has a significant role in the generalization of continuity in topological spaces. The modified form of these sets and generalized continuity were further developed by many mathematicians (\([5]\), \([6]\), \([2]\), \([13]\), \([14]\)). Many authors have tried to weaken the condition closed in this theorem. In 1978, Long and Herrington \([11]\) used almost closedness due to Singal \([19]\). Malghan \([14]\) introduced the concept of generalized closed maps in topological spaces. Devi \([7]\) introduced and studied \( sg \)-closed maps and \( gs \)-closed maps. \( wg \)-closed maps and \( rwg \)-closed maps were introduced and studied by Nagavani \([15]\). Regular closed maps, \( gpr \)-closed maps and \( rg \)-closed maps have been introduced and studied by Long \([11]\), Gnanambal \([8]\) and Arockiarani \([2]\), respectively. In 2012, \([17]\) we introduced the concepts of Strongly \( g^* \)-closed sets and Strongly \( g^* \)-open set in topological spaces. Also we have introduced the concepts of Strongly \( g^* \)-continuous functions, Strongly \( g^* \)-irresolute functions, Strongly \( g^* \)-open maps and Strongly \( g^* \)-closed maps in \([20]\), \([21]\), \([18]\).

In this paper, by deriving the properties of Strongly-\( T_0^g \)-space, Strongly-\( T_1^g \)-space and Strongly-\( T_2^g \) in topological spaces. Also, we introduce the concept of Strongly \( g^* \)-symmetric and studied some of their properties. Further various characterization are studied.

### 2. Preliminaries

Throughout this paper \((X, \tau)\) and \((Y, \sigma)\) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset \(A\) of a space \((X, \tau), cl(A), int(A)\) and \(A^c\) denote the closure of \(A\), the interior of \(A\) and the complement of \(A\) in \(X\), respectively.

**Definition 2.1.** A subset \(A\) of a topological space \((X, \tau)\) is called

(a) a preopen set \([14]\) if \(A \subseteq int(cl(A))\) and pre-closed set if \(cl(int(A)) \subseteq A\).

(b) a semiopen set \([10]\) if \(A \subseteq cl(int(A))\) and semi closed set if \(int(cl(A)) \subseteq A\).

(c) an \(\alpha\)-open set \([16]\) if \(A \subseteq int(cl(int(A)))\) and an \(\alpha\)-closed set if \(cl(int(cl(A))) \subseteq A\).

(d) a semi-preopen set \([1]\) (\(\beta\)-open set) if \(A \subseteq cl(cl(A))\) and semi-preclosed set if \(int(cl(int(A))) \subseteq A\).

**Definition 2.2.** A space \((X, \tau_X)\) is called a \(T_{1/2}^g\)-space \([9]\) if every \(g\)-closed set is closed.

**Definition 2.3** (\([17]\)). Let \((X, \tau)\) be a topological space and \(A\) be its subset, then \(A\) is Strongly \(g^*\)-closed set if \(cl(int(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g\)-open.

The complement of Strongly \(g^*\)-closed set is called Strongly \(g^*\)-open set in \((X, \tau)\).

**Definition 2.4** (\([20]\)). Let \(X\) and \(Y\) be topological spaces. A map \(f : (X, \tau) \rightarrow (Y, \sigma)\) is said to be strongly \(G^*\)-continuous (\(sg^*\)-continuous) if the inverse image of every open set \(Y\) is \(sg^*\)-open in \(X\).
Definition 2.5 ([21]). Let X and Y be topological spaces. A map \( f : (X, \tau) \Rightarrow (Y, \sigma) \) is said to be strongly \( g^* \)-irresolute map (\( sg^* \)-irresolute map) if the inverse image of every \( sg^* \)-open set in Y is \( sg^* \)-open in X.

Definition 2.6 ([21]). Let X and Y be two topological spaces. A bijection map \( f : (X, \tau) \Rightarrow (Y, \sigma) \) from a topological space X into a topological space Y is called strongly \( g^* \)-Homeomorphism (\( sg^* \)-homeomorphism) if \( f \) and \( f^{-1} \) are \( sg^* \)-continuous.

Definition 2.7 ([18]).

(a) Let X be a topological space and let \( x \in X \). A subset \( N \) of X is said to be \( Strongly g^* \)-nbhd of \( x \) if there exists an \( \text{Strongly} g^* \)-open set \( G \) such that \( x \in G \subset N \).

The collection of all \( Strongly g^* \)-nbhd of \( x \in X \) is called a \( Strongly g^* \)-nbhd system at \( x \) and shall be denoted by \( Strongly g^*N(x) \).

(b) Let X be a topological space and \( A \) be a subset of X. A subset \( N \) of X is said to be \( Strongly g^* \)-nbhd of \( A \) if there exists a \( \text{Strongly} g^* \)-open set \( G \) such that \( A \in G \subset N \).

(c) Let \( A \) be a subset of \( X \). A point \( x \in A \) is said to be a \( \text{Strongly} g^* \)-interior point of \( A \), if \( A \) is a \( \text{Strongly} g^*N(x) \). The set of all \( \text{Strongly} g^* \)-interior points of \( A \) is called a \( \text{Strongly} g^* \)-interior of \( A \) and is denoted by \( Sg^*\text{INT}(A) \).

\( Sg^*\text{INT}(A) = \cup\{G : G \text{ is \( g^* \)-open,} G \subset A\} \).

(d) Let \( A \) be a subset of \( X \). A point \( x \in A \) is said to be a \( \text{Strongly} g^* \)-closure of \( A \). Then

\( Sg^*\text{CL}(A) = \cap\{F : A \subset F \text{ is \( g^* \)-closed}\} \).

3. Strongly-\( T_{k g^*} \)-spaces: \( k = 0, 1, 2 \)

Definition 3.1. A topological space \((X, \tau)\) is said to be

(a) \( \text{Strongly-} T_{0 g^*} \) if for each pair of distinct points \( x, y \in X \), there exists a \( \text{Strongly} g^* \)-open set \( U \) such that either \( x \in U \) and \( y \notin U \) or \( x \notin U \) and \( y \in U \).

(b) \( \text{Strongly-} T_{1 g^*} \) if for each pair of distinct points \( x, y \in X \), there exist two \( \text{Strongly} g^* \)-open sets \( U \) and \( V \) such that \( x \in U \) but \( y \notin U \) and \( y \in V \) but \( x \notin V \).

(c) \( \text{Strongly-} T_{2 g^*} \) if for each distinct points \( x, y \in X \), there exist two disjoint \( \text{Strongly} g^* \)-open sets \( U \) and \( V \) containing \( x \) and \( y \) respectively.

Proposition 3.2. A topological space \((X, \tau)\) is \( \text{Strongly-} T_{0 g^*} \) if and only if for each pair of distinct points \( x, y \) of \( X \), \( Sg^*\text{CL}(\{x\}) \neq Sg^*\text{CL}(\{y\}) \).

Proof. Necessity. Let \((X, \tau)\) be a \( \text{Strongly-} T_{0 g^*} \)-space and \( x, y \) be any two distinct points of \( X \). There exists a \( \text{Strongly} g^* \)-open set \( U \) containing \( x \) or \( y \), say \( x \) but not \( y \). Then \( X/U \) is a \( \text{Strongly} g^* \)-closed set which does not contain \( x \) but contains \( y \). Since \( Sg^*\text{CL}(\{y\}) \) is the smallest \( \text{Strongly} g^* \)-closed set containing \( y \), \( Sg^*\text{CL}(\{y\}) \subseteq X/U \) and therefore \( x \notin Sg^*\text{CL}(\{y\}) \). Consequently \( Sg^*\text{CL}(\{x\}) \neq Sg^*\text{CL}(\{y\}) \).

Sufficiency. Suppose that \( x, y \in X \), \( x \neq y \) and \( Sg^*\text{CL}(\{x\}) \neq Sg^*\text{CL}(\{y\}) \). Let \( z \) be a point of \( X \) such that \( z \in Sg^*\text{CL}(\{x\}) \) but \( z \notin Sg^*\text{CL}(\{y\}) \). We claim that \( x \notin Sg^*\text{CL}(\{y\}) \). For, if \( x \in Sg^*\text{CL}(\{y\}) \)
then $Sg^* CL((x)) \subseteq Sg^* CL((y))$. This contradicts the fact that $x \notin Sg^* CL((y))$. Consequently $x$ belongs to the Strongly $g^*$-open set $X/Sg^* CL(y)$ to which $y$ does not belong.

**Proposition 3.3.** A topological space $(X, \tau)$ is Strongly-$T_1^{g^*}$ if and only if the singletons are Strongly $g^*$-closed sets.

**Proof.** Let $(X, \tau)$ be Strongly-$T_1^{g^*}$ and $x$ any point of $X$. Suppose $y \in X/x$, then $x \neq y$ and there exists a Strongly $g^*$-open set $U$ such that $y \in U$ but $x \notin U$. Consequently $y \in U \subseteq X/x$, that is $X/x = \cup(U : y \in X/x)$ which is Strongly $g^*$-open.

Conversely, suppose $(p)$ is Strongly $g^*$-closed for every $p \in X$. Let $x, y \in X$ with $x \neq y$. Now $x \neq y$ implies $y \in X/x$. Hence $X/x$ is a Strongly $g^*$-open set contains $y$ but not $x$. Similarly $X/y$ is a Strongly $g^*$-open set contains $x$ but not $y$. Accordingly $X$ is a Strongly-$T_1^{g^*}$-space.

**Theorem 3.4.** Let $A$ and $B$ be subsets of $X$. Then

(a) $Sg^* INT(X) = X$ and $Sg^* INT(\phi) = \phi$,

(b) $Sg^* INT(A) \subseteq A$,

(c) If $B$ is any Strongly $g^*$-open set contained in $A$, then $B \subseteq Sg^* INT(A)$,

(d) If $A \subseteq B$, then $Sg^* INT(A) \subseteq Sg^* INT(B)$.

**Proof.** (a) Since $X$ and $\phi$ are Strongly $g^*$-open sets, $Sg^* INT(X) = \cup\{G : G$ is Strongly $g^*$-open, $G \subseteq X\} = X \cup\{all$ Strongly $g^*$-open sets$\} = X$. That is $Sg^* INT(X) = X$. Since $\phi$ is the only Strongly $g^*$-open set contained in $\phi$, $Sg^* INT(\phi) = \phi$.

(b) Let $x \in Sg^* INT(A) \Rightarrow x$ is a Strongly $g^*$-interior point of $A$.

$\Rightarrow A$ is a Strongly $g^*$-$N(x)$.

$\Rightarrow x \in A$.

Thus $x \in Sg^* INT(A) \Rightarrow x \in A$. Hence $Sg^* INT(A) \subseteq A$.

(c) Let $B$ be any Strongly $g^*$-open set such that $B \subseteq A$. Let $x \in B$, since $B$ is a Strongly $g^*$-open set contained in $A$, then $x$ is a Strongly $g^*$-interior point of $A$. That is $x \in Sg^* INT(A)$. Hence $B \subseteq Sg^* INT(A)$.

(d) Let $A$ and $B$ be subsets of $X$ such that $A \subseteq B$. Let $x \in Sg^* INT(A)$. Then $x$ is a Strongly $g^*$-interior point of $A$ and so $A$ is an Strongly $g^*$-$N(x)$. Since $B \supseteq A$, $B$ is also a Strongly $g^*$-$N(x)$. This implies that $x \in Sg^* INT(B)$. Thus we have shown that $x \in Sg^* INT(A) \Rightarrow x \in Sg^* INT(B)$. Hence $Sg^* INT(A) \subseteq Sg^* INT(B)$.

**Proposition 3.5.** The following statements are equivalent for a topological space $(X, \tau)$:

(a) $X$ is Strongly-$T_2^{g^*}$.

(b) Let $x \in X$. For each $y \neq x$, there exists a Strongly $g^*$-open set $U$ containing $x$ such that $y \notin Sg^* CL(U)$.

(c) For each $x \in X$, $\cap\{Sg^* CL(U) : U \in Strongly\; g^* O(X)$ and $x \in U\} = \{x\}$. 

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Theorem 3.8. If $x$ is Strongly-$T_{2g^*}$, there exist disjoint Strongly $g^*$-open sets $U$ and $V$ containing $x$ and $y$ respectively. So, $U \subseteq X/V$. Therefore, $Sg^*\text{CL}(U) \subseteq X/V$. So $y \notin Sg^*\text{CL}(U)$.

(b)$\Rightarrow$(c): If possible for some $y \neq x$, we have $y \in Sg^*\text{CL}(U)$ for every Strongly $g^*$-open set $U$ containing $x$, which then contradicts (b).

(c)$\Rightarrow$(a): Let $x, y \in X$ and $x \neq y$. Then there exists a Strongly $g^*$-open set $U$ containing $x$ such that $y \notin Sg^*\text{CL}(U)$. Let $V = X/Sg^*\text{CL}(U)$, then $y \in V$ and $x \in U$ and also $U \cap V = \emptyset$.

**Theorem 3.6.** If $A$ and $B$ are subsets of $X$, then $Sg^*\text{INT}(A) \cup Sg^*\text{INT}(B) \subseteq Sg^*\text{INT}(A \cup B)$.

**Proof.** We know that $A \subseteq A \cup B$ and $B \subseteq A \cup B$, $Sg^*\text{INT}(A) \subseteq Sg^*\text{INT}(A \cup B)$ and $Sg^*\text{INT}(B) \subseteq Sg^*\text{INT}(A \cup B)$. Therefore, $Sg^*\text{INT}(A \cup B) = Sg^*\text{INT}(A) \cup Sg^*\text{INT}(B)$.

**Remark 3.7.** Let $(X, \tau)$ be a topological space, then every Strongly-$T_{2g^*}$ space is Strongly-$T_{1g^*}$.

**Theorem 3.8.** If $A$ and $B$ are subsets of a space $X$, then $Sg^*\text{INT}(A \cap B) = Sg^*\text{INT}(A) \cap Sg^*\text{INT}(B)$.

**Proof.** We know that $A \cap B \subseteq A$ and $A \cap B \subseteq B$, $Sg^*\text{INT}(A \cap B) \subseteq Sg^*\text{INT}(A)$ and $Sg^*\text{INT}(A \cap B) \subseteq Sg^*\text{INT}(B)$. This implies that $Sg^*\text{INT}(A \cap B) \subseteq Sg^*\text{INT}(A \cap B) \Rightarrow (i)$.

Again, let $x \in Sg^*\text{INT}(A) \cap Sg^*\text{INT}(B)$, then $x \in Sg^*\text{INT}(A)$ and $x \in Sg^*\text{INT}(B)$. Hence $x$ is a Strongly $g^*$-interior point of each sets $A$ and $B$. It follows that $A$ and $B$ are Strongly $g^*\text{N}(x)$, so that their intersection $A \cap B$ is also a Strongly $g^*\text{N}(x)$. Hence $x \in Sg^*\text{INT}(A \cap B)$. Thus $x \in Sg^*\text{INT}(A) \cap Sg^*\text{INT}(B)$ implies that $x \in Sg^*\text{INT}(A \cap B)$.

Therefore, $Sg^*\text{INT}(A) \cap Sg^*\text{INT}(B) \subseteq Sg^*\text{INT}(A \cap B) \Rightarrow (ii)$.

From (i) and (ii), we get $Sg^*\text{INT}(A \cap B) = Sg^*\text{INT}(A) \cap Sg^*\text{INT}(B)$.

**Definition 3.9.** A topological space $(X, \tau)$ is said to be Strongly $g^*$-symmetric if for $x$ and $y$ in $X$, $x \in Sg^*\text{CL}(y)$ implies $y \in Sg^*\text{CL}(x)$.

**Corollary 3.10.** If a topological space $(X, \tau)$ is a Strongly-$T_{1g^*}$ space, then it is Strongly $g^*$-symmetric.

**Proof.** In a Strongly-$T_{1g^*}$ space, every singleton is Strongly $g^*$-closed, $(X, \tau)$ is Strongly $g^*$-symmetric.

**Corollary 3.11.** For a topological space $(X, \tau)$, the following statements are equivalent:

(a) $(X, \tau)$ is Strongly $g^*$-symmetric and Strongly-$T_{0g^*}$.

(b) $(X, \tau)$ is Strongly-$T_{1g^*}$.

**Proof.** By the previous corollary, it suffices to prove only (a)$\Rightarrow$(b): Let $x \neq y$ and as $(X, \tau)$ is Strongly-$T_{0g^*}$, we may assume that $x \in U \subseteq X/(y)$ for some $U \in \text{Strongly } g^*\text{O}(X)$. Then $x \notin Sg^*\text{CL}(y)$ and hence $y \notin Sg^*\text{CL}(x)$. There exists a Strongly $g^*$-open set $V$ such that $y \in V \subseteq X/(x)$ and thus $(X, \tau)$ is a Strongly-$T_{1g^*}$-space.
**Proposition 3.12.** If \((X, \tau)\) is a Strongly \(g^*\)-symmetric space, then the following statements are equivalent:

(a) \((X, \tau)\) is a Strongly-\(T_0\)\(g^*\)-space.

(b) \((X, \tau)\) is a Strongly-\(T_1\)\(g^*\)-space.

**Proof.** (a)\(\iff\)(b): Obvious from the previous corollary.

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**Theorem 3.13.** If \(A\) and \(B\) are subsets of a space \(X\), then

(a) \(Sg^* CL(X) = X\) and \(Sg^* CL(\phi) = \phi\),

(b) \(A \subset Sg^* CL(A)\),

(c) If \(B\) is any Strongly \(g^*\)-closed set containing \(A\), then \(Sg^* CL(A) \subset B\),

(d) If \(A \subset B\), then \(Sg^* CL(A) \subset Sg^* CL(B)\).

**Proof.** (a) By the definition of a Strongly \(g^*\)-closure, \(X\) is the only Strongly \(g^*\)-closed set containing \(X\). Therefore \(Sg^* CL(X) = \cap\{X \in \text{Strongly } g^* \text{-closed sets containing } X = \cap\{X = X\}\). That is \(Sg^* CL(X) = X\). By the definition of a Strongly \(g^*\)-closure, \(Sg^* CL(\phi) = \cap\{X = X\}\). Therefore, \(Sg^* CL(\phi) = \phi\). That is \(Sg^* CL(\phi) = \phi\).

(b) By the definition of a Strongly \(g^*\)-closure of \(A\). It is obvious that \(A \subset Sg^* CL(A)\).

(c) Let \(B\) be any Strongly \(g^*\)-closed set containing \(A\). Since \(Sg^* CL(A)\) is the intersection of all Strongly \(g^*\)-closed sets containing \(A\), \(Sg^* CL(A)\) is contained in every Strongly \(g^*\)-closed set containing \(A\). Hence in particular \(Sg^* CL(A) \subset B\).

(d) Let \(A\) and \(B\) be subsets of \(X\) such that \(A \subset B\). By the definition of a Strongly \(g^*\)-closure, \(Sg^* CL(B) = \cap\{F : F \subset F \in \text{Strongly } g^* \text{-Closed sets in } X = \cap\{X, \tau_X\}\}\). If \(B \subset F \in \text{Strongly } g^* \text{-Closed spaces in } X, \tau_X\), then \(Sg^* CL(B) \subset F\). Since \(A \subset B\), \(A \subset B \subset F \in \text{Strongly } g^* \text{-Closed spaces in } X, \tau_X\), we have \(Sg^* CL(A) \subset F\). Therefore, \(Sg^* CL(A) \subset \cap\{F : F \subset F \in \text{Strongly } g^* \text{-Closed spaces in } X, \tau_X\}\). That is \(Sg^* CL(A) \subset Sg^* CL(B)\).

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**Theorem 3.14.** If \(A\) and \(B\) are subsets of a space \(X\), then \(Sg^* CL(A \cap B) \subset Sg^* CL(A) \cap Sg^* CL(B)\).

**Proof.** Let \(A\) and \(B\) be subsets of \(X\). Clearly \(A \cap B \subset A\) and \(A \cap B \subset B\), \(Sg^* CL(A \cap B) \subset Sg^* CL(A)\) and \(Sg^* CL(A \cap B) \subset Sg^* CL(B)\). Hence \(Sg^* CL(A \cap B) \subset Sg^* CL(A) \cap Sg^* CL(B)\).

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**Theorem 3.15.** If \(A\) and \(B\) are subsets of a space \(X\), then \(Sg^* CL(A \cup B) = Sg^* CL(A) \cup Sg^* CL(B)\).

**Proof.** Let \(A\) and \(B\) be subsets of \(X\). Clearly \(A \subset A \cup B\) and \(B \subset A \cup B\). Hence \(Sg^* CL(A) \cup Sg^* CL(B) \subset Sg^* CL(A \cup B)\) \(\Rightarrow (i)\).

Now to prove \(Sg^* CL(A \cup B) \subset Sg^* CL(A) \cup Sg^* CL(B)\). Let \(x \in Sg^* CL(A \cup B)\) and suppose \(x \notin Sg^* CL(A) \cup Sg^* CL(B)\). Then there exists a Strongly \(g^*\)-closed sets \(A_1\) and \(B_1\) with \(A \subset A_1\), \(B \subset B_1\) and \(x \notin A_1 \cup B_1\). We have \(A \cup B \subset A_1 \cup B_1\) and \(A_1 \cup B_1\) is a Strongly \(g^*\)-closed set such...
that \( x \notin A_1 \cup B_1 \). Thus \( x \notin Sg^* CL(A \cup B) \) which is a contradiction to \( x \in Sg^* CL(A \cup B) \). Hence \( Sg^* CL(A \cup B) \subset Sg^* CL(A) \cup Sg^* CL(B) \). Hence \( Sg^* CL(A \cup B) = Sg^* CL(A) \cup Sg^* CL(B) \).

From (i) and (ii), we have \( Sg^* CL(A \cup B) = Sg^* CL(A) \cup Sg^* CL(B) \). \( \square \)

**Definition 3.16.** A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called a Strongly \( g^* \)-open function if the image of every Strongly \( g^* \)-open set in \( (X, \tau) \) is a Strongly \( g^* \)-open set in \( (Y, \sigma) \).

**Proposition 3.17.** Suppose that \( f : (X, \tau) \rightarrow (Y, \sigma) \) is Strongly \( g^* \)-open and surjective. If \( (X, \tau) \) is Strongly-\( T_{k \times g^*} \), then \( (Y, \sigma) \) is Strongly-\( T_{k \times g^*} \), for \( k = 0, 1, 2 \).

**Proof.** We prove only the case for Strongly-\( T_{1 \times g^*} \)-space the others are similarly. Let \( (X, \tau) \) be a Strongly-\( T_{1 \times g^*} \)-space and let \( y_1, y_2 \in Y \) with \( y_1 \neq y_2 \). Since \( f \) is surjective, so there exist distinct points \( x_1, x_2 \) of \( (X, \tau) \) such that \( f(x_1) = y_1 \) and \( f(x_2) = y_2 \). Since \( (X, \tau) \) is a Strongly-\( T_{1 \times g^*} \)-space, there exist Strongly \( g^* \)-open sets \( G \) and \( H \) such that \( x_1 \in G \) but \( x_2 \notin G \) and \( x_2 \in H \) but \( x_1 \notin H \). Since \( f \) is a Strongly \( g^* \)-open function, \( f(G) \) and \( f(H) \) are \( \alpha^m \)-open sets of \( (Y, \sigma) \) such that \( y_1 = f(x_1) \in f(G) \) but \( y_2 = f(x_2) \notin f(G) \), and \( y_2 = f(x_2) \in f(H) \) but \( y_1 = f(x_1) \notin f(H) \). Hence \( (Y, \sigma) \) is a Strongly-\( T_{1 \times g^*} \)-space. \( \square \)

### 4. Conclusion

In this paper, we studied that the set Strongly-\( T_{0 \times g^*} \)-space, Strongly-\( T_{1 \times g^*} \)-space,\( T_{2 \times g^*} \) and Strongly \( g^* \)-symmetric spaces in topological spaces. Also we discussed some of their characters.

**Competing Interests**

The authors declare that they have no competing interests.

**Authors’ Contributions**

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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