Journal of Informatics and Mathematical Sciences
Vol. 11, Nos. 3-4, pp. 375–382, 2019
ISSN 0975-5748 (online); 0974-875X (print)
Published by RGN Publications
http://www.rgnpublications.com
DOI: 10.26713/jims.v11i3-4.930

Research Article

Strongly g^*-Closed Graph Function with Strongly-$T_i{}^{g^*}$ Spaces

K. Sivakumar[1], T. Arunachalam[2] and R. Parimelazhagan[3,*]

[1] *Department of Mathematics, Sathyabama University, Chennai 119, Tamilnadu, India*
[2] *Department of Mathematics, Kumaraguru College of Technology, Coimbatore 641049, Tamilnadu, India*
[3] *Department of Mathematics, RVS Technical Campus, Coimbatore 641402, Tamilnadu, India*
[*] **Corresponding author:** parimelazhagankce@gmail.com

Abstract. In this paper, we introduce the graph function called Strongly g^*-closed graph function and studied some of their properties.

Keywords. Strongly g^*-closed graph function

MSC. 57N40

Received: June 7, 2018 **Accepted:** March 25, 2019

1. Introduction

We say that a function f from a space X into a space Y has a closed graph if the graph of the function f, i.e., the set $\{(x,y) \in X \times Y : y = f(x)\}$, is a closed subset of the product $X \times Y$. It is well known that any continuous function f from a space X into a Hausdorff space Y has a closed graph.

Topology, sometimes referred to as "the mathematics of continuity", or "rubber sheet geometry", or "the theory of abstract topological spaces", is all of these, but, above all, it is a language, used by mathematicians in practically all branches of our science. The notion homeomorphism plays a very important role in topology. By definition, a homeomorphism between two topological spaces X and Y is a bijective map f. It is well known that as Janich says correctly: homeomorphisms play the same role in topology that linear isomorphisms play in linear algebra, or that biholomorphic maps play in function theory, or group isomorphisms in group theory, or isometries in Riemannian geometry. In the course of generalizations of the notion of homeomorphism. A space which has an associated family of subsets that constitute a topology. The relationships between members of the space are mathematically analogous to those between points in ordinary two- and three-dimensional space. In topology and related branches of mathematics, a topological space may be defined as a set of points, along with a set of neighbourhoods for each point, satisfying a set of axioms relating points and neighbourhoods. The definition of a topological space relies only upon set theory and is the most general notion of a mathematical space that allows for the definition of concepts such as continuity, connectedness, and convergence. Other spaces, such as manifolds and metric spaces, are specializations of topological spaces with extra structures or constraints. Being so general, topological spaces are a central unifying notion and appear in virtually every branch of modern mathematics.

The branch of mathematics that studies topological spaces in their own right is called point-set topology or general topology. Andrijevic [1] introduced the notion of b-open sets in a topological space and obtained their various properties. El-Etik [12] introduced the same concept in the name of γ-open sets. El-Etik also introduced the concept of γ-continuous (b-continuous) functions with the aid of b-open sets. In 2004, Ekici and Caldas [13] introduced the notion of slightly γ-continuity (slightly b-continuity) which is a weakened form of b-continuity. In their paper, the authors have studied basic properties and preservation theorems of slightly b-continuous functions. The relationships of slightly b-continuity with other weaker forms of continuity have also been studied. The concept of generalized closed sets (briefly g-closed) in topological spaces was introduced by Levine [16] and a class of topological spaces called $T_{\frac{1}{2}}$ spaces. Arya and Nour [3], Bhattacharya and Lahiri [7], Levine [17], Mashhour [19], Njastad [22], and Andrijevic ([2], [1]) introduced and investigated generalized semi-open sets, semi generalized open sets, generalized open sets, semi-open sets, pre-open sets and α-open sets, semi pre-open sets and b-open sets which are some of the weak forms of open sets and the complements of these sets are called the same types of closed sets.

Tong ([29], [30]) has introduced A-sets, B-sets and t-sets. A-sets and B-sets are also weak forms of open sets whereas t-sets is a weak form of a closed sets. Ganster and Reilly [14] have introduced locally closed sets, which are weaker than both open and closed sets. Cameron [9] has introduced regular semi-open sets which are weaker than regular open sets.

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the variously modified forms of continuity, separation axioms etc. by utilizing generalized open sets. For a subset A of a topological space (X,τ), $Cl(A)$ and $Int(A)$ denote the closure of A and the interior of A, respectively. Wilansky [31] has introduced the concept of US spaces. Aull [6] studied some separation axioms between the T_1 and T_2 spaces, namely, S_1 and S_2. Next, Arya et al. [5] have introduced and studied the concept of semi-US spaces in the year 1982 and also they made study of s-convergence, sequentially semi-closed sets, sequentially s-compact notions. Navlagi studied P-Normal Almost-P-Normal and Mildly-P-Normal spaces. Closedness are basic concept for the study and investigation in general topological spaces. This concept has been generalized and studied by many authors from different points of views. Njastad [22] introduced and defined an α-open and α-closed set. After the works of Njastad on α-open sets, various mathematicians turned their attention to the generalizations of various concepts in topology by considering semi-open, α-open sets. The concept of g-closed [16], s-open [17] and α-open sets has a significant role in the generalization of continuity in topological spaces. The modified form of these sets and generalized continuity were further developed by many mathematicians ([8], [10], [4], [20], [19]). Many authors have tried to weaken the condition closed in this theorem. In 1978, Long and Herrington [18] used almost closedness due to Singal [28]. Malghan [19] introduced the concept of generalized closed maps in topological spaces. Devi [11] introduced and studied sg-closed maps and gs-closed maps. wg-closed maps and rwg-closed maps were introduced and studied by Nagavani [21]. Regular closed maps, gpr-closed maps and rg-closed maps have been introduced and studied by Long [18], Gnanambal [15] and Arockiarani [4] respectively. In 2012, [23] we introduced the concepts of Strongly g^*-closed sets and Strongly g^*-open set in topological spaces. Also we have introduced the concepts of Strongly g^*-continuous functions, Strongly g^*-irresolute functions, Strongly g^*-open maps and Strongly g^*-closed maps in ([25], [27], [25], [24]).

In this paper, by deriving the properties of Strongly g^*-closed graph function and studied some of their properties. Further various characterisation are studied.

2. Preliminaries

Throughout this paper (X,τ) and (Y,σ) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), $cl(A)$, $int(A)$ and A^c denote the closure of A, the interior of A and the complement of A in X, respectively.

Definition 2.1. A subset A of a topological space (X,τ) is called

(a) a preopen set [19] if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.

(b) a semiopen set [17] if $A \subseteq cl(int(A))$ and semi closed set if $int(cl(A)) \subseteq A$.

(c) an α-open set [22] if $A \subseteq int(cl(int(A)))$ and an α-closed set if $cl(int(cl(A))) \subseteq A$.

(d) a semi-preopen set [2] (β-open set) if $A \subseteq cl(int(cl(A)))$ and semi-preclosed set if $int(cl(int(A))) \subseteq A$.

Definition 2.2. A space (X,τ_X) is called a $T_{\frac{1}{2}}$-space [16] if every g-closed set is closed.

Definition 2.3. [23] Let (X,τ) be a topological space and A be its subset, then A is Strongly g^*-closed set if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open.
The complement of Strongly g^*-closed set is called Strongly g^*-open set in (X,τ).

Definition 2.4. [27] Let X and Y be topological spaces. A map $f:(X,\tau) \to (Y,\sigma)$ is said to be strongly g^*-irresolute map (sg^*-irresolute map) if the inverse image of every sg^*-open set in Y is sg^*-open in X.

Definition 2.5 ([25])**.** (a) Let X be a topological space and let $x \in X$. A subset N of X is said to be Strongly g^*-nbbd of x if there exists an Strongly g^*-open set G such that $x \in G \subset N$.

The collection of all Strongly g^*-nbhd of $x \in X$ is called a Strongly g^*-nbhd system at x and shall be denoted by Strongly $g^*N(x)$.

(b) Let X be a topological space and A be a subset of X, A subset N of X is said to be Strongly g^*-nbhd of A if there exists a Strongly g^*-open set G such that $A \in G \subseteq N$.

(c) Let A be a subset of X. A point $x \in A$ is said to be a Strongly g^*-interior point of A, if A is a Strongly $g^*N(x)$. The set of all Strongly g^*-interior points of A is called a Strongly g^*-interior of A and is denoted by $Sg^*INT(A)$.

$$Sg^*INT(A) = \bigcup\{G : G \text{ is Strongly } g^*\text{-open}, G \subset A\}.$$

(d) Let A be a subset of X. A point $x \in A$ is said to be a Strongly g^*-closure of A. Then

$$Sg^*CL(A) = \bigcap\{F : A \subset F \in S\text{ trongly } g^*C(X,\tau)\}.$$

Definition 2.6 ([24])**.** A topological space (X,τ) is said to be Strongly g^*-symmetric if for x and y in X, $x \in Sg^*CL(\{y\})$ implies $y \in Sg^*CL(\{x\})$.

Definition 2.7 ([26])**.** A subset A of a topological space (X,τ) is called a Strongly g^*-Difference set (briefly, Strongly $g^*d_\#$-set) if there are $U,V \in Strongly\ g^*O(X,\tau)$ such that $U \neq X$ and $A = U/V$.

It is true that every Strongly g^*-open set U different from X is a Strongly $g^*d_\#$-set if $A = U$ and $V = \phi$. So, we can observe the following.

Definition 2.8 ([26])**.** A topological space (X,τ) is said to be

(a) Strongly $\widetilde{d}^{\,0}$-G^* if for any pair of distinct points x and y of X there exists a Strongly $g^*d_\#$-set of X containing x but not y or Strongly $g^*d_\#$-set of X containing y but not x.

(b) Strongly $\widetilde{d}^{\,1}$-G^* if for any pair of distinct points x and y of X there exists a Strongly $g^*d_\#$-set of X containing x but not y and Strongly $g^*d_\#$-set of X containing y but not x.

(c) Strongly $\widetilde{d}^{\,2}$-G^* if for any pair of distinct points x and y of X there exist disjoint Strongly $g^*d_\#$-set G and E of X containing x and y, respectively.

3. Strongly g^*-Closed Graph Function

Definition 3.1. A function $f : (X,\eta_1) \to (Y,\eta_2)$ is said to have a Strongly g^*-closed graph if for each $(x,y) \in X \times Y - G(f)$, there exist $U \in$ Strongly $g^*O(X,x)$, $V \in$ Strongly $g^*O(Y,y)$ such that $U \times V \cap G(f) = \phi$.

Lemma 3.2. *Let $f : (X,\eta_1) \to (Y,\eta_2)$ be a function then the graph $G(f)$ is Strongly g^*-closed in $X \times Y$ if and only if for each $(x,y) \in X \times Y - G(f)$ there exist $U \in Strongly\ g^*O(X,x)$, $V \in$ Strongly $g^*O(Y,y)$ such that $f(U) \cap V = \phi$.*

Proof. *Necessity:* since f has a Strongly g^*-closed graph, then for each $x \in X$ and $y \in Y$ such that $y \neq f(x)$ there exist Strongly g^*-open sets U and V containing x and y, such that $U \times V \cap G(g) = \phi$. This implies that for every $x \in X$ and $y \in Y$ such that $y \neq f(x)$. So, $f(U) \cap V = \phi$.

Sufficiency: Let $(x, y) \notin G(f)$ then there exist two Strongly g^*-open sets U and V containing x and y respectively such that $f(U) \cap V = \phi$. This implies that, for each $x \in X$ and $y \in Y$ such that $f(x) \neq y$. So $U \times V \cap G(f) = \phi$. Hence f has a Strongly g^*-closed graph. □

Example 3.3. Let $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2, y_3\}$ be endowed with the topology $\eta_1 = \{X, \phi, \{x_1\}\}$ and $\eta_2 = \{Y, \phi, \{y_1, y_2\}\}$ respectively. Let $f : (X, \eta_1) \to (Y, \eta_2)$ be the mapping defined by $f(x_1) = y_1$ and $f(x_2) = y_2$. Then f has Strongly g^*-closed graph.

Remark 3.4. Evidently every Strongly g^*-closed graph is closed graph. But the converse is not true is seen from the following example.

Example 3.5. Let $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3, y_4\}$ be endowed with the discrete topology $\eta_1 = \{X, \phi, \{x_3, x_4\}\}$ and $\eta_2 = \{Y, \phi, \{y_3, y_4\}\}$ respectively. Let $f : (X, \eta_1) \to (Y, \eta_2)$ be the mapping defined by $f(x_1) = y_1$, $f(x_2) = y_2$ and $f(x_3) = y_3$. Then f has closed graph but it is not Strongly g^*-closed graph.

Definition 3.6 ([24])**.** A topological space (X, τ) is said to be

(a) Strongly-$T_0{}^{g^*}$ if for each pair of distinct points x, y in X, there exists a Strongly g^*-open set U such that either $x \in U$ and $y \notin U$ or $x \notin U$ and $y \in U$.

(b) Strongly-$T_1{}^{g^*}$ if for each pair of distinct points x, y in X, there exist two Strongly g^*-open sets U and V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$.

(c) Strongly-$T_2{}^{g^*}$ if for each distinct points x, y in X, there exist two disjoint Strongly g^*-open sets U and V containing x and y respectively.

Remark 3.7. (a) Discrete topological space with two point sets is Strongly-$T_1{}^{g^*}$-space.

(b) Indiscrete topological space with at least two points is not Strongly-$T_1{}^{g^*}$-space.

(c) X is the Strongly-$T_1{}^{g^*}$-space if and only if for each $x \in X$, the singleton sets $\{x\}$ is Strongly g^*-closed.

Theorem 3.8. *If $f : (X, \eta_1) \to (Y, \eta_2)$ an injection function with the Strongly g^*-closed graph $G(f)$ then X is Strongly-$T_1{}^{g^*}$.*

Proof. Let x and y be two distinct points of X then $f(x) \neq f(y), (x, f(y)) \in X \times Y - G(f)$, but $G(f)$ is Strongly g^*-closed graph. So there exist Strongly g^*-open sets U and V containing x and $f(y)$ respectively. Such that $f(U) \cap V = \phi$. Hence $y \notin U$. Similarly their exist Strongly g^*-open sets M and N containing y and $f(x)$ such that $f(M) \cap N = \phi$. Hence $x \notin M$. It follows that X is Strongly-$T_1{}^{g^*}$-space. □

Theorem 3.9. *If $f : (X, \eta_1) \to (Y, \eta_2)$ a surjection function with the Strongly g^*-closed graph $G(f)$ then Y is Strongly-$T_1{}^{g^*}$.*

Proof. Let y and z be two distinct points of y. Since f is surjection, there exist a point x in X such that $f(x) = z$. Therefore $(x,y) \notin G(f)$ by the previous lemma, there exist Strongly g^*-open sets U and V Containing x and y respectively such that $f(U) \cap V = \phi$. It follows that $z \notin V$.

Similarly there exist $w \in X$ such that $f(w) = y$. Hence $(w,z) \notin G(f)$. Similarly there exist Strongly g^*-open sets M and N containing w and z respectively such that $f(M) \cap N = \phi$. Thus $y \notin N$, hence the space Y is Strongly-$T_1{}^{g^*}$. □

Remark 3.10. If $f : (X,\eta_1) \to (Y,\eta_2)$ a bijective function with the Strongly g^*-closed graph $G(f)$. Then both X and Y is Strongly-$T_1{}^{g^*}$.

Definition 3.11. [25] Let X and Y be topological spaces. A map $f : (X,\tau) \to (Y,\sigma)$ is said to be strongly G^*-continuous (sg^*-continuous) if the inverse image of every open set Y is sg^*-open in X.

Theorem 3.12. *If a function $f : (X,\eta_1) \to (Y,\eta_2)$ is a Strongly g^*-continuous and Y is Strongly-$T_2{}^{g^*}$-space, then $G(f)$ is Strongly g^*-closed.*

Proof. Let $(x,y) \notin G(f)$ or $(x,y) \in X \times Y - G(f)$, then $y \notin f(x)$ and Y is Strongly-$T_2{}^{g^*}$-space. There exist two Strongly g^*-open sets U and V such that $f(x) \in U$, $y \in V$ in Y and $U \cap V = \phi$. since f is Strongly g^*-continuous, there exist a Strongly g^*-open neighbourhood w of x such that $f(w) \subset U$. Hence $f(w) \cap V = \phi$. This implies that $W \times V \cap G(f) = \phi$. Hence f has a Strongly g^*-closed graph. □

Definition 3.13. [24] A function $f : (X,\tau) \to (Y,\sigma)$ is called a Strongly g^*-open function if the image of every Strongly g^*-open set in (X,τ) is a Strongly g^*-open set in (Y,σ).

Theorem 3.14. *If $f : (X,\eta_1) \to (Y,\eta_2)$ is a Strongly g^*-open surjection function with the Strongly g^*-closed graph $G(f)$ then Y is Strongly-$T_2{}^{g^*}$ Space.*

Proof. Let y and w be distinct points in Y. Then there are distinct points x and z in X such that $f(x) = y$ and $f(z) = w$. Since $(x,w) \notin G(f)$ and $G(f)$ is Strongly g^*-closed graph, there exist Strongly g^*-open sets U and V containing x and w respectively such that $f(U) \cap V = \phi$. But $f(U)$ is Strongly g^*-open and contains y. Consequently, Y is Strongly-$T_2{}^{g^*}$ Space. □

Theorem 3.15. *If $f : (X,\eta_1) \to (Y,\eta_2)$ is an injective and Strongly g^*-continuous with the Strongly g^*-closed graph $G(f)$ and Y is Strongly-$T_2{}^{g^*}$ Space, and then X is Strongly-$T_2{}^{g^*}$ Space.*

Proof. Let x and y in X be any pair of points. Then there exist disjoint open sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$. since f is Strongly g^*-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are Strongly g^*-open in X containing x and y respectively. We have $f^{-1}(U) \cap f^{-1}(V) = \phi$. Thus X is Strongly-$T_2{}^{g^*}$ Space. □

Definition 3.16 ([27])**.** Let X and Y be two topological spaces. A bijection map $f:(X,\tau)\to(Y,\sigma)$ from a topological space X into a topological space Y is called strongly g^*-Homeomorphism (sg^*-homeomorphism) if f and f^{-1} are sg^*-continuous.

Remark 3.17. let $f:(X,\eta_1)\to(Y,\eta_2)$ be a sg^*-homeomorphism of X onto Y having $G(f)$ Strongly g^*-closed. Then both X and Y are Strongly-$T_2{}^{g^*}$ space.

4. Conclusion

In this paper, we studied that the graph function called Strongly g^*-closed graph function and studied some of their properties.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] D. Andrijevic, On b-open sets, *Mat. Vesink* **48** (1996), 59 – 64.

[2] D. Andrijevic, Semi-preopen sets, *Mat. Vesink* **38** (1986), 24 – 32.

[3] S. P. Arya and T. Nour, Characterizations of s-normal spaces, *Indian J. Pure Appl. Math.* **21** (1990), 717 – 719.

[4] I. Arockiarani, *Studies on Generalizations of Generalized Closed Sets and Maps in Topological Spaces*, Ph.D Thesis, Bharathiar University, Coimbatore (1997), http://hdl.handle.net/10603/101249.

[5] S. P. Arya and M. P. Bhamini, A note on semi-US spaces, *Ranchi Uni. Math. J.* **13** (1982), 60 – 68.

[6] C. E. Aull, Sequences in topological spaces, *Comm. Math.* (1968), 329 – 336.

[7] P. Bhattacharyya and B. K. Lahiri, Semi-generalizsed closed sets in topology, *Indian J. Math.* **29** (1987), 375 – 382.

[8] K. Balachandran, P. Sundran and H. Maki, On generalized continuous maps in topological spaces, *Mem. Fac.Sci. Kochi Univ., Ser. A. Math.* **12** (1991), 5 – 13.

[9] D. E. Cameron, Some maximal topologies which are Q.H.C., *Proc. Amer. Math. Soc.* **75** (1979), 149 – 156, DOI: 10.1090/S0002-9939-1979-0529232-8.

[10] R. Devi, H. Maki and K. Balachandran, Semi-generalised closed maps and generalized semi-closed maps, *Mem. Fac. Sci. Kochi Univ. Ser. A. Math.* **14** (1993), 41 – 54.

[11] R. Devi, *Studies on Generalizations of Closed Maps and Homeomorphisms in Topological Spaces*, Ph.D Thesis, Bharathiar University, Coimbatore (1994), http://hdl.handle.net/10603/102774.

[12] A. A. El-Atik, *A Study on Some Types of Mappings on Topological Spaces*, M.Sc. thesis, Tanta University, Egypt (1997).

[13] E. Ekici and M. Caldas, Slightly γ-continuous functions, *Bol. Soc. Paran. Mat.* **22** (2004), 63 – 74.

[14] M. Ganster and I. L. Reilly, Locally closed sets and LC-continuous functions, *Internat. J. Math. Math. Sci.* **12**(3) (1989), 417 – 424, DOI: 10.1155/S0161171289000505.

[15] Y. Gnanambal, On generalized pre-regular closed sets in topological spaces, *Indian J. Pure Appl. Math.* **28** (1997), 351 – 360.

[16] N. Levine, Generalized closed sets in topology, *Rend. Circ. Math. Palermo* **19** (1970), 89 – 96, DOI: 10.1007/BF02843888.

[17] N. Levine, Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly* **70** (1963), 36 – 41, DOI: 10.1080/00029890.1963.11990039.

[18] P. E. Long and L. L. Herington, Basic properties of regular closed functions, *Rend. Cir. Mat. Palermo* **27** (1978), 20 – 28, DOI: 10.1007/BF02843863.

[19] A. S. Mashhour, M. E. Abd EI-Monsef and S. N. EI-Deeb, On precontinuous and weak pre-continuous mapping, *Proc. Math., Phys. Soc. Egypt* **53** (1982), 47 – 53.

[20] A. S. Mashhour, I. A. Hasanein and S. N. El-Deeb, α-continuous and α-open mappings, *Acta Math. Hung.* **41** (1983), 213 – 218, DOI: 10.1007/BF01961309.

[21] N. Nagaveni, *Studies on Generalizations of Homeomorphisms in Topological Spaces*, Ph.D Thesis, Bharathiar University, Coimbatore (1999).

[22] O. Njastad, On some classes of nearly open sets, *Pacific J. Math.* **15** (1965), 961 – 970, DOI: 10.2140/pjm.1965.15.961.

[23] R. Parimelazhagan and V. S. Pillai, Strongly g^*-closed sets in topological spaces, *Int. Journal of Math. Analysis*. **6**(30) (2012), 1481 – 1489.

[24] R. Parimelazhagan, Strongly $T_k^{g^*}$-spaces, *Journal of Informatics and Mathematical Sciences* **11**(2) (2019), 147 – 154, DOI: 10.26713/jims.v11i2.958.

[25] V. S. Pillai and R. Parimelazhagan, On strongly g^*-continuous maps and pasting lemma in topological spaces, *International Journal of Computer Applications* **63**(6) (2013), 46 – 48.

[26] R. Parimelazhagan, Strongly $g^*d_\#$-sets in topological spaces, (submitted).

[27] V. S. Pillai and R. Parimelazhagan, Strongly g^*-irresolute and homeomorphism in topological spaces, *International Journal of Recent Scientific Research* **4**(1) (2013),005 – 007.

[28] M. K. Singal and A. R. Singal, Almost-continuous mappings, *Yokohama Math. J.* **16** (1968), 63 – 73.

[29] J. Tong, A decomposition of continuity, *Acta Math. Hungar.* **48** (1986), 11 – 15, DOI: 10.1007/BF01949045.

[30] J. Tong, On decomposition of continuity in topological spaces, *Acta Math. Hungar.* **54** (1-2) (1989), 51 – 55, DOI: 10.1007/BF01950708.

[31] A. Wilansky, Between T_1 and T_2, *Amer. Math. Monthly* **74** (1967), 261 – 266, DOI: 10.1080/00029890.1967.11999950.