# Yield Spread Estimation with Credit Events 

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#### Abstract

We study a problem of modeling impacts of credit rating migration on spread curves. We consider the construction of credit curves adopting specific groups of representative issuers for credit rating by taking their average spread. Then, we introduce the method of grouping in full grade to handle the case for which a statistically representative set of issuers does not exist. Finally, empirical experiment shows the efficiency of our approach.


Keywords. Migration Risk; Default risk; Spreads; Spread shock; Multiplicative shock; Additive shock; Credit rating Migration

MSC. 62-XX

Received: January 30, 2016
Accepted: August 14, 2016
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## 1. Introduction

The essential role of banks is to invest into corporate and sovereign economic sectors, banks face risks that their counterparties or issuers will be incapable of returning the capital, or the
interest, or both. These risks are called counterparty risk or credit risk. Under guidelines in Basel I and II, banks are required to evaluate their portfolios in the presence of credit events. Many research are concerned with defaultable asset evaluation and credit risk (see [10], [3], and [8]). Nevertheless, after the financial crisis in 2007-2009, one recognizes that the valuation of default risk was not sufficient or relevant to all credit risks. Since regulatory capital for bank trading portfolios was too small, losses were not only due to defaults, but also to downgrades. Regulatory capital requirement calculated from Value at Risk (VaR) was a heavily underestimated risk as well. They did not fully capture default and migration risks for portfolios over a 1-year horizon because of the lack of reliable information on credit curves volatility from illiquid positions (see [4]). Regulators have required banks to evaluate their assets by taking into account both migration risk and liquidity risk over a one-year capital horizon. This requirement could be presented in the calculations of Incremental Risk Charge (IRC) and Credit Value Adjustment (CVA) measure (see [2], [1]). The paper considers the demand of an advanced approach in Basel requirement. As with many financial products, the price of financial securities such as bonds, Credit Default Swap (CDS) vary gradually and continuously in reaction to market information. However, two bonds which have the same cash-flow and the same maturity could be traded at different prices. The reason is that financial securities are subject to not only market risk but also credit risk. The bond price and CDS spread and other financial instruments traded on markets depend on the credit quality of issuers. In practice, credit quantity is represented by credit rating that is provided by rating agencies such as S\&P and Moody's, as well as others. A change of the issuer's rating could result in a possible significant loss on bond and CDS position. Most research adopts classical models such as the Gaussian copula model, Vasicek's model, and the Credit Metrics model to build the credit curves (see [5], [6], [7]). Tschirhart et al. constructed credit curves via mean spreads, median spreads at each maturity, and "best fit" method which minimizes the sum of squared differences between the individual spreads and the estimated generic curve spreads, at for most commercial and in-house development. However, a main drawback is that the paper did not show the method to handle the lack of the existence of a statistically valid representative set of issuers (see [9]). Therefore, in this paper, we propose the method of grouping in full grade to handle the drawback. The objective of this paper is to estimate spread shock when credit events happen. From the point of view of risk manager, this model will be calibrated into market data in order to capture the possible loss induced by a migration.

The paper is organized as follows. Section 1 introduces the importance of spread shock estimation with credit events. In Section 2 and 3, we discuss methodologies of spread shock calibration and grouping in full grade that establishes consistency with the default risk and migration risk of the wider credit market. These curves may be used to calculate VaR, other risk measures, and hedging strategies because they play an important role as proxies for live information on market volatility otherwise unavailable. Section 4 shows some results of spread shock at 7 full grades and at 17 notch grades in maturity $1,3,5,7,10$ and 20 years. Finally, section 5 provides some conclusions and expansions.

## 2. Calibration of Spread shock

The calibration is realised in full grades and shock of spread for notch grades is obtained by interpolation. We construct spread curve for each rating as the average of all issuers' spread emitting the same rating, i.e.,

$$
\bar{S}_{i}(t)=\frac{1}{\# \mathscr{R}_{i}(t)} \sum_{k \in \mathscr{R}_{i}(t)} S_{k}(t),
$$

where $R_{i}(t)$ is the set of all issuers in the sample whose notation is $i$ at time $t$. In Basel guideline, spread shock is supposed to be multiplicative. However, in this paper, the multiplicative shock is applied for corporate issuers while the additive shock is used for sovereign issuers. We use additive spread shock for all sovereign issuers for several reasons. Sovereign spreads are calibrated from bond spreads. Sovereign spreads are usually very small, multiplication could cause an exposure of spread shock. A rating migration of an issuer causes a jump in its spread curve. A full grade is to set up to obtain a sufficient number to increase a statistical representative set and to improve the quality of the estimate. There are seven full grades: AAA, AA, A, BBB, BB, B and CCC. For example, notch grades such as AA+, AA and AA- are considered identical and are grouped in the AA rating as follows

$$
A=A_{\text {notch }}^{+} \cup A_{\text {notch }} \cup A_{\text {notch }}^{-} .
$$

Average Spread for the full grade is defined by

$$
\left.\bar{S}_{A}(t)=P\left(A_{\text {notch }}^{+}\right) \bar{S}_{\left(A_{\text {notec }}^{+}\right)}(t)+P\left(A_{\text {notch }}\right) \bar{S}_{\left(A_{\text {notch }}\right)}\right)(t)+P\left(A_{\text {notch }}^{-}\right) \bar{S}_{\left(A_{\text {notch }}^{-}\right)}(t),
$$

where $P\left(A_{\text {notch }}^{+}\right)+P\left(A_{\text {notch }}^{-}\right)+P\left(A_{\text {notch }}\right)=1$, and $P\left(A_{\text {notch }}^{+}\right), P\left(A_{\text {notch }}^{-}\right), P\left(A_{\text {notch }}\right)$ denote a proportion of number of issuers of each notch grade and number of issuers of full grade. Segmentation of the notch grades can greatly reduce the number of individuals per class but it does not reduce the dispersion within each category. This will lead to undermine the quality of the estimate. Grouping in full grade allows to increase the number of individuals contributing to the estimates. The curve which is obtained from the combination presents a better regularity. The additive spread shock is calibrated from the historical market data in the following way

$$
\Delta_{i j}(T)=\bar{S}_{i}(T)-\bar{S}_{j}(T)
$$

so that the spread of a sovereign issuer after migration from $i$ to $j$ is

$$
S_{j}(T)=S_{i}(T)+\Delta_{i j}(T)
$$

The multiplicative spread shock is calibrated from the historical market data in the following way

$$
\Delta_{i j}(T)=\frac{\bar{S}_{i}(T)}{\bar{S}_{j}(T)},
$$

so that the spread of a corporate issuer after migration from $i$ to $j$ is

$$
S_{j}(T)=S_{i}(T) \Delta_{i j}(T)
$$

## 3. Transform From Full Grades to Notch Grades by Interpolation

A log-linear interpolation is realized for corporate because the spread curve is in the exponential form. In contrast, a linear interpolation is applied in case of sovereigns, for which an additive shock is used. A notch grade is considered as a unit.

Let spread shock $\Delta(s)_{2}^{1}$ be between $\Delta(s)_{1}^{1}$ and $\Delta(s)_{3}^{1}$; and $\Delta(s)_{1}^{2}$ be between $\Delta(s)_{1}^{1}$ and $\Delta(s)_{1}^{3}$. Sovereign spread shock at notch grades can be derived from the following equation systems

$$
\left\{\begin{array}{l}
\Delta(s)_{2}^{1}=w \Delta(s)_{1}^{1}+(1-w) \Delta(s)_{3}^{1} \\
\Delta(s)_{2}^{2}=w \Delta(s)_{1}^{2}+(1-w) \Delta(s)_{3}^{2} \\
\Delta(s)_{2}^{3}=w \Delta(s)_{1}^{3}+(1-w) \Delta(s)_{3}^{3}, \\
\Delta(s)_{1}^{2}=w \Delta(s)_{1}^{1}+(1-w) \Delta(s)_{1}^{3}, \\
\Delta(s)_{3}^{2}=w \Delta(s)_{3}^{1}+(1-w) \Delta(s)_{3}^{3} .
\end{array}\right.
$$

Similarly, corporate spread shock at notch grades can be obtained by the following equation systems

$$
\left\{\begin{array}{l}
\Delta(s)_{2}^{1}=\exp \left(w \ln \left(\Delta(s)_{1}^{1}\right)+(1-w) \ln \left(\Delta(s)_{3}^{1}\right)\right) \\
\Delta(s)_{2}^{2}=\exp \left(w \ln \left(\Delta(s)_{1}^{2}\right)+(1-w) \ln \left(\Delta(s)_{3}^{2}\right)\right) \\
\Delta(s)_{2}^{3}=\exp \left(w \ln \left(\Delta(s)_{1}^{3}\right)+(1-w) \ln \left(\Delta(s)_{3}^{3}\right)\right) \\
\Delta(s)_{1}^{2}=\exp \left(w \ln \left(\Delta(s)_{1}^{1}\right)+(1-w) \ln \left(\Delta(s)_{1}^{3}\right)\right) \\
\Delta(s)_{3}^{2}=\exp \left(w \ln \left(\Delta(s)_{3}^{1}\right)+(1-w) \ln \left(\Delta(s)_{3}^{3}\right)\right)
\end{array}\right.
$$

In the case that the spread shocks locate at the four corners, $\Delta(s)_{3}^{2}$ will be calculated from the spread shock at the upper right corner $\Delta(s)_{4}^{1}$ and the diagonal; $\Delta(s)_{2}^{3}$ and will be calculated from the spread shock at the lower left corner $\Delta(s)_{1}^{4}$ and the diagonal.

The term structure is interpolated as the following methodology. Plots that are used for the construction of the term structure of shocks involve six maturities: $1,3,5,7,10$, and 20 years. These plots correspond to the blocks for which the internal system offers a number satisfying calibration curves for the shock. The method of shock spread calibration will be used to calculate the PnL. Shock calibration of corporate and sovereign issuers are obtained by log linear interpolation and linear interpolation respectively between two shock tenors. Spread shock at maturity $t$ between $T_{1}$ and $T_{2}$ for corporate issuers can be obtained by log linear interpolation

$$
\ln \left(\Delta_{i j}(t)\right)=\ln \left(\Delta_{i j}\left(T_{1}\right)\right) \frac{T_{2}-t}{T_{2}-T_{1}}+\ln \left(\Delta_{i j}\left(T_{2}\right)\right) \frac{t-T_{1}}{T_{2}-T_{1}}
$$

The impact of migration from rating $i$ to $j$ for sovereign issuers at maturity $t$ between and can
be calculated by linear interpolation

$$
\Delta_{i j}(t)=\Delta_{i j}\left(T_{1}\right) \frac{T_{2}-t}{T_{2}-T_{1}}+\Delta_{i j}\left(T_{2}\right) \frac{t-T_{1}}{T_{2}-T_{1}}
$$

For any spread less than 1 year or beyond 20 years, we use a flat extrapolation. Spread shocks with maturity less than 1 year or beyond 20 years are subject to the same shocks at 1 years or 20 years respectively.

## 4. Results

We use the historical rating data set from Credit Pro. The period of historical data for the calibration of spread shock is from July 1st, 2000 to June 21st, 2014, which is correspondent to 5,040 days. Data consists CDS spread curves of corporate products: senior, subordinate, senior coverage and bond spread curves of sovereign products with 132 sovereign credit rating from 44 countries in Euro region and 88 countries in other regions at specific days over the period; and with 13,863 corporate ratings including 8,655 issuers in US region, 2105 issuers from Euro region and 3,103 issuers from other region. One important characteristic is that a 14 -years period is long enough to allow a calibration; and the sample is large enough to provide a valid representation. CDS spreads and bond spreads are available for both investment grade and non-investment grade obligors with a range of maturity for individual obligors. Moreover, if shock matrices are directly calibrated at notch grades, the smaller number of spreads at notch grades can lead to less precise estimates, and shock curves are less regular. This is one of shortcomings that the previous method was less effective in comparison with our method (for example, see [3]). In Table 1, we calibrate six matrices of spread shock at seven full grades in seven maturities. Then, in Table 2, six matrices of spread shock at seventeen notch grades in 7 maturities are derived from matrices in Table 1 by interpolation. Following matrices of shock of spread, spread shocks are in ascending order from left to right. For example, spread shock of upgrade from AA to CCC is greater than spread shock of upgrade from AA to BBB. Spread shocks are in descending order from top to bottom. For example, spread shock of downgrade from AA to BBB is greater than spread shock of downgrade from A to BBB. The reason is that the riskier the issuer is, the greater the spread is. Hence, spread at rating BBB is less than the spread at rating CCC. It implies that the ratio of spread at BBB to spread at AA is less than the ratio of spread at CCC to spread at AA for corporate or the difference between spread at BBB and spread at AA is less than the difference between spread at CCC and spread at AA for sovereign. By the same reasoning, we can explain why spread shock is in descending order from top to bottom.

Table 1. The levels of spread shock at full grades from the calibration corporate senior products in maturity $1,3,5,7,10,20$ years

| 1Y | AAA | AA | A | BBB | BB | B | CCC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AAA | 1 | 1.25 | 2.01 | 4.66 | 10.04 | 16.87 | 18.53 |
| AA | 0.8 | 1 | 1.6 | 3.71 | 8.01 | 13.5 | 17.53 |
| A | 0.5 | 0.63 | 1 | 2.32 | 5 | 8.42 | 9.94 |
| BBB | 0.21 | 0.27 | 0.43 | 1 | 2.16 | 3.63 | 4.4 |
| BB | 0.1 | 0.12 | 0.2 | 0.46 | 1 | 1.68 | 2.23 |
| B | 0.06 | 0.07 | 0.12 | 0.28 | 0.59 | 1 | 1.48 |
| CCC | 0.05 | 0.06 | 0.1 | 0.23 | 0.45 | 0.68 | 1 |
| 3Y | AAA | AA | A | BBB | BB | B | CCC |
| AAA | 1 | 1.29 | 1.92 | 4.67 | 10.53 | 17.12 | 20.1 |
| AA | 0.77 | 1 | 1.48 | 3.61 | 8.13 | 13.23 | 15.31 |
| A | 0.52 | 0.68 | 1 | 2.44 | 5.49 | 8.93 | 10.19 |
| BBB | 0.21 | 0.28 | 0.41 | 1 | 2.25 | 3.66 | 4.19 |
| BB | 0.1 | 0.12 | 0.18 | 0.44 | 1 | 1.63 | 1.9 |
| B | 0.06 | 0.08 | 0.11 | 0.27 | 0.61 | 1 | 1.29 |
| CCC | 0.05 | 0.07 | 0.1 | 0.24 | 0.53 | 0.78 | 1 |
| 5Y | AAA | AA | A | BBB | BB | B | CCC |
| AAA | 1 | 1.36 | 1.85 | 4.09 | 11.21 | 17.47 | 20.04 |
| AA | 0.74 | 1 | 1.36 | 3.01 | 8.26 | 12.87 | 15.3 |
| A | 0.54 | 0.74 | 1 | 2.22 | 6.07 | 9.47 | 10.72 |
| BBB | 0.24 | 0.33 | 0.45 | 1 | 2.74 | 4.26 | 4.49 |
| BB | 0.09 | 0.12 | 0.16 | 0.37 | 1 | 1.56 | 1.87 |
| B | 0.06 | 0.08 | 0.11 | 0.23 | 0.64 | 1 | 1.39 |
| CCC | 0.05 | 0.07 | 0.09 | 0.22 | 0.53 | 0.72 | 1 |
| 7Y | AAA | AA | A | BBB | BB | B | CCC |
| AAA | 1 | 1.44 | 1.85 | 4.26 | 12.09 | 17.95 | 21.2 |
| AA | 0.7 | 1 | 1.29 | 2.96 | 8.41 | 12.48 | 15.17 |
| A | 0.54 | 0.78 | 1 | 2.3 | 6.54 | 9.71 | 11.47 |
| BBB | 0.23 | 0.34 | 0.43 | 1 | 2.84 | 4.22 | 5.1 |
| BB | 0.08 | 0.12 | 0.15 | 0.35 | 1 | 1.49 | 1.84 |
| B | 0.06 | 0.08 | 0.1 | 0.24 | 0.67 | 1 | 1.4 |
| CCC | 0.05 | 0.07 | 0.09 | 0.2 | 0.54 | 0.71 | 1 |
| 10Y | AAA | AA | A | BBB | BB | B | CCC |
| AAA | 1 | 1.7 | 2.03 | 4.19 | 13.24 | 19.35 | 21.94 |
| AA | 0.59 | 1 | 1.19 | 2.46 | 7.76 | 11.38 | 14.82 |
| A | 0.49 | 0.84 | 1 | 2.06 | 6.52 | 9.54 | 11.64 |
| BBB | 0.24 | 0.41 | 0.48 | 1 | 3.15 | 4.61 | 5.21 |
| BB | 0.08 | 0.13 | 0.15 | 0.32 | 1 | 1.46 | 1.75 |
| B | 0.05 | 0.09 | 0.1 | 0.22 | 0.68 | 1 | 1.48 |
| CCC | 0.05 | 0.07 | 0.09 | 0.19 | 0.57 | 0.68 | 1 |
| 20Y | AAA | AA | A | BBB | BB | B | CCC |
| AAA | 1 | 2.58 | 2.63 | 3.99 | 17.07 | 24.01 | 24.39 |
| AA | 0.23 | 1 | 0.88 | 0.78 | 5.61 | 7.71 | 13.66 |
| A | 0.33 | 1.04 | 1 | 1.27 | 6.45 | 8.99 | 12.22 |
| BBB | 0.25 | 0.64 | 0.65 | 1 | 4.2 | 5.92 | 5.56 |
| BB | 0.05 | 0.16 | 0.16 | 0.2 | 1 | 1.39 | 1.46 |
| B | 0.04 | 0.11 | 0.11 | 0.15 | 0.72 | 1 | 1.73 |
| CCC | 0.04 | 0.07 | 0.08 | 0.18 | 0.66 | 0.56 | 1 |

Table 2. The levels of spread shock at notches from the calibration corporate senior products in maturity $1,3,5,7,10,20$ years

| 1Y | AAA | AA+ | AA | AA- | A+ | A | A- | BBB+ | BBB | BBB- | BB+ | BB | BB- | B+ | B | B- | CCC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AAA | 1 | 1.12 | 1.25 | 1.47 | 1.71 | 2.01 | 2.66 | 3.52 | 4.66 | 6.02 | 7.77 | 10.04 | 11.93 | 14.19 | 16.87 | 17.68 | 18.53 |
| AA+ | 0.89 | 1 | 1.12 | 1.31 | 1.53 | 1.79 | 2.37 | 3.14 | 4.16 | 5.37 | 6.94 | 8.97 | 10.67 | 12.69 | 15.09 | 16.49 | 18.02 |
| AA | 0.8 | 0.89 | 1 | 1.17 | 1.37 | 1.6 | 2.12 | 2.8 | 3.71 | 4.8 | 6.2 | 8.01 | 9.53 | 11.34 | 13.5 | 15.38 | 17.53 |
| AA- | 0.68 | 0.76 | 0.86 | 1 | 1.17 | 1.37 | 1.81 | 2.4 | 3.17 | 4.1 | 5.3 | 6.85 | 8.15 | 9.69 | 11.53 | 12.93 | 14.51 |
| A+ | 0.58 | 0.65 | 0.73 | 0.86 | 1 | 1.17 | 1.55 | 2.05 | 2.71 | 3.51 | 4.53 | 5.85 | 6.96 | 8.28 | 9.85 | 10.88 | 12.01 |
| A | 0.5 | 0.56 | 0.63 | 0.73 | 0.86 | 1 | 1.32 | 1.75 | 2.32 | 3 | 3.87 | 5 | 5.95 | 7.08 | 8.42 | 9.15 | 9.94 |
| A- | 0.38 | 0.42 | 0.47 | 0.55 | 0.65 | 0.76 | 1 | 1.32 | 1.75 | 2.26 | 2.93 | 3.78 | 4.5 | 5.35 | 6.36 | 6.94 | 7.57 |
| BBB+ | 0.28 | 0.32 | 0.36 | 0.42 | 0.49 | 0.57 | 0.76 | 1 | 1.32 | 1.71 | 2.21 | 2.86 | 3.4 | 4.04 | 4.81 | 5.26 | 5.77 |
| BBB | 0.21 | 0.24 | 0.27 | 0.32 | 0.37 | 0.43 | 0.57 | 0.76 | 1 | 1.29 | 1.67 | 2.16 | 2.57 | 3.05 | 3.63 | 3.99 | 4.4 |
| BBB- | 0.17 | 0.19 | 0.21 | 0.24 | 0.29 | 0.33 | 0.44 | 0.58 | 0.77 | 1 | 1.29 | 1.67 | 1.99 | 2.36 | 2.81 | 3.14 | 3.5 |
| BB+ | 0.13 | 0.14 | 0.16 | 0.19 | 0.22 | 0.26 | 0.34 | 0.45 | 0.6 | 0.77 | 1 | 1.29 | 1.54 | 1.83 | 2.18 | 2.47 | 2.79 |
| BB | 0.1 | 0.11 | 0.12 | 0.15 | 0.17 | 0.2 | 0.26 | 0.35 | 0.46 | 0.6 | 0.77 | 1 | 1.19 | 1.42 | 1.68 | 1.94 | 2.23 |
| BB- | 0.08 | 0.09 | 0.1 | 0.12 | 0.14 | 0.17 | 0.22 | 0.29 | 0.39 | 0.5 | 0.65 | 0.84 | 1 | 1.19 | 1.42 | 1.66 | 1.94 |
| B+ | 0.07 | 0.08 | 0.09 | 0.1 | 0.12 | 0.14 | 0.19 | 0.25 | 0.33 | 0.42 | 0.55 | 0.71 | 0.84 | 1 | 1.19 | 1.42 | 1.69 |
| B | 0.06 | 0.07 | 0.07 | 0.09 | 0.1 | 0.12 | 0.16 | 0.21 | 0.28 | 0.36 | 0.46 | 0.59 | 0.71 | 0.84 | 1 | 1.21 | 1.48 |
| B- | 0.06 | 0.06 | 0.07 | 0.08 | 0.09 | 0.11 | 0.14 | 0.19 | 0.25 | 0.32 | 0.41 | 0.52 | 0.6 | 0.7 | 0.82 | 1 | 1.21 |
| CCC | 0.05 | 0.06 | 0.06 | 0.07 | 0.08 | 0.1 | 0.13 | 0.17 | 0.23 | 0.29 | 0.36 | 0.45 | 0.51 | 0.59 | 0.68 | 0.82 | 1 |
| 3Y | AAA | AA+ | AA | AA- | A+ | A | A- | BBB+ | BBB | BBB- | BB+ | BB | BB- | B+ | B | B- | CCC |
| AAA | 1 | 1.14 | 1.29 | 1.48 | 1.68 | 1.92 | 2.58 | 3.47 | 4.67 | 6.12 | 8.03 | 10.53 | 12.38 | 14.56 | 17.12 | 18.55 | 20.1 |
| AA+ | 0.88 | 1 | 1.14 | 1.3 | 1.48 | 1.68 | 2.27 | 3.05 | 4.11 | 5.38 | 7.06 | 9.25 | 10.88 | 12.8 | 15.05 | 16.25 | 17.54 |
| AA | 0.77 | 0.88 | 1 | 1.14 | 1.3 | 1.48 | 1.99 | 2.68 | 3.61 | 4.73 | 6.2 | 8.13 | 9.57 | 11.25 | 13.23 | 14.23 | 15.31 |
| AA- | 0.68 | 0.77 | 0.88 | 1 | 1.14 | 1.3 | 1.75 | 2.35 | 3.17 | 4.15 | 5.44 | 7.14 | 8.39 | 9.87 | 11.61 | 12.45 | 13.36 |
| A+ | 0.59 | 0.68 | 0.77 | 0.88 | 1 | 1.14 | 1.53 | 2.06 | 2.78 | 3.64 | 4.77 | 6.26 | 7.36 | 8.66 | 10.18 | 10.9 | 11.67 |
| A | 0.52 | 0.59 | 0.68 | 0.77 | 0.88 | 1 | 1.35 | 1.81 | 2.44 | 3.2 | 4.19 | 5.49 | 6.46 | 7.6 | 8.93 | 9.54 | 10.19 |
| A- | 0.39 | 0.44 | 0.5 | 0.57 | 0.65 | 0.74 | 1 | 1.35 | 1.81 | 2.37 | 3.11 | 4.08 | 4.8 | 5.64 | 6.64 | 7.09 | 7.58 |
| BBB+ | 0.29 | 0.33 | 0.37 | 0.43 | 0.48 | 0.55 | 0.74 | 1 | 1.35 | 1.76 | 2.31 | 3.03 | 3.57 | 4.19 | 4.93 | 5.27 | 5.64 |
| BBB | 0.21 | 0.24 | 0.28 | 0.32 | 0.36 | 0.41 | 0.55 | 0.74 | 1 | 1.31 | 1.72 | 2.25 | 2.65 | 3.12 | 3.66 | 3.92 | 4.19 |
| BBB- | 0.16 | 0.19 | 0.21 | 0.24 | 0.27 | 0.31 | 0.42 | 0.57 | 0.76 | 1 | 1.31 | 1.72 | 2.02 | 2.38 | 2.8 | 3 | 3.22 |
| BB+ | 0.12 | 0.14 | 0.16 | 0.18 | 0.21 | 0.24 | 0.32 | 0.43 | 0.58 | 0.76 | 1 | 1.31 | 1.54 | 1.81 | 2.13 | 2.3 | 2.47 |
| BB | 0.1 | 0.11 | 0.12 | 0.14 | 0.16 | 0.18 | 0.25 | 0.33 | 0.44 | 0.58 | 0.76 | 1 | 1.18 | 1.38 | 1.63 | 1.76 | 1.9 |
| BB- | 0.08 | 0.09 | 0.1 | 0.12 | 0.14 | 0.15 | 0.21 | 0.28 | 0.38 | 0.49 | 0.65 | 0.85 | 1 | 1.18 | 1.38 | 1.52 | 1.67 |
| B+ | 0.07 | 0.08 | 0.09 | 0.1 | 0.12 | 0.13 | 0.18 | 0.24 | 0.32 | 0.42 | 0.55 | 0.72 | 0.85 | 1 | 1.18 | 1.31 | 1.47 |
| B | 0.06 | 0.07 | 0.08 | 0.09 | 0.1 | 0.11 | 0.15 | 0.2 | 0.27 | 0.36 | 0.47 | 0.61 | 0.72 | 0.85 | 1 | 1.14 | 1.29 |
| B- | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.1 | 0.14 | 0.19 | 0.26 | 0.33 | 0.44 | 0.57 | 0.66 | 0.76 | 0.88 | 1 | 1.14 |
| CCC | 0.05 | 0.06 | 0.07 | 0.07 | 0.09 | 0.1 | 0.13 | 0.18 | 0.24 | 0.31 | 0.4 | 0.53 | 0.6 | 0.68 | 0.78 | 0.88 | 1 |
| 5 Y | AAA | AA+ | AA | AA- | A+ | A | A- | BBB+ | BBB | BBB- | BB+ | BB | BB- | B+ | B | B- | CCC |
| AAA | 1 | 1.17 | 1.36 | 1.5 | 1.67 | 1.85 | 2.41 | 3.14 | 4.09 | 5.72 | 8.01 | 11.21 | 13 | 15.07 | 17.47 | 18.71 | 20.04 |
| AA+ | 0.86 | 1 | 1.17 | 1.29 | 1.43 | 1.58 | 2.06 | 2.69 | 3.51 | 4.91 | 6.87 | 9.62 | 11.16 | 12.94 | 15 | 16.21 | 17.51 |
| AA | 0.74 | 0.86 | 1 | 1.11 | 1.23 | 1.36 | 1.77 | 2.31 | 3.01 | 4.21 | 5.9 | 8.26 | 9.57 | 11.1 | 12.87 | 14.04 | 15.3 |
| AA- | 0.66 | 0.77 | 0.9 | 1 | 1.11 | 1.23 | 1.6 | 2.08 | 2.72 | 3.8 | 5.32 | 7.45 | 8.64 | 10.02 | 11.62 | 12.57 | 13.59 |
| A+ | 0.6 | 0.7 | 0.82 | 0.9 | 1 | 1.11 | 1.44 | 1.88 | 2.45 | 3.43 | 4.81 | 6.73 | 7.8 | 9.05 | 10.49 | 11.25 | 12.07 |
| A | 0.54 | 0.63 | 0.74 | 0.82 | 0.9 | 1 | 1.3 | 1.7 | 2.22 | 3.1 | 4.34 | 6.07 | 7.04 | 8.17 | 9.47 | 10.07 | 10.72 |
| A- | 0.42 | 0.48 | 0.56 | 0.63 | 0.69 | 0.77 | 1 | 1.3 | 1.7 | 2.38 | 3.33 | 4.66 | 5.4 | 6.26 | 7.26 | 7.63 | 8.02 |
| BBB+ | 0.32 | 0.37 | 0.43 | 0.48 | 0.53 | 0.59 | 0.77 | 1 | 1.3 | 1.82 | 2.55 | 3.57 | 4.14 | 4.8 | 5.56 | 5.78 | 6 |
| BBB | 0.24 | 0.29 | 0.33 | 0.37 | 0.41 | 0.45 | 0.59 | 0.77 | 1 | 1.4 | 1.96 | 2.74 | 3.17 | 3.68 | 4.26 | 4.37 | 4.49 |
| BBB- | 0.17 | 0.2 | 0.24 | 0.26 | 0.29 | 0.32 | 0.42 | 0.55 | 0.71 | 1 | 1.4 | 1.96 | 2.27 | 2.63 | 3.05 | 3.2 | 3.35 |
| BB+ | 0.12 | 0.15 | 0.17 | 0.19 | 0.21 | 0.23 | 0.3 | 0.39 | 0.51 | 0.71 | 1 | 1.4 | 1.62 | 1.88 | 2.18 | 2.34 | 2.51 |
| BB | 0.09 | 0.1 | 0.12 | 0.13 | 0.15 | 0.16 | 0.21 | 0.28 | 0.37 | 0.51 | 0.71 | 1 | 1.16 | 1.34 | 1.56 | 1.71 | 1.87 |
| BB- | 0.08 | 0.09 | 0.1 | 0.12 | 0.13 | 0.14 | 0.19 | 0.24 | 0.32 | 0.44 | 0.62 | 0.86 | 1 | 1.16 | 1.34 | 1.51 | 1.7 |
| B+ | 0.07 | 0.08 | 0.09 | 0.1 | 0.11 | 0.12 | 0.16 | 0.21 | 0.27 | 0.38 | 0.53 | 0.74 | 0.86 | 1 | 1.16 | 1.34 | 1.54 |
| B | 0.06 | 0.07 | 0.08 | 0.09 | 0.1 | 0.11 | 0.14 | 0.18 | 0.23 | 0.33 | 0.46 | 0.64 | 0.74 | 0.86 | 1 | 1.18 | 1.39 |
| B- | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.1 | 0.13 | 0.17 | 0.23 | 0.31 | 0.43 | 0.59 | 0.66 | 0.75 | 0.85 | 1 | 1.18 |
| CCC | 0.05 | 0.06 | 0.07 | 0.07 | 0.08 | 0.09 | 0.12 | 0.17 | 0.22 | 0.3 | 0.4 | 0.53 | 0.59 | 0.65 | 0.72 | 0.85 | 1 |

(Table Contd.)

| $7 Y$ | AAA | AA + | AA | AA- | A+ | A | A- | BBB + | BBB | BBB- | BB + | BB | BB- | B+ | B | B- | CCC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AAA | 1 | 1.2 | 1.44 | 1.56 | 1.7 | 1.85 | 2.44 | 3.22 | 4.26 | 6.03 | 8.54 | 12.09 | 13.79 | 15.73 | 17.95 | 19.51 | 21.2 |
| AA+ | 0.83 | 1 | 1.2 | 1.3 | 1.42 | 1.54 | 2.04 | 2.69 | 3.55 | 5.03 | 7.12 | 10.08 | 11.5 | 13.12 | 14.97 | 16.38 | 17.94 |
| AA | 0.7 | 0.83 | 1 | 1.09 | 1.18 | 1.29 | 1.7 | 2.24 | 2.96 | 4.19 | 5.94 | 8.41 | 9.59 | 10.94 | 12.48 | 13.76 | 15.17 |
| AA- | 0.64 | 0.77 | 0.92 | 1 | 1.09 | 1.18 | 1.56 | 2.06 | 2.72 | 3.85 | 5.46 | 7.73 | 8.82 | 10.06 | 11.48 | 12.6 | 13.82 |
| A+ | 0.59 | 0.71 | 0.85 | 0.92 | 1 | 1.09 | 1.44 | 1.9 | 2.5 | 3.55 | 5.02 | 7.11 | 8.11 | 9.25 | 10.56 | 11.53 | 12.59 |
| A | 0.54 | 0.65 | 0.78 | 0.85 | 0.92 | 1 | 1.32 | 1.74 | 2.3 | 3.26 | 4.62 | 6.54 | 7.46 | 8.51 | 9.71 | 10.55 | 11.47 |
| A- | 0.41 | 0.49 | 0.59 | 0.64 | 0.7 | 0.76 | 1 | 1.32 | 1.74 | 2.47 | 3.5 | 4.95 | 5.65 | 6.44 | 7.35 | 8.02 | 8.76 |
| BBB+ | 0.31 | 0.37 | 0.45 | 0.49 | 0.53 | 0.57 | 0.76 | 1 | 1.32 | 1.87 | 2.65 | 3.75 | 4.28 | 4.88 | 5.57 | 6.1 | 6.68 |
| BBB | 0.23 | 0.28 | 0.34 | 0.37 | 0.4 | 0.43 | 0.57 | 0.76 | 1 | 1.42 | 2.01 | 2.84 | 3.24 | 3.7 | 4.22 | 4.64 | 5.1 |
| BBB- | 0.17 | 0.2 | 0.24 | 0.26 | 0.28 | 0.31 | 0.4 | 0.53 | 0.71 | 1 | 1.42 | 2.01 | 2.29 | 2.61 | 2.98 | 3.29 | 3.63 |
| BB+ | 0.12 | 0.14 | 0.17 | 0.18 | 0.2 | 0.22 | 0.29 | 0.38 | 0.5 | 0.71 | 1 | 1.42 | 1.62 | 1.84 | 2.1 | 2.33 | 2.58 |
| BB | 0.08 | 0.1 | 0.12 | 0.13 | 0.14 | 0.15 | 0.2 | 0.27 | 0.35 | 0.5 | 0.71 | 1 | 1.14 | 1.3 | 1.49 | 1.65 | 1.84 |
| BB- | 0.07 | 0.09 | 0.1 | 0.11 | 0.12 | 0.13 | 0.18 | 0.23 | 0.31 | 0.44 | 0.62 | 0.88 | 1 | 1.14 | 1.3 | 1.48 | 1.68 |
| B+ | 0.06 | 0.08 | 0.09 | 0.1 | 0.11 | 0.12 | 0.16 | 0.2 | 0.27 | 0.38 | 0.54 | 0.77 | 0.88 | 1 | 1.14 | 1.32 | 1.53 |
| B | 0.06 | 0.07 | 0.08 | 0.09 | 0.09 | 0.1 | 0.14 | 0.18 | 0.24 | 0.34 | 0.48 | 0.67 | 0.77 | 0.88 | 1 | 1.18 | 1.4 |
| B- | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.09 | 0.12 | 0.16 | 0.22 | 0.3 | 0.43 | 0.61 | 0.68 | 0.76 | 0.84 | 1 | 1.18 |
| CCC | 0.05 | 0.06 | 0.07 | 0.07 | 0.08 | 0.09 | 0.11 | 0.15 | 0.2 | 0.28 | 0.39 | 0.54 | 0.6 | 0.65 | 0.71 | 0.84 | 1 |


| 10Y | AAA | AA + | AA | AA- | A+ | A | A- | BBB + | BBB | BBB- | BB + | BB | BB- | B+ | B | B- | CCC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AAA | 1 | 1.3 | 1.7 | 1.81 | 1.91 | 2.03 | 2.59 | 3.29 | 4.19 | 6.15 | 9.03 | 13.24 | 15.02 | 17.05 | 19.35 | 20.6 | 21.94 |
| AA+ | 0.77 | 1 | 1.3 | 1.38 | 1.47 | 1.56 | 1.98 | 2.52 | 3.21 | 4.71 | 6.91 | 10.14 | 11.51 | 13.07 | 14.84 | 16.36 | 18.03 |
| AA | 0.59 | 0.77 | 1 | 1.06 | 1.12 | 1.19 | 1.52 | 1.93 | 2.46 | 3.61 | 5.29 | 7.76 | 8.82 | 10.02 | 11.38 | 12.99 | 14.82 |
| AA- | 0.55 | 0.72 | 0.94 | 1 | 1.06 | 1.12 | 1.43 | 1.82 | 2.32 | 3.4 | 4.99 | 7.32 | 8.32 | 9.45 | 10.73 | 12.12 | 13.68 |
| A+ | 0.52 | 0.68 | 0.89 | 0.94 | 1 | 1.06 | 1.35 | 1.72 | 2.19 | 3.21 | 4.71 | 6.91 | 7.85 | 8.91 | 10.12 | 11.3 | 12.62 |
| A | 0.49 | 0.64 | 0.84 | 0.89 | 0.94 | 1 | 1.27 | 1.62 | 2.06 | 3.03 | 4.44 | 6.52 | 7.4 | 8.4 | 9.54 | 10.54 | 11.64 |
| A- | 0.39 | 0.51 | 0.66 | 0.7 | 0.74 | 0.79 | 1 | 1.27 | 1.62 | 2.38 | 3.49 | 5.12 | 5.81 | 6.6 | 7.49 | 8.17 | 8.9 |
| BBB+ | 0.3 | 0.4 | 0.52 | 0.55 | 0.58 | 0.62 | 0.79 | 1 | 1.27 | 1.87 | 2.74 | 4.02 | 4.56 | 5.18 | 5.88 | 6.33 | 6.81 |
| BBB | 0.24 | 0.31 | 0.41 | 0.43 | 0.46 | 0.48 | 0.62 | 0.79 | 1 | 1.47 | 2.15 | 3.15 | 3.58 | 4.06 | 4.61 | 4.9 | 5.21 |
| BBB- | 0.16 | 0.21 | 0.28 | 0.29 | 0.31 | 0.33 | 0.42 | 0.54 | 0.68 | 1 | 1.47 | 2.15 | 2.44 | 2.77 | 3.14 | 3.38 | 3.62 |
| BB+ | 0.11 | 0.14 | 0.19 | 0.2 | 0.21 | 0.23 | 0.29 | 0.37 | 0.46 | 0.68 | 1 | 1.47 | 1.66 | 1.89 | 2.14 | 2.32 | 2.52 |
| BB | 0.08 | 0.1 | 0.13 | 0.14 | 0.14 | 0.15 | 0.2 | 0.25 | 0.32 | 0.46 | 0.68 | 1 | 1.14 | 1.29 | 1.46 | 1.6 | 1.75 |
| BB- | 0.07 | 0.09 | 0.11 | 0.12 | 0.13 | 0.14 | 0.17 | 0.22 | 0.28 | 0.41 | 0.6 | 0.88 | 1 | 1.14 | 1.29 | 1.46 | 1.65 |
| B+ | 0.06 | 0.08 | 0.1 | 0.11 | 0.11 | 0.12 | 0.15 | 0.19 | 0.25 | 0.36 | 0.53 | 0.78 | 0.88 | 1 | 1.14 | 1.33 | 1.56 |
| B | 0.05 | 0.07 | 0.09 | 0.09 | 0.1 | 0.1 | 0.13 | 0.17 | 0.22 | 0.32 | 0.47 | 0.68 | 0.78 | 0.88 | 1 | 1.22 | 1.48 |
| B- | 0.05 | 0.06 | 0.08 | 0.08 | 0.09 | 0.09 | 0.12 | 0.16 | 0.2 | 0.3 | 0.43 | 0.62 | 0.68 | 0.75 | 0.82 | 1 | 1.22 |
| CCC | 0.05 | 0.06 | 0.07 | 0.07 | 0.08 | 0.09 | 0.11 | 0.15 | 0.19 | 0.28 | 0.4 | 0.57 | 0.6 | 0.64 | 0.68 | 0.82 | 1 |


| $20 Y$ | AAA | AA + | AA | AA- | A+ | A | A- | BBB + | BBB | BBB- | BB + | BB | BB- | B+ | B | B- | CCC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AAA | 1 | 1.66 | 2.58 | 2.61 | 2.63 | 2.63 | 3.07 | 3.53 | 3.99 | 6.57 | 10.65 | 17.07 | 19.13 | 21.43 | 24.01 | 24.25 | 24.39 |
| AA+ | 0.54 | 1 | 1.66 | 1.65 | 1.63 | 1.6 | 1.8 | 1.97 | 2.08 | 3.66 | 6.21 | 10.32 | 11.54 | 12.89 | 14.4 | 16.27 | 18.36 |
| AA | 0.23 | 0.54 | 1 | 0.97 | 0.93 | 0.88 | 0.91 | 0.89 | 0.78 | 1.65 | 3.14 | 5.61 | 6.24 | 6.94 | 7.71 | 10.41 | 13.66 |
| AA- | 0.27 | 0.58 | 1.02 | 1 | 0.97 | 0.93 | 1 | 1.02 | 0.98 | 1.89 | 3.43 | 5.96 | 6.65 | 7.4 | 8.24 | 10.51 | 13.2 |
| A+ | 0.3 | 0.6 | 1.04 | 1.02 | 1 | 0.97 | 1.06 | 1.13 | 1.14 | 2.09 | 3.67 | 6.24 | 6.96 | 7.77 | 8.67 | 10.54 | 12.72 |
| A | 0.33 | 0.62 | 1.04 | 1.04 | 1.02 | 1 | 1.12 | 1.21 | 1.27 | 2.26 | 3.86 | 6.45 | 7.21 | 8.05 | 8.99 | 10.51 | 12.22 |
| A- | 0.31 | 0.55 | 0.89 | 0.89 | 0.89 | 0.88 | 1 | 1.12 | 1.21 | 2.08 | 3.46 | 5.67 | 6.34 | 7.1 | 7.94 | 8.64 | 9.4 |
| BBB+ | 0.28 | 0.48 | 0.76 | 0.76 | 0.76 | 0.76 | 0.88 | 1 | 1.12 | 1.86 | 3.04 | 4.91 | 5.5 | 6.16 | 6.9 | 7.08 | 7.23 |
| BBB | 0.25 | 0.41 | 0.64 | 0.64 | 0.65 | 0.65 | 0.76 | 0.88 | 1 | 1.63 | 2.64 | 4.2 | 4.71 | 5.28 | 5.92 | 5.77 | 5.56 |
| BBB- | 0.15 | 0.26 | 0.41 | 0.41 | 0.41 | 0.41 | 0.47 | 0.54 | 0.6 | 1 | 1.63 | 2.64 | 2.95 | 3.3 | 3.7 | 3.66 | 3.59 |
| BB+ | 0.09 | 0.16 | 0.26 | 0.26 | 0.26 | 0.25 | 0.29 | 0.32 | 0.35 | 0.6 | 1 | 1.63 | 1.83 | 2.04 | 2.28 | 2.3 | 2.3 |
| BB | 0.05 | 0.1 | 0.16 | 0.16 | 0.16 | 0.16 | 0.17 | 0.19 | 0.2 | 0.35 | 0.6 | 1 | 1.12 | 1.25 | 1.39 | 1.43 | 1.46 |
| BB- | 0.05 | 0.09 | 0.14 | 0.14 | 0.14 | 0.14 | 0.16 | 0.17 | 0.18 | 0.32 | 0.54 | 0.9 | 1 | 1.12 | 1.25 | 1.4 | 1.57 |
| B+ | 0.04 | 0.08 | 0.13 | 0.13 | 0.13 | 0.12 | 0.14 | 0.15 | 0.16 | 0.29 | 0.48 | 0.8 | 0.9 | 1 | 1.12 | 1.36 | 1.66 |
| B | 0.04 | 0.07 | 0.11 | 0.11 | 0.11 | 0.11 | 0.13 | 0.14 | 0.15 | 0.26 | 0.44 | 0.72 | 0.8 | 0.9 | 1 | 1.32 | 1.73 |
| B- | 0.04 | 0.06 | 0.09 | 0.09 | 0.09 | 0.1 | 0.12 | 0.14 | 0.17 | 0.27 | 0.43 | 0.69 | 0.71 | 0.73 | 0.75 | 1 | 1.32 |
| CCC | 0.04 | 0.05 | 0.07 | 0.08 | 0.08 | 0.08 | 0.11 | 0.14 | 0.18 | 0.28 | 0.43 | 0.66 | 0.63 | 0.6 | 0.56 | 0.75 | 1 |

Positive jump of an issuer's spread reduces market prices of instrument written on this issuer, hence produces a loss for policyholders.


Figure 1. P\&L Calculation in Single Period Simulation1 P\&L Calculation in Single Period Simulation.

## 5. Conclusion and Future Work

In this paper, we discussed the importance of modeling impacts of migration on spread curves in order to value, estimate profit and loss as well as manage credit risk to credit derivatives. Our proposed approach is to introduce a simple method at which spread curves are formed via mean spread at tenors, and find out the way to handle the case for which a statistically representative set of issuers does not exist. We would like to continue our research in improvement of other methods and models; step-back and verification; calculations of Incremental Risk Charge (IRC) and Credit Value Adjustment (CVA).

## Competing Interests

The authors declare that they have no competing interests.

## Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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