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Terahertz Radiation Emission from A Surface Wave Pumped Free Electron Laser

Research Article

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Abstract. We develop an analytical formalism for tunable coherent terahertz (THz) radiation generation from bunched relativistic electron beam (REB) counter -propagating to the surface wave in the vacuum region Compton backscatters the surface wave. Plasma supports the surface wave that acquires a large wave number around pump wave frequency. The surface wave extends into vacuum region that can be employed as a wiggler for the generation of terahertz radiation. As the beam bunches pass through the surface plasma wave wiggler, they acquire a transverse velocity, constituting a transverse current producing coherent THz radiation. It was found that the terahertz power increases with electric field as well as with the thermal velocity of electrons. It was also found that the growth rate and efficiency of the instability both increases with the modulation index of the density modulated beam.

Keywords. Terahertz radiation; FEL; SPW; Density modulation

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1. Introduction

Recently, there has been a great deal of interest in generation of high power terahertz radiation due to its non-ionizing behavior and good penetrating power and hence its applications in wide areas including medical imaging, chemical and security identification, spectroscopy analysis, remote sensing [1–3]. The main aspects of generating terahertz radiation of high power lies in using a prebunched or a modulated electron beam interacting with the wiggler in a free electron laser where it exchanges the energy with the electromagnetic radiation. The beam transfers the energy to the radiation wave and the wave gets amplified [4–11]. The kind of the wave excited due to beam-plasma interaction depends on the parameters of beam and plasma, mainly the three parameters: wiggler period, beam velocity and the phase velocity of the radiation [6].

Chen et al. [7] investigated the generation of the radiation by the interaction of relativistic circularly polarized laser pulse with overdense plasma which allows large plasma density and high pump laser intensity.

Kumar and Tripathi [8] have studied the coupling of a relativistic electron beam, modulated by two laser beams with a magnetic wiggler. They observed that THz power increases with the beam velocity and varies linearly with the bunch length and scales as square of the bunch radius.

Neumann et al. [9] have experimentally demonstrated the generation of multiple sub picoseconds pulses repeated at the terahertz frequencies with a modified drive laser by creating the highly modulated electron beams at the photocathode.

Sharma et al. [10] have examined the growth rate, efficiency of the radiation by employing an interaction of an energy modulated electron beam with the surface plasma wave as wiggler at vacuum plasma interface. They observed that the phase velocity of the radiation can be reduced by plasma which helps in the decrease of beam energy required for generating the radiation.

Yugami et al. [11] have experimentally demonstrated the generation of the radiation by the coupling of the wake field with the magnetic field and observed that the relative power of the radiation varied with the square of the applied magnetic field strength and increases with the increase of the density. They also suggested that the frequency can be tuned by varying the pressure of the supersonic gas, used to create the vacuum-plasma interface.

Singh et al. [12] observed the coupling of an oscillatory velocity of electron beam produced by the laser beam incident on the metal surface with the density perturbation produced by the surface plasma wave propagating along a metal-free space boundary via the generation of nonlinear current and observed that power of the radiated beam increases monotonically with the frequency of the probing laser and the SPW field.

Kraft et al. [13] have studied the generation of the wave by injecting the beam at an angle to the magnetic field and concluded that the emission is very weak in case if parallel propagation and is not affected by the beam modulation where as the emission is improved upto 50% if incident obliquely.

For the generation of the radiation by the upper hybrid wave FEL, the plasma density must be greater than the beam density, otherwise the frequency of the upper hybrid wave will approach the cyclotron frequency and hence it will lead to cyclotron damping, investigated by Sharma and Tripathi [14].

Pathak et al. [15] have reported the generation of terahertz radiation using free electron laser in rippled density plasma and observed that the growth rate is affected by the plasma density, amplitude, depth of density ripple and the transverse component of the ripple wave vector.

Liu and Tripathi [16] employed a layered dielectric with periodic permittivity in space as a wiggler for the generation of the radiation and observed that the beam energy requirement is considerably less as compared to one without dielectric.

Sharma and Singh [17] investigated that the two cross focused lasers in the presence of collisional plasma were able to produce nonlinear current at the terahertz frequency. They observed that the amplitude and power of the wave increases with the amplitude of the static electric field.

Leemans et al. [18] observed that the radiation is emitted at the plasma boundary when the pre-bunched beam is highly dense and scales quadratically with the density for larger wavelengths as compared to the bunch length.

In this paper, we study the Terahertz radiation emission from the surface wave pumped FEL and examine the growth rate and efficiency of the generated terahertz radiation by a density modulated relativistic electron beam counter-propagating surface plasma wave. The surface wave degrades as it moves away from the surface ($k_{ox} \ll k_{oz}$) and hence they have large amplitude at low powers. The SPW is a guided electromagnetic wave which propagates between the two media with different conductivities and dielectric properties along the boundary [19, 20]. We have considered the vacuum-plasma interface and calculated the growth rate, power and efficiency of the generated wave in the later section.

2. Theoretical Dispersion Relation

We consider a relativistic pre-bunched electron beam which is density modulated traveling in the z-direction and a vacuum-plasma interface with a surface plasma wave (SPW) propagating above the interface. The interface is taken at y=0, with vacuum present above $x = 0$ and plasma below $x = 0$, both occupying half of the spaces. The vacuum has permittivity unity i.e. $\epsilon_0 = 1$ and plasma with effective permittivity, $\epsilon_{\text{eff}} = 1 - \frac{\omega_p^2}{\omega_0^2}$, where $\omega_p^2 = \frac{4\pi n_0 e^2}{m_e}$, ω_p being the plasma frequency, ω^0 is the SPW frequency. The plasma is characterized by lattice dielectric constant ϵ_L and a.c conductivity σ, where $\sigma = \frac{n_0 e^2}{m_e \omega^0}$ (n_0 is the plasma density, e is the electronic charge and m_e is the mass of the electron).

The electric field of the SPW propagating along the interface is given as

$$E = E(x) e^{-i(\omega_0^0 t - k_{wz} z)}. \tag{2.1}$$

Electric field is polarized in the x-z plane. Using the Maxwell's III and IV equations, we obtain

$$\nabla^2 \vec{E} + \omega_0^{02}/c^2 \epsilon \vec{E} = 0 \tag{2.2}$$

which can be written as

$$\partial^2 E_z / \partial x^2 - k_p E_z = 0 \quad \text{for plasma} \tag{2.3}$$

and

$$\partial^2 E_z/\partial x^2 - k_v E_z = 0 \quad \text{for vacuum} \tag{2.4}$$

where $k_p^2 = k_{wz}^2 - \frac{\omega_0^{02}}{c^2}\epsilon$ and $k_v^2 = k_{wz}^2 - \frac{w_0^{02}}{c^2}$, k_p, k_v is the plasma wave vector and vacuum wave vector respectively.

The solutions of above equations can be given as

$$E_z = A_p e^{(k_p x)} e^{-i(\omega_0^0 t - k_{wz} z)} \quad \text{for plasma} \tag{2.5}$$

and

$$E_z = A_v e^{-(k_v x)} e^{-i(\omega_0^0 t - k_{wz} z)} \quad \text{for vacuum.} \tag{2.6}$$

The dispersion relation for SPW can be obtained by applying the boundary conditions at $x = 0$ for E_z and ϵE_zcontinuous at the boundary

$$k_{wz}^0 = \omega^2 \epsilon / c^2 (1+\epsilon), \tag{2.7}$$

where $\epsilon = k_p / k_v$.

In order to ensure that the surface wave is spatially damped away from the interface, we require $k_z^2 < 0$ i.e. $1+\epsilon < 0$ or $\omega < \omega_p/\sqrt{(1+\epsilon)}$ or $\omega < \omega_p/\sqrt{2}$ such that $k_p, k_v \to \infty$ as 1+ $\epsilon < 0$. Thus the waves are strongly localized near the surface.

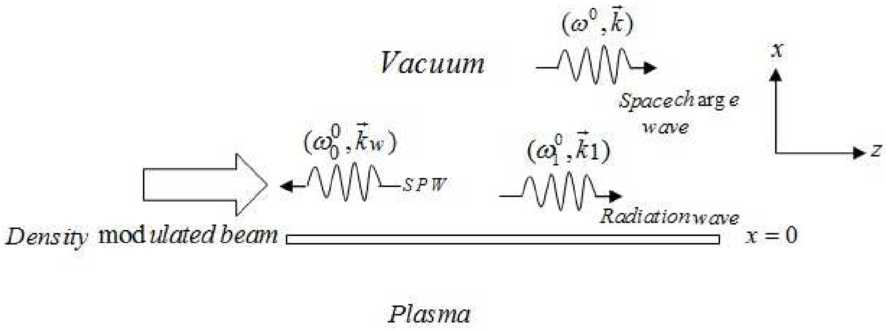

Figure 1. Schematic diagram of SPW-pumped FEL.

3. Beam Wiggler Interaction

The density modulated pre bunched electron beam interacts with the surface plasma wave which acts as a wiggler in the present case. The initial parameters of the beam are n_b^0 as the beam density, velocity $\vec{v}_b^0$. In the free electron laser, we have considered the propagation of surface wave over a wiggler with wave vector k_{wz} and frequency ω^0. The electron beam when interacts with the SPW, acting as a wiggler acquires an oscillatory velocity $\vec{v}_0^0$. This velocity can be obtained by using the equation of motion for an electromagnetic wave as

$$m_e \left[\frac{\partial(\gamma \vec{v}_0^0)}{\partial t} + (\vec{v}_b^0 \cdot \nabla)(\gamma \vec{v}_0^0) \right] = -e \left(\vec{E} + \frac{1}{c} \vec{v}_0^0 \times \vec{B} \right) \tag{3.1}$$

On solving this equation with $\vec{B} = \frac{c}{w_0^0}(\vec{k}_w \times \vec{E})$ and taking different components of velocity, we obtain the x-component of oscillatory velocity as

$$v_{0x}^0 = \frac{eE_{0z}}{m_e\omega_0^0\gamma_0}\left[\frac{k_{wz}}{k_{0v}} + \frac{k_{0v}v_b^0}{(\omega_0^0 - k_{wz}v_b^0)}\right], \tag{3.2}$$

where m_e, e are the electronic mass and charge, respectively, c is the speed of light and k_z, ω_0^0 being the SPW wave vector and frequency, respectively.

The equilibrium can be perturbed by applying the electromagnetic perturbation with perturbed values as

$$E_{1x} = A_1 e^{-i(\omega_1^0 t - k_{1z}z)}, \quad \vec{B}_1 = \frac{c}{\omega_1^0}(\vec{k}_1 \times \vec{E}_1), \quad \vec{v}_1^0 = \frac{e\vec{E}_1}{im\omega_1^0\gamma_0}.$$

Pondermotive force is exerted by the FEL radiation and the surface wave on the beam electrons with the equation of motion as

$$m\frac{\partial\vec{v}}{\partial t} = -e\vec{E} + \vec{F}_p - \frac{1}{n}\nabla(nT_e), \tag{3.3}$$

where $\vec{F}_p = -m(\vec{v}\cdot\nabla)\vec{v} - \frac{e}{c}(\vec{v}\times\vec{B})$ is the pondermotive force. The z-component of the pondermotive force is

$$F_{p_z} = e\nabla\phi_p$$

The pondermotive force produces perturbed oscillatory velocity v_{2b}^0 of the electrons in the axial direction with density perturbation as n_{2b}, obtained using the equation of motion and continuity respectively.

$$v_{2b}^0 = -\frac{ek_z\phi_p}{m_e\gamma_0^3(\omega - k_z v_b^0)}, \tag{3.4}$$

$$n_{2b} = \frac{k_z^2}{4\pi e}\chi_b\phi_p, \tag{3.5}$$

where $k_z = k_{wz} + k_{1z}$, $\omega^0 = \omega_0^0 + \omega_1^0$ are the wave number and frequency of the space charge wave, respectively.

$$\phi_p = \frac{m\gamma_0}{2e}v_{1x}^0 v_{0x}^0 \quad \text{is the pondermotive potential}$$

and

$$\chi_b = -\frac{\omega_{pb}^2}{(\omega - k_z v_b^0)^2\gamma_0^3} \quad \text{is the beam susceptibiliiy.}$$

In case of Compton regime, the potential of the beam mode is much less than the pondermotive potential and hence can be neglected ($\phi \ll \phi_p$). The beam density is $n_b^0 =$

$n^0_{0b}+n^0_{1b}+n^0_{bm}$, where n^0_{bm} is the modulated beam density such that $n^0_{bm}=\Delta n^0_{0b}$, Δ being the modulation index. The phase of the perturbed beam as $e^{-i(\omega^0 t-k_z z)}$ and that of the modulated beam is $e^{-i(\omega^0_0 t-k_{wz}z)}$.

On solving the equation of continuity, we get

$$n_{1b}=\frac{n_{0b}\vec{v}^0_{1b}}{(\omega^0-k_z v^0_{0b})}\left[k_z(1+\Delta)+k_{wz}\Delta\right]. \tag{3.6}$$

The perturbed non-linear current density can be given as

$$\vec{J}^{NL}_1=-en^0_{0b}\vec{v}^0_{1b}-en^0_{1b}\vec{v}^0_{0b}-en^0_{bm}\vec{v}^0_{1b}. \tag{3.7}$$

On substituting the values, we obtain

$$\vec{J}^{NL}_1=\frac{ie^2n^0_{0b}k_z v^0_{0x}\vec{A}_1}{2m_e\omega^0_1\gamma^3_0(\omega-v^0_{0b}k_z)}\left[(1+\Delta)+\frac{v^0_{0b}}{(\omega-v^0_{0b}k_z)}(k_z(1+\Delta)+k_{wz}\Delta)\right]\left[e^{-i\left(\omega^0_1 t-\left(k_{1z}+\frac{\omega^0_1}{v^0_{0b}}z\right)\right)}\right]. \tag{3.8}$$

In using Maxwell's III and IV equations

$$\nabla\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t},$$

$$\nabla\times\vec{H}=\frac{4\pi}{c}\vec{J}_1+\frac{1}{c}\frac{\partial\vec{E}_1}{\partial t},$$

$$(\omega^{02}_1-k^2_{1z}c^2)(\omega^0-v^0_{0b}k_z)^2=\frac{4\pi\omega^0_1e^2n^0_{0b}k_z v^0_{0x}v^0_{0b}}{2m_e\omega^0_1\gamma^3_0}\left[k_z(1+\Delta)+k_{wz}\Delta\right].$$

On assuming that $\omega^0=k_z v^0_{0b}+\delta$ and $\omega^0_1=k_{1z}c+\delta$ we obtain

$$\delta=\left[\frac{\pi e^2n^0_{0b}k_z v^0_{0x}v^0_{0b}}{m_e\omega^0_1\gamma^3_0}(k_z(1+\Delta)+k_{wz}\Delta)\right]^{\frac{1}{3}}e^{\frac{2in\pi}{3}}. \tag{3.9}$$

The growth rate is obtained by taking the imaginary part of δ i.e. $\Gamma=\mathrm{Im}\,\delta$, where Γ is the growth rate of the generated wave instability.

Hence, the growth rate is given as

$$\Gamma=\left[\frac{\omega^2_{pb}v^0_{0x}v^0_{0b}}{c\gamma^3_0}\left((1+\Delta)\frac{\omega^0_1}{c}+k_{wz}\Delta\right)\right]^{\frac{1}{3}}\frac{\sqrt{3}}{2}, \tag{3.10}$$

where $\omega^2_{pb}=\frac{4\pi n^0_{0b}e^2}{m_e}$.

The power of the wave generated can be calculated using

$$P=\omega^0_0\frac{\partial\epsilon_0}{\partial\omega_0}\frac{E^2_0}{8\pi}v_g\pi r^2_b, \tag{3.11}$$

where $v_g \simeq v_{th} = \sqrt{\frac{T_e}{m_e}}$ is the group velocity of the beam, v_{th} is the electron thermal velocity. T_e is the temperature of the plasma, m_e is the mass of the electron. πr_b^2 is the area of the beam.

The efficiency of the beam is given as

$$\eta = \frac{\Gamma}{\omega_1} \frac{\gamma_0^3}{\sqrt{3}(\gamma_0 - 1)},$$

on substituting the value of growth rate, efficiency is obtained as

$$\eta = \left[\frac{\omega_{pb}^2 v_{0x}^0 v_{0b}^0}{c} \left((1+\Delta)\frac{\omega_1^0}{c} + k_{wz}\Delta \right) \right]^{\frac{1}{3}} \frac{\gamma_0^2}{2\omega_1^0(\gamma_0 - 1)}. \tag{3.12}$$

4. Results and Discussions

Standard parameters of THz radiation have been used for calculating the required parameters of the radiation which is obtained from the surface plasma wave pumped FEL. In Figure 2 the variation of the growth rate of the unstable wave instability has been plotted against the modulation index using the typical parameters: beam energy $E_b = 0.12$ MeV, beam current $I_b = 1.4$ A, beam radius $r_b = 2.4$ cm, electric field $E_0 = 100$ esu, radiation frequency $\omega_1^0 = 3 \times 10^{12}$ rad/sec, and modulation frequency $\omega' = 2.5 \times 10^{10}$ rad/sec. From Figure 2 we can see that the growth rate increases with the increase in the modulation index Δ and has reached the largest value when $\Delta \to 1$ and when the frequency and wave number of the generated radiation wave is of the order of the pre-bunched modulated electron beam. For the largest value of Δ i.e., $\Delta = 1$, the growth rate comes out to be $\Gamma = 2.87 \times 10^9$ (rad/sec). When $\Delta = 0$ i.e., without pre-modulated beam, the growth rate of the THz radiation is $\Gamma = 1.85 \times 10^9$ (rad/sec).

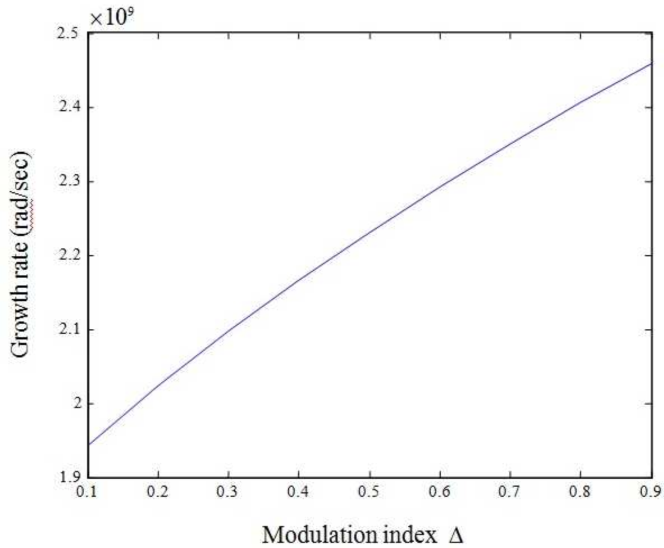

Figure 2. The figure shows the variation of growth rate (rad/sec) as a function of the modulation index for $E_b = 0.12$ Mev, $I_b = 1.4$ A, $\omega_1^0 = 3 \times 10^{10}$ (rad/sec).

In Figure 3, we have plotted the effect of electric field E_0 (esu) on the radiated power P_0 (in Watt) of the radiation obtained using electron thermal velocity $v_g = 5.9 \times 10^7$ (cm/sec), $\omega_0^0 \frac{\partial \epsilon_0}{\partial \omega_0} \simeq 1$ and the other parameters are same as taken for the Figure 2. It was observed that the power increases as we increase the electric field and comes out to be the order of kilo watt. Hence by providing the suitable magnetic field, we can obtain an electric field of optimum value and therefore can be able to obtain the radiation of more high power, which is our motive.

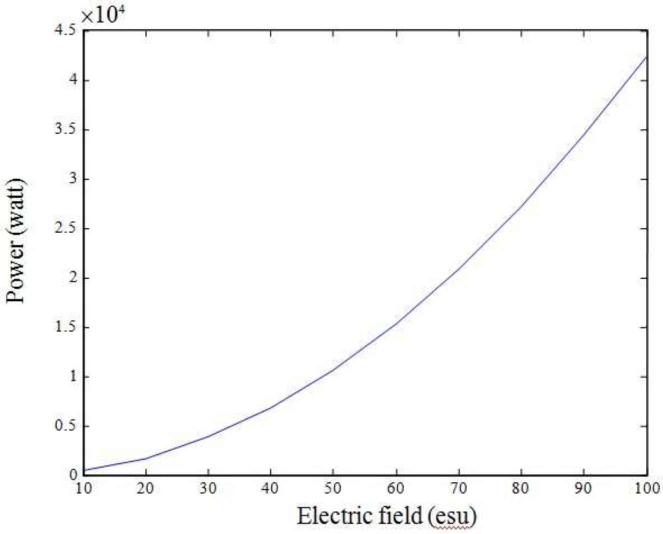

Figure 3. The figure shows the variation of the power (erg/sec) with the electric field (esu) with the parameters $v_g = 5.9 \times 10^7$ (cm/sec) and $r_b = 2.4$ cm.

Figure 4 shows that the power P_0 (in Watt) of the beam increases as the electron thermal velocity v_g (cm/sec) of electrons increases with $E_0 = 100$ esu. The plot shows that the power increases linearly with the electron thermal velocity.

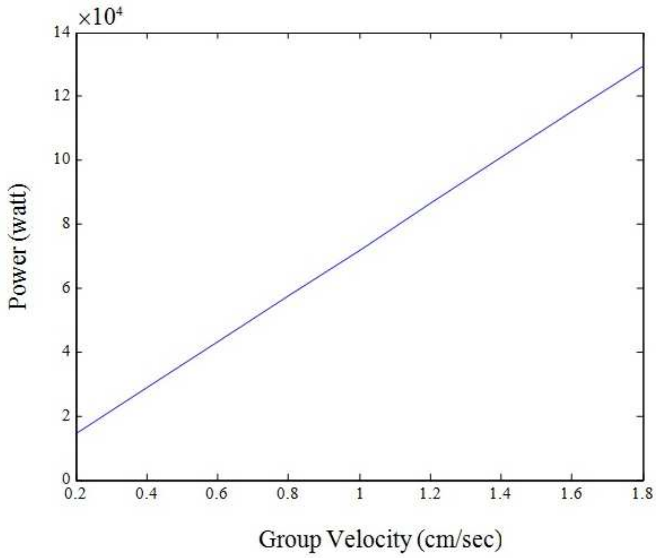

Figure 4. The figure shows the variation of the power (erg/sec) with the group velocity (cm/sec) i.e. electron thermal velocity with $E_0 = 100$ esu.

Figure 5 shows the variation of the efficiency of the SPW-pumped FEL instability with respect to the modulation index. From Figure 5 it can be seen that the efficiency increases with the modulation index. The growth rate and efficiency scales as two-third power of the plasma frequency. Hence, it improves with an increase in plasma frequency, thus the plasma density.

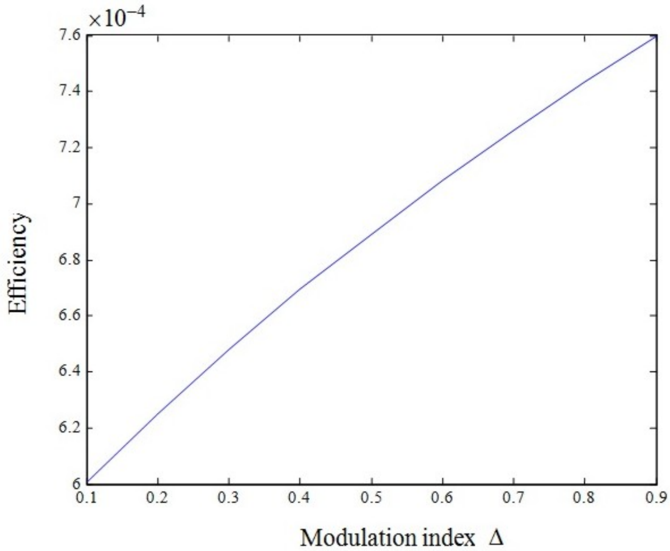

Figure 5. The figure shows the effect of modulation index on the efficiency radiation with $\omega' = 2.5 \times 10^{10}$ (rad/sec), $E_b = 0.12$ Mev, $I_b = 1.4$ A.

5. Conclusion

It can be concluded that the SPW are driven to instability by pre-bunched relativistic beam, which is density modulated leading to the increase in growth rate, amplitude and efficiency of the radiation wave with the modulation index and attained the largest value when Δ approached unity. The beam and plasma interaction leads to the growth of the radiation beam and the growth rate increases with the increase in plasma density. Using the SPW as a wiggler, it has been possible to generate high power radiation even at lower beam current. The density modulation has led to an improvement in the radiation generation of FEL instability and the radiation of desired frequency and power can be obtained by varying the frequency and modulation index of the incident relativistic beam. Further plasma can help in reducing the phase velocity of the radiation wave significantly, thus helping in the decrease of beam energy required for generating THz radiation.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed equally and significantly in writing this article. All the authors read and approved the final manuscript.

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