A Study on Probe Transmission through an Inverted Y-type Atomic System in Presence of Three Coherent Laser Fields

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Abstract. A four-level inverted Y-type atomic system is subjected to three coherent laser fields simultaneously. The system has two closely spaced ground energy levels, one intermediate energy level and an uppermost energy level. Transition from the two ground levels to the intermediate level as well as from the intermediate level to the uppermost level are dipole allowed. Transition from the two ground energy levels to the uppermost level is dipole forbidden. A weak laser beam, known as the probe beam, acts between one of the ground levels and the intermediate level. A strong laser field, known as the control or pump field, couples the intermediate level with the uppermost level. A repump field of intensity usually higher than the probe field acts between the other ground level and the intermediate level. The probe transmission through such a medium is investigated at different values of the control and repump Rabi frequencies under Doppler free condition as well as with thermal averaging. The nonradiative population transfer rates between the ground energy levels have been incorporated.

Keywords. Four-level system; Inverted Y; Density matrix; three coherent fields; Probe transmission; EIT

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1. Introduction

The propagation of a low power coherent radiation field in presence of a high power coherent radiation beam through an otherwise absorbing atomic vapour medium has been the subject of interest to the researchers worldwide. The phenomenon of electromagnetically induced transparency (EIT) [1–4], a field induced transparency, not only makes an absorbing vapour medium transparent to a coherent radiation field, but also makes the medium highly dispersing to the probe pulse thereby altering the group velocity of the probe pulse propagating through the vapour medium to a large extent [5,6]. In fact, based on this effect it has been demonstrated experimentally how the velocity of light in a medium can be manipulated and light pulse may almost be stopped and stored in a dispersing medium [5,6]. This has opened up a number of new possibilities for future applications. These include development of optical data storage devices, all optical switches, optical delay generators, quantum computers etc. The physical mechanism responsible for the occurrence of EIT has been explained in details by many researchers [1–4]. The theoretical studies as well as experimental observation on the three-level as well as more complicated cascade type systems have been reported [7–10]. In an earlier work, Joshi et al. [11] showed how an inverted Y type system can be formed by adding an extra ground level in a three-level cascade type system. They have studied the probe response theoretically under Doppler free condition by using density matrix method. In this article we shall describe a four-level inverted Y type system in some details and show how the probe beam propagation can get affected in presence of a strong control field and a repump field. The formation of EIT window under both Doppler free conditions as well as in hot atoms will be presented.

2. Theoretical Model and Simulation

In this work we have taken an inverted Y type four level system as shown in Figure 1. The outermost electron of the alkali atoms executes \( nS \rightarrow nP \) and \( nP \rightarrow nD/n'D \) transitions as well as \( nS \rightarrow nP \) and then \( nP \rightarrow (n+1)S \) where \( n, n' \) represents the principal quantum numbers with \( n' > n \). The ground level of the alkali atoms is the \( S \) level which has two hyperfine components. Hence the two ground hyperfine levels and the next \( P \) level form the inverted head of the ‘Y’ type system and the \( nP \rightarrow nD/n'D \) or \( nP \rightarrow n'S \) form the upper part of the inverted Y level. As for example, electron from the \( ^5S_{1/2} \) ground state of \(^{87}\)Rb can be transferred to the \( ^5P_{3/2} \) excited state by using a laser radiation of wavelength around 780 nm [11]. The decay of the excited electron from the \( P \) state to the two ground hyperfine levels is possible subject to the fulfilment of the selection rule \( \Delta F = 0, \pm 1 \). Another laser radiation of wavelength around 775.7 nm can couple the \( ^5P_{3/2} \) level to the \( ^5D_{5/2} \) level of \(^{87}\)Rb, thereby forming the upper part of the inverted system [12]. The details of the hyperfine structure of the \( P \) and \( D \) levels are not considered at present to keep the model simple.
The two ground levels have been represented by levels $|0\rangle$ and $|1\rangle$ as can be seen in Figure 1. The intermediate energy level has been represented by $|2\rangle$ and the uppermost energy level is the level $|3\rangle$. The levels $|1\rangle$ and $|2\rangle$ are coupled by a weak probe field of Rabi frequency $\Omega_1 = \frac{\mu_{12}E_1}{\hbar}$. The strong control field with a Rabi frequency $\Omega_2 = \frac{\mu_{23}E_2}{\hbar}$ acts between the energy levels $|2\rangle$ and $|3\rangle$. Here $\mu_{ij}$ represents the electric dipole moment associated with transition $|i\rangle \rightarrow |j\rangle$. The transfer of population between the two ground levels has been assumed to be slow ($\gamma_0$ from $|0\rangle \rightarrow |1\rangle$). This has been included in the theoretical model. In reality nonradiative process like collision causes transfer of population between the ground hyperfine levels of the alkali atoms. Increase in the temperature of the sample vapour cell enhances the rate of collision and thereby becomes a very important factor in controlling the coherence between the energy levels. This affects the EIT line shape. A coherent repump field of Rabi frequency $\Omega_0 = \frac{\mu_{02}E_0}{\hbar}$ is assumed to be applied between the levels $|0\rangle$ and $|2\rangle$. The intensities of the control and repump fields have been kept greater than the probe field. The probe response ($\rho_{21}$) is calculated analytically by solving a set of fifteen density matrix based equations following rotating wave approximation assuming $\rho_{20} = 0$ and $\rho_{32} = 0$ (this assumption simplifies the calculation to some extent and gives an overall physical picture of the chosen system without much loss in accuracy although a more rigorous treatment demands the removal of this assumption). The imaginary part of $\rho_{21}$ determines the probe absorption through the atomic vapour whereas the real part of $\rho_{21}$ gives the dispersion. We have chosen the $^{5}S_{1/2} \rightarrow ^{5}P_{3/2} \rightarrow ^{5}D_{5/2}$ transition of $^{87}$Rb to simulate the probe response and accordingly the radiative decay rates of population from $|2\rangle \rightarrow |1\rangle$ and $|2\rangle \rightarrow |0\rangle$ ($\gamma_1$ and $\gamma_2$ respectively) have been set equal to be 3 MHz [4] whereas the radiative decay rate from $|3\rangle \rightarrow |2\rangle$ ($\gamma_3$) has been chosen to be 4.19 MHz [12]. The analytical expression for $\rho_{21}$ is written below:

$$\rho_{21} = i \frac{\Omega_1}{2} \left( \rho_{11} - \rho_{22} \right) \left\{ \frac{\gamma_1 + \gamma_2}{2} - i \Delta_1 \right\} + \left\{ \frac{\Omega_1^2}{4} \right\} + \left\{ \frac{\Omega_2^2}{4} \right\} + \left\{ \frac{\gamma_3}{2} - i (\Delta_1 + \Delta_2) \right\}. \tag{2.1}$$
Here $\rho_{ii}$ represents the population of the level $|i\rangle$; $\Omega_1$, $\Delta_0$, $\Delta_1$ and $\Delta_2$ are the frequency detunings of the repump, probe and pump lasers. $\gamma_0$ is the population transfer rate from level $|0\rangle$ to the lowest ground level $|1\rangle$. $\gamma_1$ and $\gamma_2$ are the spontaneous decay rates (Einstein’s A coefficient) from level $|2\rangle$ to the two ground levels $|1\rangle$ and $|0\rangle$ respectively and $\gamma_3$ is that from the uppermost level $|3\rangle$ to the level $|2\rangle$. In the simulation $\gamma_0$ has been kept fixed at 0.1 MHz. In Figure 2 we have shown the probe transmission at different values of the control Rabi frequency ($\Omega_2$) by using the imaginary part of eqn. (2.1). The probe Rabi frequency ($\Omega_1$) has been kept fixed at 1 MHz throughout the simulation process. The thermal averaging has been included in the simulation assuming a temperature ($T$) of 300 K [4]. The repump Rabi frequency is held constant at 10 MHz. The control, probe and repump fields have been taken to be copropagating. Very narrow EIT window appears when Raman resonance condition is satisfied. Since the pump field detuning ($\Delta_2$) is kept zero in the simulation, the EIT window appears at and around zero probe field detuning ($\Delta_1 = 0$). Figure 2 shows that the EIT width is only few hundred kHz and remains almost immune to the variation of the Rabi frequency of the control field. This is the most unique feature of this theoretical simulation. This is attributed to the velocity averaging. With the alteration in the Rabi frequency of the repump field, the width of the EIT signal varies accordingly. This shows the dominance of the lambda type emission-absorption cycle formed by the repump and the probe fields over the entire system. But still when the control and repump Rabi frequencies have been assigned higher values of few tens of MHz the EIT width remains not only sub-natural but in the sub MHz region. Such low EIT linewidth is not obtained for the three-level lambda type system under similar values of the parameters. This would correspond to very steep dispersion for the probe signal in the vapour medium. We shall discuss this point in details in some future communication very soon.

Figure 2. (Colour online) Plot of probe transmission vs. probe frequency detuning at different values of control Rabi frequency ($\Omega_2$). Probe Rabi frequency ($\Omega_1$) and repump Rabi frequency ($\Omega_0$) are held fixed at 1 MHz and 10 MHz respectively during the simulation.
In Figure 3, we have shown the variation of the width of the EIT window at different values of the repump Rabi frequency by keeping the probe and control Rabi frequencies fixed at 1 MHz and 10 MHz respectively. We must mention here that for the three-level cascade type system, there is no signature of any transparency in the probe response signal for copropagating probe and control fields under thermal averaging but for the inverted ‘Y’ scheme under similar configuration presence of an extra coherent field acting as the repump field alters the situation by affecting the coherence terms as well as the population of the energy levels, especially the two ground levels. In fact, the controlling factor for the width and size of EIT window is now the repump field. Figure 3 establishes this. On the other hand, the presence of the control field between level |2⟩ and level |3⟩ has also played important role in keeping the width of the EIT window low compared to that observed in the three-level conventional Lambda type system. From this figure it is clear that the width of the EIT window remains sub-natural, in fact, lower than 1 MHz even when the probe and pump Rabi frequencies are 1 MHz and 10 MHz respectively and the repump Rabi frequency has been varied from 1 MHz to 9 MHz. If this is compared with a similar graph for the lambda type system, it would be seen that the width of the EIT window under similar condition (the control field being withdrawn of course) is far greater. The analysis, being an approximate one, would not be valid for comparable intensities of control, probe and repump fields. We have used this model in determining the probe response at various values of the repump Rabi frequency by keeping the probe Rabi frequency at a low value (1 MHz) and the control Rabi frequency higher (10 MHz or above).

Figure 3. (Colour online) Plot of probe transmission vs. probe frequency detuning at different values of the repump Rabi frequency (Ω₀) under Doppler broadened regime. The probe Rabi frequency (Ω₁) and the control Rabi frequency (Ω₂) are held fixed at 1 MHz and 10 MHz respectively during the simulation. The complete Doppler background is missing here since we have shown the zoomed part around zero probe detuning.
We have plotted the probe transmission versus probe detuning for inverted Y and three-level lambda type systems in Figure 4 to compare the EIT line shape for the two different level schemes. It is evident that the width of the EIT window under inverted Y type scheme is far less than that of the EIT window for a three-level lambda type system when the values of the control and probe Rabi frequencies of 10 MHz and 1 MHz respectively, (for both inverted Y and lambda type systems), repump Rabi frequency of 10 MHz (applicable for inverted Y system only) have been used. The radiative decay rates from the intermediate level $|2\rangle$ to the ground levels $|1\rangle$ and $|0\rangle$ ($\gamma_1$ and $\gamma_2$ respectively) have been set equal to be 3 MHz each [4] whereas the radiative decay rate from $|3\rangle \rightarrow |2\rangle$ ($\gamma_3$) has been chosen to be 4.19 MHz [12] (please refer to Figure 1). Ideally we could have set the values of $\gamma_1$ and $\gamma_2$ equal to 6 MHz each. The non-radiative population transfer rate ($\gamma_0$) from $|0\rangle \rightarrow |1\rangle$ has been assumed to be 0.1 MHz as usual.

![Figure 4](image-url)

**Figure 4.** (Colour online) Plot of probe transmission vs. probe frequency detuning for a four-level inverted Y type system and a three-level $\Lambda$ type system. The values of the control, probe and repump Rabi frequencies have been kept fixed at 10 MHz, 1 MHz and 10 MHz respectively during the simulation.

In Figure 5, we have shown the variation of the probe signal of the four-level inverted Y-type system and the three-level $\Lambda$ type system as a function of probe frequency detuning under Doppler free condition ($T = 0$). The values of the parameters used in this simulation are same as those used in the last graph (Figure 4). By comparing the two plots (Figure 4 and Figure 5) we can easily conclude that under velocity averaging, the EIT window is narrowed further as compared to the Doppler free condition. A narrower EIT peak corresponds to a steeper dispersion in the medium, which in turn results in reduction in the group velocity of a probe pulse around the steeper dispersive window. The formation of very narrow EIT window in a Doppler broadened medium thus provides an excellent testing bed for studying the effect of a
dispersive medium on the probe pulse propagation and consequently manipulating the velocity of the probe pulse by controlling the dispersive properties of the medium.

![Plot of probe transmission vs. probe frequency detuning under Doppler free condition for a four-level inverted Y-type system and a three-level lambda type system for a fixed set of Rabi frequencies of the control, probe and repump fields (mentioned in the graph).](image)

**Figure 5.** (Colour online) Plot of probe transmission vs. probe frequency detuning under Doppler free condition for a four-level inverted Y-type system and a three-level lambda type system for a fixed set of Rabi frequencies of the control, probe and repump fields (mentioned in the graph).

Let us now come to the probe transmission shown in the figure for the four-level inverted ‘Y’ and three-level lambda type systems. For the inverted ‘Y’ type system, the separation between the two absorption peaks appearing on either sides of the EIT window is found to be (from plot) 14.13 MHz whereas that between the two peaks for the lambda type system is as usual equal to the Rabi frequency of the control field, i.e., 10 MHz in this case. The separation between the peaks for the inverted ‘Y’ system comes out to be \( \sqrt{\Omega_2^2 + \Omega_0^2} \). The interesting part is however the reduction in the width of the transparency window for the two different level schemes from Doppler free condition (Figure 3) to the thermally broadened situation (Figure 4).

## 3. Conclusion

The addition of an extra ground level to a three-level cascade type system makes it an inverted ‘Y’ type system and use of three coherent radiation fields to couple four different energy levels alters the probe transmission through an alkali vapour medium compared to what one gets under simple three-level cascade or lambda type configuration. In fact, under Doppler free condition the transparency reaches almost 100% at fairly low values of control and probe field intensities under the three-level lambda type configuration but under similar condition the inverted type system shows greater width of the EIT window. We have used practical values for the radiative decay rates \( \gamma_1, \gamma_2, \gamma_3 \) and included Doppler averaging in order to get realistic results in the simulation. The experiments performed at room temperature using alkali vapour
cells provide spectra having thermal broadening unless special techniques are used to remove the Gaussian background of the probed transition. The occurrence of a transparency window in presence of a repump field under copropagating control, repump and probe fields is significant since this is not observed in the cascade configuration. Furthermore, the width of the EIT window is sub-natural, in fact in the sub MHz region, even at appreciably high intensities of the control and repump fields. The thermal averaging plays a significant role in producing such narrow transparency window. In an earlier work by Joshi et al. [11] under Doppler free condition, the variation of probe transmission under different probe, pump and repump field intensities had been studied. But in this article, the probe transmission through an atomic vapour cell at room temperature has been found to differ from that under Doppler free condition [11], rather resulted in an extremely narrow EIT window immune to the variation of the control field intensity. This also shows the way of producing a line shape independent of the intensity of the control field. The occurrence of such narrow EIT window makes the otherwise absorbing medium highly dispersive to the probe field and detail investigation on the dispersive property of the medium is required. It can be easily predicted that the very steep dispersion around the EIT window will reduce the velocity of the probe pulse drastically within a very short probe field detuning; hence there will be a possibility to manipulate the propagation of a probe pulse through the medium. This will be discussed in details in a future communication shortly. We firmly believe that the outcome of this work will be useful in slowing down, stopping and storage of light pulse in a medium, optical delay generator, frequency locking of lasers, optical logic gates etc.

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**Competing Interests**

The authors declare that they have no competing interests.

**Authors’ Contributions**

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.
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