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Some New Contra Continuous Functions in Topology

Research Article

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Abstract. In this paper we apply the notion of *sgp*-open sets in topological space to present and study a new class of functions called contra and almost contra *sgp*-continuous functions as a generalization of contra continuity which was introduced and investigated by Dontchev [5]. We also discuss the relationships between them and with some other related functions.

Keywords. *sgp*-closed set; Contra-continuous; Contra *sgp*-continuous; Almost contra-*sgp*-continuous function

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1. Introduction

In General Topology, generalized open sets plays an important role. Indeed a significant theme in General Topology and Real Analysis concerns that variously modified forms of continuity, separation axioms etc. by utilizing generalized open sets. In 1996, Dontchev [5] introduced the notion of contra continuity and strong S-closedness in topological spaces. A new weaker form of functions called contra semi continuous function was introduced and investigated by Dontchev and Noiri [6]. Recently in [9] the notion of semi-generalized preopen (briefly, *sgp*-open)set was introduced. In [12, 13] the concept of Almost contra θgs-continuous and Contra θgs-continuous functions has been discussed. Also, in [14] notion of contra and almost contra continuity has been discussed using g^*p-closed sets. The aim of this paper is to introduce and study new generalization of contra continuity called Contra and Almost contra *sgp*-continuous functions utilizing *sgp*-open sets.

2. Preliminaries

Throughout this paper (X,τ), (Y,σ) (or simply X, Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X the closure and interior of A with respect to τ are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$ respectively.

Definition 2.1. A subset A of a space X is called

(1) a semi-open set [8] if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$.

(2) a semi-closed set [2] if $\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A))) \subseteq A$.

(3) a regular open [19] if $A = \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$.

Definition 2.2 ([9])**.** A topological space X is called semi-generalized preclosed (briefly, *sgp*-closed) set if $\mathrm{pCl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.

Definition 2.3 ([1])**.** Intersection of all *sgp*-closed closed sets containing A is called semi-generalized preclosure (briefly, $sgp\,\mathrm{Cl}$).
A set A is *sgp*-clsoed if and only if $A = sgp\,\mathrm{Cl}(A)$.

Definition 2.4 ([4])**.** A topological space X is called $_{sgp}Tc$-space if every *sgp*-closed set is closed set.

Definition 2.5 ([4])**.** A topological space X is said to be

(i) *sgp*-T_0 space if for each pair of distinct points in X there exists *sgp*-open set of X containing one point but not the other.

(ii) *sgp*-T_1 space if for any pair of distinct points x and y there exist *sgp*-open sets G and H such that $x \in G$, $y \notin G$ and $x \notin H$, $y \in H$.

(iii) *sgp*-T_2 space if for each pair of distinct points x and y of X there exist disjoint *sgp*-open sets, one containing x and the other containing y.

Definition 2.6 ([9])**.** A function f :X→ Y is called *sgp*-continuous (briefly, *sgp*-continuous) if $f^{-1}(F)$ is *sgp*-closed set in X for every closed set F of Y.

Definition 2.7 ([1])**.** A function $f : X \to Y$ is said to be *sgp*-open (resp., *sgp*-closed) if $f(V)$ is *sgp*-open (resp., *sgp*-closed) in Y for every open set (resp., closed) V in X.

Definition 2.8 ([18])**.** (i) A topological space X is called Ultra Hausdroff space if every pair of distinct points of x and y in X there exist disjoint clopen sets U and V in X containing x and y respectively.

(ii) A topological space X is called Ultra normal if each pair of disjoint closed sets can be separated by disjoint clopen sets.

Definition 2.9 ([1])**.** A topological space X is said to be

(i) *sgp*-normal if each pair of disjoint closed sets can be separated by disjoint *sgp*-open sets.

(ii) *sgp*-connected if X cannot be written as union of two non empty disjoint *sgp*-open sets.

(iii) *sgp*-compact if every *sgp*-open cover of X has a finite subcover.

Definition 2.10 ([16])**.** A topological space X is said to be hyperconnected if every open set is dense.

Definition 2.11 ([11])**.** A space X is said to be weakly Hausdorff if each element of X is an intersection of regular closed sets.

Definition 2.12. A space X is said to be

(i) Nearly compact [16] if every regular open cover of X has a finite subcover.

(ii) Nearly countably compact [16] if every countable cover of X by regular open sets has a finite subcover.

(iii) Nearly Lindelöf [16] if every regular open cover of X has a countable subcover.

(iv) S-Lindelöf [3] if every cover of X by regular closed sets has a countable subcover.

(v) Countably S-closed [4] if every countable cover of X by regular closed sets has a finite subcover.

(vi) S-closed [20] if every regular closed cover of X has a finite subcover.

3. Contra *sgp*-Continuous Functions

In this section, the notion of a new class of function called contra *sgp*-continuous functions is introduced and obtain some of their characterizations and properties. Also, the relationships with some other existing functions are discussed.

Definition 3.1. A function $f : X \to Y$ is said to be contra *sgp*-continuous if $f^{-1}(F)$ is *sgp*-closed set in X for every open set F of Y.

Remark 3.2. From the following example it is clear that both contra *sgp*-continuous and *sgp*-continuous are independent notions of each other.

Example 3.3. Let $X = Y = \{a,b,c\}$, $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{a,c\}\}$. $\sigma = \{Y,\phi,\{a\},\{b\},\{a,b\}\}$. Define a map $f : X \to Y$ by $f(a) = b$, $f(b) = c$, $f(c) = a$. Then f is contra *sgp*-continuous but not continuous as for the closed set $\{b,c\}$ in Y, $f^{-1}(\{b,c\}) = \{a,b\}$ which is not *sgp*-closed in X.

Example 3.4. Let $X = Y = \{a,b,c\}$, $\tau = \{X,\phi,\{a\},\{a,b\}\}$ and $\sigma = \{Y,\phi,\{a\}\}$. Then the identity map $f : X \to Y$ is *sgp*-continuous but not contra *sgp*-continuous since for the open set $\{a\}$ in Y, $f^{-1}(\{a\}) = \{a\}$ which is not *sgp*-closed set in X.

Theorem 3.5. *If $f : X \to Y$ is contra continuous then f is contra* sgp*-continuous.*

Proof. Let V be an open set in Y. Since f is contra continuous, $f^{-1}(V)$ is closed in X. Since every closed set is *sgp*-closed, $f^{-1}(V)$ is *sgp*-closed in X. Therefore f is contra *sgp*-continuous. □

Remark 3.6. Converse of the above theorem need not be true in general as seen from the following example.

Example 3.7. In Example 3.3 the function f is contra *sgp*-continuous but not contra continuous since the open set $\{a\}$ in Y, $f^{-1}(\{a\}) = \{b\}$ is not closed in X.

Theorem 3.8. *For $x \in X$, $x \in \mathrm{sgp}\,\mathrm{Cl}(A)$ if and only if $U \cap A \neq \phi$ for every* sgp*-open set U containing x.*

Proof. Necessity: Suppose there exists a *sgp*-closed set U containing x such that $U \cap A = \phi$. Since $A \subset X - U$, $sgp\,\mathrm{Cl}(A) \subset X - U$. This implies $x \notin sgp\,\mathrm{Cl}(A)$, a contradiction.

Sufficiency: Suppose $x \notin sgp\,\mathrm{Cl}(A)$. Then there exists a *sgp*-closed subset F containing A such that $x \notin F$. Then $x \in X - F$ and $X - F$ is *sgp*-open, also $(X - F) \cap A = \phi$, a contradiction. □

Lemma 3.9 ([7])**.** *The following properties holds for subsets A and B of a space X*

(i) *$x \in \ker(A)$ if and only if $A \cap F \neq \phi$ for any closed set F of X containing x.*

(ii) *$A \subset \ker(A)$ and $A = \ker(A)$ if A is open in X.*

(iii) *If $A \subset B$, then $\ker(A) \subset \ker(B)$.*

Theorem 3.10. *If $f : X \to Y$ is a function then the following are equivalent*

(i) *f is contra* sgp*-continuous.*

(ii) *For every closed set F of Y, $f^{-1}(F)$ is* sgp*-open set of X.*

(iii) *For each $x \in X$ and each closed set F of Y containing $f(x)$ there exists* sgp*-open set U containing x such that $f(U) \subset F$.*

(iv) *$f(\mathrm{sgp}\,\mathrm{Cl}(A)) \subset \ker(f(A))$ for every subset A of X.*

(v) $\mathrm{sgp}\,\mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\ker(B))$.

Proof. The implications (i)→(ii) and (ii)→(iii) are obvious.

(iii)→(ii) Let F be closed set in Y containing $f(x)$. Then $x \in f^{-1}(F)$. From (iii), there exists *sgp*-open set Ux in X containing x such that $f(Ux) \subset F$. That is $Ux \subset f^{-1}(F)$. Thus $f^{-1}(F) = \cup\{Ux : x \in f^{-1}(F)\}$, which is union of *sgp*-open sets. Since union of *sgp*-open sets is a *sgp*-open set, $f^{-1}(F)$ is *sgp*-open set of X.

(ii)→(iv) Let A be any subset of X. Suppose $y \notin \ker(f(A))$. Then by Lemma 3.9, there exists a closed set F in Y containing $f(x)$ such that $f(A) \cap F = \phi$. Thus, $A \subset f^{-1}(F) = \phi$. Therefore $A \subset X - f^{-1}(F)$. By (ii), $f^{-1}(F)$ is *sgp*-open set in X and hence $X - f^{-1}(F)$ is *sgp*-closed set in X. Therefore $sgp\,\mathrm{Cl}(X - f^{-1}(F)) = X - f^{-1}(F)$. Now $A \subset X - f^{-1}(F)$, which implies $sgp\,\mathrm{Cl}(A) \subset sgp\,\mathrm{Cl}(X - f^{-1}(F)) = X - f^{-1}(F)$. Therefore $sgp\,\mathrm{Cl}(A) \cap f^{-1}(F) = \phi$ which implies $f(sgp\,\mathrm{Cl}(A)) \cap F = \phi$ and hence $y \notin sgp\,\mathrm{Cl}(A)$. Therefore $f(sgp\,\mathrm{Cl}(A)) \subset \ker(f(A)$ for every subset A of X.

(iv)→(v) Let F be closed subset of Y. By (iv) and by Lemma 3.9, we have $f(spgCl(f^{-1}(F)) \subset \ker(f(f^{-1}(F))) \subset \ker(F)$ and $sgp\,\mathrm{Cl}(f^{-1}(F)) \subset f^{-1}(\ker(F))$.

(v)→(i) Let V be any open subset of Y. Then, by Lemma 3.9, we have $sgp\,\mathrm{Cl}(f^{-1}(V)) \subset f^{-1}(\ker(V)) = f^{-1}(V)$ and $sgp\,\mathrm{Cl}(f^{-1}(V)) = f^{-1}(V)$. Thus $f^{-1}(V)$ is *sgp*-closed set in X. This shows that f is contra *sgp*-continuous. □

Remark 3.11. If a function $f : X \to Y$ is contra-*sgp*-continuous and X is ${}_{sgp}T_c$-space, then f is contra continuous.

Definition 3.12. A space X is called locally *sgp*-indiscrete space if every *sgp*-open set is closed in X.

Theorem 3.13. *If a function $f : X \to Y$ is contra-*sgp*-continuous and X is locally *sgp*-indiscrete space, then f is continuous.*

Proof. Let U be an open set in Y. Since f is contra *sgp*-continuous and X is locally *sgp*-indiscrete space, $f^{-1}(U)$ is an open set in X. Therefore f is continuous. □

4. Almost Contra *sgp*-Continuous Functions

In this section, new type of continuity called an almost contra *sgp*-continuity, which is weaker than contra *sgp*-continuity is introduced and studied some of their properties.

Definition 4.1. A function $f : X \to Y$ is said to be a almost contra-semi generalized pre-continuous (briefly, almost contra *sgp*-continuius) if $f^{-1}(V)$ is *sgp*-closed in X for each regular open set V in Y.

Theorem 4.2. *If $f : X \to Y$ is contra-*sgp*-continuous then it is almost contra-*sgp*-continuous.*

Proof. Follows from every regular open set is open set. □

Remark 4.3. Converse of the above theorem is not true as seen from the following example.

Example 4.4. Let $X = \{a,b,c\} = Y$, $\tau = \{X,\phi,\{a,b\}\}$ and $\sigma = \{Y,\phi,\{a\},\{b\},\{a,b\},\{a,c\}\}$. Then the identity map $f : X \to Y$ is almost contra-*sgp*-continuous but not contra-*sgp*-continuous as open set $\{a,b\}$ in Y, $f^{-1}(\{a,b\}) = \{a,b\}$ which is not *sgp*-closed set in X.

Theorem 4.5. *If X is ${}_{\mathrm{sgp}}T_c$-space and $f : X \to Y$ is almost contra-*sgp*-continuous then f is almost contra continuous.*

Proof. Let U be a regular open set in Y. Since f is almost contra *sgp*-continuous $f^{-1}(U)$ is *sgp*-closed set in X and X is ${}_{sgp}T_c$-space, which implies $f^{-1}(U)$ is closed set in X. Therefore f is almost contra continuous. □

Definition 4.6 ([10]). A function $f : X \to Y$ is said to be almost continuous if $f^{-1}(V)$ is open in X for each regular open set V in Y.

Theorem 4.7. *If a function $f : X \to Y$ is almost contra-*sgp*-continuous and X is locally *sgp*-indiscrete space, then f is almost continuous.*

Proof. Let U be a regular open set in Y. Since f is almost contra *sgp*-continuous $f^{-1}(U)$ is *sgp*-closed set in X and X is locally *sgp*-indiscrete space, which implies $f^{-1}(U)$ is an open set in X. Therefore f is almost continuous. □

Remark 4.8. If $f : X \to Y$ is almost contra *sgp*-continuous and X is ${}_{sgp}T_c$-space then f is almost contra continuous.

Theorem 4.9. *For a function $f : X \to Y$ the followings are equivalent:*

(i) *f is almost contra* sgp*-continuous.*

(ii) *For every regular closed set F of Y, $f^{-1}(F)$ is* sgp*-open set of X.*

Proof. (i)→(ii) Let F be a regular closed set in Y, then $Y - F$ is a regular open set in Y. By (i), $f^{-1}(Y - F) = X - f^{-1}(F)$ is *sgp*-closed set in X. This implies $f^{-1}(F)$ is *sgp*-open set in X. Therefore, (ii) holds.

(ii)→(i) Let G be a regular open set of Y. Then $Y - G$ is a regular closed set in Y. By (ii), $f^{-1}(Y - G)$ is open set in X. This implies $X - f^{-1}(G)$ is *sgp*-open set in X, which implies $f^{-1}(G)$ is *sgp*-closed set in X. Therefore, (i) holds. □

Theorem 4.10. *For a function $f : X \to Y$ the followings are equivalent:*

(i) *f is almost contra* sgp*-continuous.*

(ii) *$f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))$ is* sgp*-closed set in X for every open subset G of Y.*

(iii) *$f^{-1}(\mathrm{Cl}(\mathrm{Int}(F)))$ is* sgp*-open set in X for every closed subset F of Y.*

Proof. (i)→(ii) Let G be an open set in Y. Then $\mathrm{Int}(\mathrm{Cl}(G))$ is regular open set in Y. By (i), $f^{-1}(\mathrm{Int}(\mathrm{Cl}(G))) \in SGPC(X)$.

(ii)→(i) Proof is obvious.

(i)→(iii) Let F be a closed set in Y. Then $\mathrm{Cl}(\mathrm{Int}(G))$ is regular closed set in Y. By (i), $f^{-1}(\mathrm{Cl}(\mathrm{Int}(G))) \in SGPO(X)$.

(iii)→(i). Proof is obvious. □

Theorem 4.11. *If $f : X \to Y$ is an almost contra* sgp*-continuous injection and Y is Weakly Hausdorff then X is* sgp*-T_1.*

Proof. For any distinct points X and Y in X, there exist V and W regular closed sets in Y such that $f(x) \in V$, $f(y) \notin V$, $f(y) \in W$ and $f(x) \notin W$ as Y is Weakly Hausdorff. Since f is almost contra *sgp*-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are *sgp*-open subsets of X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $y \in f^{-1}(W)$ and $x \notin f^{-1}(W)$. This shows that X is *sgp*-T_1. □

Corollary 4.12. *If $f : X \to Y$ is a contra* sgp*-continuous injection and Y is weakly Hausdorff then X is* sgp*-T_1.*

Theorem 4.13. *If $f : X \to Y$ is an almost contra* sgp*-continuous injective function from a space X into an Ultra Hausdroff space Y then X is* sgp*-T_2.*

Proof. Let X and Y be any two distinct points in X. Since f is an injective $f(x) \neq f(y)$ and Y is Ultra Hausdroff space, there exist disjoint clopen sets U and V of Y containing $f(x)$ and $f(y)$ respectively. Then $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint *sgp*-open sets in X. Therefore X is *sgp*-T_2. □

Theorem 4.14. *If $f : X \to Y$ is an almost contra* sgp*-continuous closed injection and Y is ultra normal then X is* sgp*-normal.*

Proof. Let E and F be disjoint closed subsets of X. Since f is closed and injective $f(E)$ and $f(F)$ are disjoint closed sets in Y. Since Y is ultra normal there exist disjoint clopen sets U and V in Y such that $f(E) \subset U$ and $f(F) \subset V$. This implies $E \subset f^{-1}(U)$ and $F \subset f^{-1}(V)$. Since f is an almost contra *sgp*-continuous injection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint *sgp*-open sets in X. This shows X is *sgp*-normal. □

Theorem 4.15. *If $f : X \to Y$ is an almost contra* sgp*-continuous surjection and X is* sgp*-connected space then Y is connected.*

Proof. Let $f : X \to Y$ be an almost contra *sgp*-continuous surjection and X is *sgp*-connected space. Suppose Y is a not a connected space. Then there exist disjoint open sets U and V such that $Y = U \cup V$. Therefore U and V are clopen in Y. Since f is almost contra-*sgp*-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are *sgp*-open sets in X. Moreover $f^{-1}(U)$ and $f^{-1}(V)$ are non empty disjoint and $X = f^{-1}(U) \cup f^{-1}(V)$. This is contradiction to the fact that X is *sgp*-connected space. Therefore Y is connected. □

Definition 4.16 ([11])**.** A function $f : X \to Y$ is said to be perfectly continuous if $f^{-1}(V)$ is clopen in X for each open set V of Y.

Theorem 4.17. *For two functions $f : X \to Y$ and $g : Y \to Z$, let $g \circ f : X \to Z$ is a composition of f and g. Then the following properties hold:*

(i) *If f is almost contra-*sgp*-continuous and g is an R-map then $g \circ f$ is almost contra-*sgp*-continuous.*

(ii) *If f is almost contra-*sgp*-continuous and g is perfectly continuous then $g \circ f$ is* sgp*-continuous and contra-*sgp*-continuous.*

(iii) *If f is contra-*sgp*-continuous and g is almost continuous then $g \circ f$ is almost contra-*sgp*-continuous.*

Proof. (i) Let V be any regular open set in Z. Since g is an R-map, $g^{-1}(V)$ is regular open in Y. Since f is an almost contra-*sgp*-continuous $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is *sgp*-closed set in X. Therefore $g \circ f$ is almost contra-*sgp*-continuous.

(ii) Let V be any open set in Z. Since g is perfectly continuous, $g^{-1}(V)$ is clopen in Y. Since f is an almost contra-*sgp*-continuous $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is *sgp*-open and *sgp*-closed set in X. Therefore, $g \circ f$ is *sgp*-continuous and contra-*sgp*-continuous.

(iii) Let V be any regular open set in Z. Since g is almost continuous, $g^{-1}(V)$ is open in Y. Since f is contra *sgp*-continuous $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is *sgp*-closed set in X. Therefore $g \circ f$ is almost contra-*sgp*-continuous. □

Theorem 4.18. *Let $f : X \to Y$ is a contra-*sgp*-continuous and $g : Y \to Z$ is* sgp*-continuous. If Y is ${}_{\text{sgp}}T_c$-space then $g \circ f : X \to Z$ is an almost contra* sgp*-continuous.*

Proof. Let V be any regular open and hence open set in Z. Since g is sgp-continuous $g^{-1}(V)$ is sgp-open in Y and Y is ${}_{sgp}T_c$-space implies $g^{-1}(V)$ open in Y. Since f is contra-sgp-continuous $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is sgp-closed set in X. Therefore $g\circ f$ is an almost contra-sgp-continuous. □

Definition 4.19. A function $f : X \to Y$ is said to be strongly sgp-open (resp. strongly sgp-closed) if image of every sgp-open (resp. sgp-closed) set of X is sgp-open (resp. sgp-closed) set in Y.

Theorem 4.20. *If $f : X \to Y$ is surjective strongly* sgp*-open (or strongly* sgp*-closed) and $g : Y \to Z$ is a function such that $g\circ f : X \to Z$ is an almost contra* sgp*-continuous then g is an almost contra-*sgp*-continuous.*

Proof. Let V be any regular closed (resp. regular open) set in Z. Since $g\circ f$ is an almost contra sgp-continuous, $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is sgp-open (resp. sgp-closed) in X. Since f is surjective and strongly sgp-open (or strongly sgp-closed), $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is sgp-open(or sgp-closed). Therefore g is an almost contra-sgp-continuous. □

Definition 4.21. A topological space X is said to be sgp-ultra-connected if every two non empty sgp-closed subsets of X intersect.

Theorem 4.22. *If X is* sgp*-ultra-connected and $f : X \to Y$ is an almost contra-*sgp*-continuous surjection then Y is hyperconnected.*

Proof. Let X be a sgp-ultra-connected and $f : X \to Y$ is an almost contra-sgp-continuous surjection. Suppose Y is not hyperconnected. Then there exists an open set V such that V is not dense in Y. Therefore, there exist nonempty regular open subsets $B_1 = \mathrm{Int}(\mathrm{Cl}(V))$ and $B_2 = Y - \mathrm{Cl}(V)$ in Y. Since f is an almost contra-sgp-continuous surjection, $f^{-1}(B_1)$ and $f^{-1}(B_2)$ are disjoint sgp-closed sets in X. This is contrary to the fact that X is sgp-ultra-connected. Therefore Y is hyperconnected. □

Definition 4.23. A space X is said to be

(i) Countably sgp-compact if every countable cover of X by sgp-open sets has a finite subcover.

(ii) sgp-Lindelöf if every sgp-open cover of X has a countable subcover.

(iii) mildly sgp-compact if every sgp-clopen cover of X has a finite subcover.

(iv) mildly countably sgp-compact if every countable cover of X by sgp-clopen sets has a finite subcover.

(v) mildly sgp-Lindelöf if every sgp-clopen cover of X has a countable subcover.

Theorem 4.24. *Let $f : X \to Y$ be an almost contra* sgp*-continuous surjection. Then the following properties hold.*

(i) *If X is* sgp*-compact then Y is S-closed.*

(ii) *If X is countably* sgp*-closed then Y is is countably S-closed.*

(iii) *If X is* sgp*-Lindelöf then Y is S-Lindelöf.*

Proof. (i) Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra *sgp*-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is *sgp*-open cover of X. Since X is *sgp*-compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite subcover for Y. Therefore Y is S-closed.

(ii) Let $\{V_\alpha : \alpha \in I\}$ be any countable regular closed cover of Y. Since f is almost contra *sgp*-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is countable *sgp*-open cover of X. Since X is countably *sgp*-compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite subcover for Y. Therefore Y is countably S-closed.

(iii) Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra *sgp*-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is *sgp*-open cover of X. Since X is *sgp*-Lindelöf, there exists a countable subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite subcover for Y. Therefore Y is S-Lindelöf. □

Definition 4.25. A function $f : X \to Y$ is said to be almost *sgp*-continuous if $f^{-1}(V)$ is *sgp*-open in X for each regular open set V of Y.

Theorem 4.26. *Let $f : X \to Y$ be an almost contra-*sgp*-continuous and almost *sgp*-continuous surjection. Then the following properties hold.*

(i) *If X is mildly *sgp*-closed then Y is nearly compact.*

(ii) *If X is mildly countably *sgp*-compact then Y is nearly countabaly compact.*

(iii) *If X is mildly *sgp*-Lindelöf then Y is nearly Lindelöf.*

Proof. Proof is similar to Theorem 4.24. □

5. *sgp*-Regular Graphs and Strongly Contra-*sgp*-Closed Graphs

In this section, we define the notions of *sgp*-regular graphs and strongly contra-*sgp*-closed graphs and investigate the relationships between the graphs and almost contra-*sgp*-continuous functions.

Recall that, for a function $f : X \to Y$, the subset $G_f = \{x, f(x) : x \in X\} \subset X \times Y$ is said to be graph of f.

Definition 5.1. A graph G_f of a function $f : X \to Y$ is said to be *sgp*-regular (resp. strongly contra-*sgp*-closed) if for each $(x, y) \in (X \times Y) - G_f$, there exist a *sgp*-closed (resp. *sgp*-open) set U in X containing x and $V \in RO(Y)$ (resp. $V \in RC(Y)$ containing y such that $(U \times V) \cap G_f = \phi$.

Theorem 5.2. *For a graph G_f of a function $f : X \to Y$ the following properties are equivalent:*

(i) *G_f is *sgp*-regular (resp. strongly contra *sgp*-closed);*

(ii) *For each point $(x, y) \in (X \times Y) - G_f$, there exist *sgp*-closed (resp. *sgp*-open) set U in X containing x and $V \in RO(Y)$ (resp. $V \in RC(Y)$) containing y such that $f(U) \cap V = \phi$.*

Proof. Follows from the Definition 5.1 and the fact that for any subsets $A \subset X$ and $B \subset Y$, $(A \times B) \cap G_f = \phi$ if and only if $f(A) \cap B = \phi$. □

Proof. Let $(x,y) \in (X \times Y) - G_f$. It follows that $f(x) \neq y$. Since Y is T_2, there exist regular open sets V and W such that $f(x) \in V$, $y \in W$ and $V \cap W = \phi$. Since f is almost contra-*sgp*-continuous function, $f^{-1}(V)$ is a *sgp*-closed set in X containing x. Let $U = f^{-1}(V)$, then we have $f(U) \subset V$. Therefore, $f(U) \cap W = \phi$ and G_f is *sgp*-regular. □

Theorem 5.3. *Let $f : X \to Y$ be a* sgp*-regular graph G_f. If f is injective then X is* sgp-T_0.

Proof. Let x and y be any two distinct points of X. Then, we have $(x, f(y)) \in (X \times Y) - G_f$. Since G_f is *sgp*-regular, then there exist a *sgp*-closed set U of X and $V \in RO(Y)$ such that $(x, f(y)) \in (U \times V)$ and $f(U) \cap = \phi$ by Theorem 5.2 and hence $U \cap f^{-1}(V) = \phi$. Therefore, $y \notin U$. Thus, $y \in (X - U)$ and $x \notin (X - U)$. We get $(X - U) \in SGPO(X)$. This implies that X is *sgp*-T_0. □

Remark 5.4. Let $f : X \to Y$ be a *sgp*-regular graph G_f. If f is surjective then Y is weakly Hausdorff.

Theorem 5.5. *Let $f : X \to Y$ be a strongly contra-*sgp *graph G_f. If f is almost contra-*sgp*-continuous injection then X is* sgp-T_2.

Proof. Let x and y be any two distinct points of X. Since f is injective, we have $f(x) \neq f(y)$. Then, $(x, f(y)) \in (X \times Y) - G_f$. Since G_f is strongly contra-*sgp*-closed, by Theorem 5.2, we have $U \in SGPO(X, x)$ and a regular closed set V containing $f(y)$ such that $f(U) \cap V = \phi$. Therefore, $U \cap f^{-1}(V) = \phi$. Since f is almost contra-*sgp*-continuous function, $f^{-1}(V) \in SGPO(X, y)$. This implies X is *sgp*-T_2. □

6. Conclusion

Sets and functions in topological spaces are developed and used in many engineering problems, information systems and computational topology. By researching generalizations of closed sets, some new separation axioms have founded and are turned to be useful in the study of digital topology. Therefore, almost contra-*sgp*-continuous functions defined using *sgp*-closed set will have many possibilities of application in computer graphics and digital topology.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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