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Research Article

The Wiener Index for Neutrosophic Labeling Graphs and Its Application in River Ecosystem Analysis

M. Gomathi[1], S. Prema*[2] and N. Preethi[3]

[1] *Department of Mathematics, Bannari Amman Institute of Technology (affiliated to Anna University), Sathyamangalam 638401, Erode, Tamil Nadu, India*
[2] *Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women (Deemed to be University), Coimbatore 641043, Tamil Nadu, India*
[3] *Department of Science and Humanities, Sri Krishna College of Engineering and Technology (affiliated to Anna University), Coimbatore 641008, Tamil Nadu, India*
***Corresponding author:** premaparameswaran14@gmail.com

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Abstract. This paper explores several topological indices in neutrosophic labeling graphs, including Randić's, Zagreb, connectivity, and harmonic indices. Theorems linked to concepts are proved using appropriate instances. Application of the Wiener index is analyzed in river basin ecosystems across different seasons to understand seasonal changes in the ecosystem's structure and interactions. This approach gives detailed information about the uncertainties and natural variations in river systems.

Keywords. Neutrosophic labeling graphs, Randić index, Zagreb indices, Connectivity index, Harmonic index, Neutrosophic labeling Weiner index

Mathematics Subject Classification (2020). 18B30, 03E72

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1. Introduction

Traditional graph theory, rooted in classical set theory, offers a binary perspective where objects are either present or absent, thus failing to effectively address this uncertainty. To overcome these limitations, fuzzy set theory, introduced by Zadeh [14], and further, Bellman and Zadeh [6] provided a means to handle uncertainty by allowing partial membership of elements in sets. This development led to significant advancements in decision-making in uncertain environments. Subsequent research by Ashraf *et al*. [4], and Mahmood *et al*. [12] extended these concepts to spherical fuzzy sets, demonstrating their applicability in multi-attribute decision-making problems and medical diagnosis.

Building on the foundations of fuzzy sets, Atanassov [5] introduced intuitionistic fuzzy sets, which incorporate degrees of membership, non-membership, and hesitation, offering a more nuanced approach to uncertainty. Neutrosophic sets, proposed by Smarandache [13], generalize intuitionistic fuzzy sets by introducing a third component, indeterminacy, which provides a more comprehensive framework for modeling uncertainty. This advancement has paved the way for the development of neutrosophic graphs, which enhance traditional fuzzy and intuitionistic fuzzy graphs by incorporating neutrosophic labels on edges and vertices. These graphs are particularly effective in capturing the degrees of truth, indeterminacy, and falsity simultaneously (Alblowi and Salama [1] and Ansari *et al*. [3]).

Recent research has applied neutrosophic graphs in various domains. For instance, Ghods and Rostami [10, 11] explored topological indices, including the Wiener Index, in neutrosophic graphs, highlighting their potential in wireless networks and pattern recognition. Similarly, Fei [9] demonstrated the application of neutrosophic graphs in wireless network analysis. Further extended to intuitionistic fuzzy graphs (Davvaz *et al*. [8]) and neutrosophic graphs (Ghods and Rostami [11]). These adaptations have significantly enhanced the analytical capabilities of the Wiener Index, making it a robust tool for analyzing complex systems characterized by uncertainty.

In this paper, we aim to explore the application of neutrosophic topological indices, including the Ranic index, Zagreb index, harmonic index, and Wiener index by using neutrosophic labeling graphs. Neutrosophic labeling graph is used to give more detailed way to characterize uncertainty.

2. Preliminaries

Definition 2.1 ([7])**.** A *neutrosophic graph* (NG) of pair $G^* = (Y, Z)$ is called an *neutrosophic labeling graph* (NLG) if

(i) $v = \{\ell_{p_1}, \ell_{p_2}, \dots, \ell_{p_n}\}$ with $v \xrightarrow{T_a^Y} [0,1]$, $v \xrightarrow{I_b^Y} [0,1]$, $v \xrightarrow{F_c^Y} [0,1]$ denotes the truth-membership, indeterminacy-membership, and falsity-membership functions $0 \le T_a^Y(\ell_{p_i}) + I_b^Y(\ell_{p_i}) + F_c^Y(\ell_{p_i}) \le 3$, for each $\ell_{p_i} \in v$.

(ii) $E \subseteq v \times v$ with $E \xrightarrow{T_a^Y} [0,1]$, $E \xrightarrow{I_b^Y} [0,1]$, $E \xrightarrow{F_c^Y} [0,1]$ as

$$T_a^Z(\ell_{p_i}, \ell_{p_j}) \le \min[T_a^Y(\ell_{p_i}), T_a^Y(\ell_{p_j})],$$
$$I_b^Z(\ell_{p_i}, \ell_{p_j}) \le \min[I_b^Y(\ell_{p_i}), I_b^Y(\ell_{p_j})],$$
$$F_c^Z(\ell_{p_i}, \ell_{p_j}) \ge \max[T_c^Y(\ell_{p_i}), T_c^Y(\ell_{p_j})]$$

and $0 \le T_a^Z(\ell_{p_i}) + I_b^Z(\ell_{p_i}) + F_c^Z(\ell_{p_i}) \le 3$, for all edges $(\ell_{p_i}, \ell_{p_j}) \in E$.

3. Topological Indices in Neutrosophic Labeling Graph $G^* = (v, \sigma, \mu)$

The following notations are used in this paper

Notation	*Abbreviation*
R_{G^*}	Randić index
H_{G^*}	Harmonic index

Notation	*Abbreviation*
$M1_{G^*}$	1st-Zagreb index
$M2_{G^*}$	2nd-Zagreb index
C_{G^*}	Connectivity index
d^*	Degree of vertices
d_2	Second degree of vertices
C_s	Strength of connectedness

Definition 3.1. Let $G^* = (v, \sigma, \mu)$ be the neutrosophic labeling graph, then the constant R_{G^*} given by

$$R_{G^*} = \sum \frac{1}{\sqrt{t(T_1(\ell_i), I_1(\ell_i)F_1(\ell_i)) \cdot d(\ell_i) \times (T_1(\ell_j), I_1(\ell_j)F_1(\ell_j)) \cdot d(\ell_j)}},$$

where $\ell_i \neq \ell_j$, $(\ell_i, \ell_j) \in v$, denotes the *Randić index* of neutrosophic labeling graph.

Example 3.1. Let $G^* = (v, \sigma, \mu)$ be the neutrosophic labeling graph with 4 vertices $v = \{a, b, c, d\}$ and 5 edges $E = \{(a,b),(b,a),(b,c),(c,d),(d,c)\}$ whose membership, non-membership and Indeterminacy function values are given in the following graph:

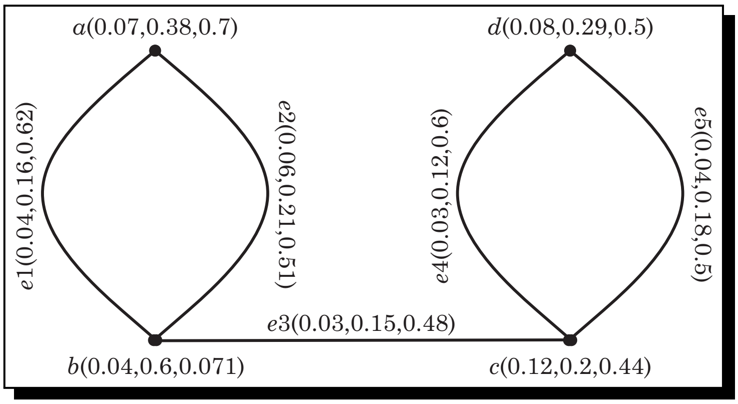

Figure 1. Neutrosophic labeling graph

For Figure 1,

$$d^*(a) = (0.04 + 0.06, 0.16 + 0.21, 0.62 + 0.51) = (0.1, 0.37, 1.13),$$

$$d^*(b) = (0.04 + 0.06 + 0.03, 0.16 + 0.21 + 0.15, 0.62 + 0.51 + 0.48) = (0.13, 0.52, 1.61),$$

$$d^*(c) = (0.03 + 0.04 + 0.03, 0.12 + 0.18 + 0.15, 0.6 + 0.5 + 0.48) = (0.1, 0.45, 1.58),$$

$$d^*(d) = (0.03 + 0.04, 0.12 + 0.18, 0.6 + 0.5) = (0.07, 0.3, 1.1).$$

Then, the Randić index is given by

$$R_{G^*} = \frac{1}{\sqrt{(0.07,0.38,0.7)\cdot(0.1,0.37,1.13) \times (0.04,0.6,0.071)\cdot(0.13,0.52,1.61)}}$$
$$+ \frac{1}{\sqrt{(0.04,0.6,0.071)\cdot(0.13,0.52,1.61) \times (0.12,0.2,0.44)\cdot(0.1,0.45,1.58)}}$$
$$+ \frac{1}{\sqrt{(0.12,0.2,0.44)\cdot(0.1,0.45,1.58) \times (0.08,0.29,0.5)\cdot(0.07,0.3,1.1)}},$$

$$
\begin{aligned}
R_{G^*} &= \frac{1}{\sqrt{0.9386\times 0.43151}} + \frac{1}{\sqrt{0.43151\times 0.7972}} + \frac{1}{\sqrt{0.7972\times 0.6426}}\\
&= \frac{1}{0.6364} + \frac{1}{0.5865} + \frac{1}{0.7157}\\
&= 4.6736.
\end{aligned}
$$

Definition 3.2. The *harmonic index* H_{G^*} of *neutrosophic labeling graph* (NLG) $G^* = (\nu,\sigma,\mu)$ is simply like the harmonic mean of the labeling values of adjacent vertices along with its vertex degrees,

$$
H_{G^*} = \sum \frac{1}{(T_1(\ell_i), I_1(\ell_i)F_1(\ell_i))\cdot d(\ell_i) + (T_1(\ell_j), I_1(\ell_j)F_{1i}(\ell_j))\cdot d(\ell_j)},
$$

where $\ell_i \neq \ell_j$, $(\ell_i,\ell_j)\in \nu$ and $\cdot$ is the scalar product operator.

Example 3.2. For the graph (Figure 1) taken in Example 3.1, the harmonic index is given by

$$
\begin{aligned}
H_{G^*} &= \frac{1}{(0.07,0.38,0.7)\cdot(0.1,0.37,1.13)+(0.04,0.6,0.071)\cdot(0.13,0.52,1.61)}\\
&\quad + \frac{1}{(0.04,0.6,0.071)\cdot(0.13,0.52,1.61)+(0.12,0.2,0.44)\cdot(0.1,0.45,1.58)}\\
&\quad + \frac{1}{(0.12,0.2,0.44)\cdot(0.1,0.45,1.58)+(0.08,0.29,0.5)\cdot(0.07,0.3,1.1)}\\
&= \frac{1}{0.9386+0.43151} + \frac{1}{0.43151+0.7972} + \frac{1}{0.7972+0.6426}\\
&= \frac{1}{1.37} + \frac{1}{1.229} + \frac{1}{1.4398}\\
&= 2.2381.
\end{aligned}
$$

Definition 3.3. For the neutrosophic labeling graph $G^* = (\nu,\sigma,\mu)$, the *first Zagreb index*, $M1_{G^*}$ is defined as $M1_{G^*} = \sum_{i=1}^{n}(T_1(\ell_i), I_1(\ell_i), F_1(\ell_i)\cdot d_2^*(\ell_i))$, for each $\ell_i \in \nu$ and the *second Zagreb index*, $M2_{G^*}$ is defined as $M2_{G^*} = \frac{1}{2}\sum_{i=1}^{n}(T_1(\ell_i), I_1(\ell_i), F_1(\ell_i)\cdot d(\ell_i)\times T_1(\ell_j), I_1(\ell_j), F_1(\ell_j)\cdot d(\ell_j))$, where $\ell_i \neq \ell_j$, $(\ell_i,\ell_j)\in\nu$.

Example 3.3. Let $G^* = (\nu,\sigma,\mu)$ be the neutrosophic labeling graph with 4 vertices $V = \{\text{I},\text{II},\text{II},\text{IV}\}$ whose labeling values are

$(T_1(\text{I}), I_1(\text{I}), F_1(\text{I})) = (0.21, 0.05, 0.3)$, $\quad (T_1(\text{II}), I_1(\text{II}), F_1(\text{II})) = (0.18, 0.17, 0.42)$,

$(T_1(\text{III}), I_1(\text{III}), F_1(\text{III})) = (0.08, 0.21, 0.52)$, $\quad (T_1(\text{IV}), I_1(\text{IV}), F_1(\text{IV})) = (0.31, 0.09, 0.4)$,

G^* consists of 3 edges $E = \{e1 = (\text{I},\text{II}), e2 = (\text{II},\text{III}), e3 = (\text{III},\text{IV}), e4 = (\text{II},\text{IV})\}$ whose labeling values are $(T_2(\text{I},\text{II}), \text{I}_2(\text{I},\text{II}), F_2(\text{I},\text{II})) = (0.11, 0.09, 0.24)$, $(T_2(\text{II},\text{III}), I_2(\text{II},\text{III}), F_2(\text{II},\text{III})) = (0.26, 0.13, 0.38)$, $(T_2(\text{II}, IV), \text{I}_2(\text{II},\text{IV}), F_2(\text{II},\text{IV})) = (0.17, 0.11, 0.38)$, and $(T_2(\text{III},\text{IV}), \text{I}_2(\text{III},\text{IV}), F_2(\text{III},\text{IV})) = (0.07, 0.19, 0.56)$.

For Figure 2, the degree of vertices are given by

$$
\begin{aligned}
d^*(\text{I}) &= (0.11, 0.09, 0.24),\\
d^*(\text{II}) &= (0.11+0.26+0.17, 0.09+0.13+0.11, 0.24+0.38+0.38) = (0.54, 0.33, 1),\\
d^*(\text{III}) &= (0.26+0.07, 0.13+0.19, 0.38+0.56) = (0.33, 0.32, 0.94),\\
d^*(\text{IV}) &= (0.07+0.17, 0.19+0.11, 0.56+0.38) = (0.24, 0.3, 0.94)
\end{aligned}
$$

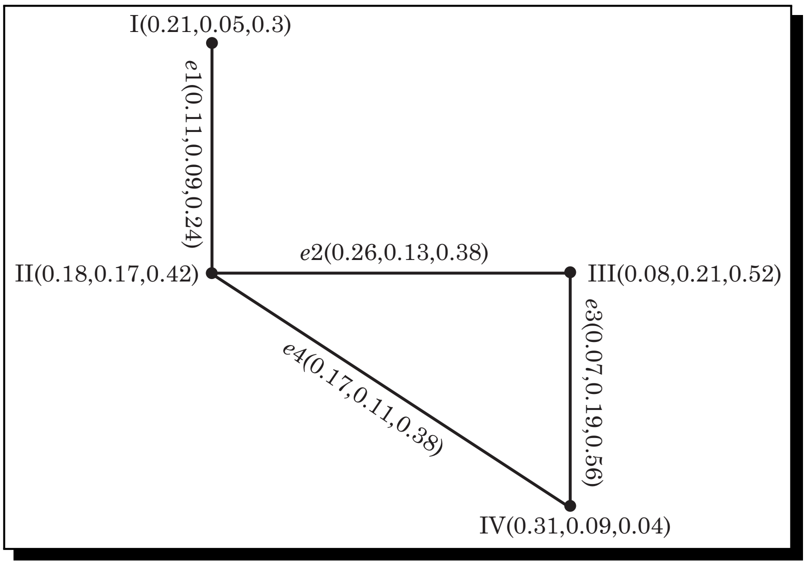

Figure 2. Neutrosophic labeling graph

and the 2-degree of the vertices are given by

$$d_2^*(\mathrm{I}) = 0,$$
$$d_2^*(\mathrm{II}) = (0.26^2+0.17^2, 0.13^2+0.11^2, 0.38^2+0.38^2) = (0.0965, 0.029, 0.2888),$$
$$d_2^*(\mathrm{III}) = (0.26^2+0.07^2, 0.13^2+0.19^2, 0.38^2+0.56^2) = (0.0725, 0.053, 0.458),$$
$$d_2^*(\mathrm{IV}) = (0.17^2+0.07^2, 0.11^2+0.19^2, 0.38^2+0.56^2) = (0.0338, 0.0482, 0.458).$$

Then, the Zagreb indices are

$$\begin{aligned} M1_{G^*} &= (0.18,0.17,0.42)\cdot(0.0965,0.029,0.2888)+(0.08,0.21,0.52)\cdot(0.0725,0.053,0.458) \\ &\quad +(0.31,0.09,0.4)\cdot(0.0338,0.0482,0.458) \\ &= 0.1436+0.2551+0.198 \\ &= 0.5967, \end{aligned}$$

$$\begin{aligned} M2_{G^*} &= \frac{1}{2}[(0.21,0.05,0.3)\cdot(0.11,0.09,0.24)\times(0.18,0.17,0.42)\cdot(0.54,0.33,1) \\ &\quad +(0.18,0.17,0.42)\cdot(0.54,0.33,1)\times(0.08,0.21,0.52)\cdot(0.33,0.32,0.94) \\ &\quad +(0.18,0.17,0.42)\cdot(0.54,0.33,1)\times(0.31,0.09,0.4)\cdot(0.24,0.3,0.94) \\ &\quad +(0.08,0.21,0.52)\cdot(0.33,0.32,0.94)\times(0.31,0.09,0.4)\cdot(0.24,0.3,0.94)] \\ &= \frac{1}{2}[(0.0996\times0.5733)+(0.5733\times0.5824)+(0.5733\times0.4774)+(0.4774\times0.5824)] \\ &= 0.4714. \end{aligned}$$

Definition 3.4. The sum of components of maximum of all the strength of path connecting the vertices ℓ_i and ℓ_j is called the strength of connectedness between the vertices ℓ_i and ℓ_j and it is denoted by $C_s(\ell_i,\ell_j)$.

Definition 3.5. Let $G^* = (\nu,\sigma,\mu)$ be the neutrosophic labeling graph, then the *Connectivity index* of G^*, denoted by C_{G^*} and it is defined as

$$C_{G^*} = \sum_{\ell_i,\ell_j\in\nu} ((T_1(\ell_i),I_1(\ell_i),F_1(\ell_i))\cdot(T_1(\ell_j),I_1(\ell_j),F_1(\ell_j)))\times C_s(\ell_i,\ell_j).$$

Example 3.4. Let the neutrosophic labeling graph given below:

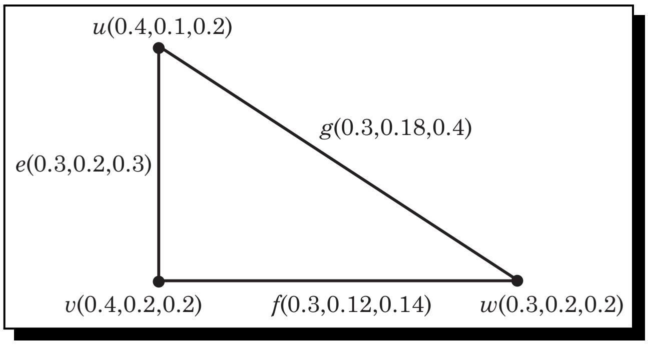

Figure 3. Neutrosophic labeling graph

For Figure 3, the strength of neutrosophic connectedness of respective vertices is given as follows:

$$C_s(u,v) = \max\{(0.3,0.2,0.3),(0.3,0.12,0.14)\} = (0.3,0.2,0.3),$$
$$C_s(v,w) = \max\{(0.3,0.12,0.14),(0.3,0.18,0.4)\} = (0.3,0.18,0.4),$$
$$C_s(u,w) = \max\{(0.3,0.18,0.4),(0.3,0.12,0.14)\} = (0.3,0.18,0.4).$$

Then, the neutosophic labeling Connectivity index of G^* for this graph is given by

$$\begin{aligned} C_{G^*} &= ((0.4,0.1,0.2)\cdot(0.4,0.2,0.2))\times(0.3+0.2+0.3)+((0.4,0.1,0.2)\cdot(0.3,0.2,0.3)) \\ &\quad \times(0.3+0.18+0.4)+((0.4,0.2,0.2)\cdot(0.3,0.2,0.3))\times(0.3+0.18+0.4) \\ &= t((0.16+0.02+0.04)\times 0.8)+((0.12+0.02+0.06)\times 0.88)+((0.12+0.04+0.06)\times 0.88) \\ &= 0.176+0.176+0.1936 \\ &= 0.5456. \end{aligned}$$

Theorem 3.1. *The topological indices remain the same for two isomorphic neutrosophic labeling graphs.*

Proof. Let $G_1^* = (v_1,\sigma_1,\mu_1)$ and $G_2^* = (v_2,\sigma_2,\mu_2)$ be the two isomorphic neutrosophic labeling graphs. Then by definition there exists a bijective function (simply identity function) which maps each vertex of G_1^* onto the corresponding vertex of G_2^* with same membership values and adjacent edges that only differs in structure which leads to the conclusion that the topological values of those two graphs remains the same. □

Theorem 3.2. *The Zagreb indices value gets reduced for the neutrosophic labeling sub graphs obtained by eliminating vertex.*

Proof. Let $G^* = (v,\sigma_1,\mu_1)$ and $H^* = G^* - v$ be the sub graph of G^* obtained by eliminating the vertex v. Since, the vertex v is removed from G^*, all the edges adjacent to v also gets eliminated. Then by the definition of Zagreb indices $M1_{H^*}$ and $M2_{H^*}$ were obtained by removing the corresponding membership values from the Zagreb indices $M1_{G^*}$ and $M2_{G^*}$ which in turn reduces the sum value. Hence, $M1_{H^*} < M1_{H^*}$ and $M2_{H^*} < M2_{H^*}$. □

Theorem 3.3. *The connectivity index value gets reduced for the neutrosophic labeling sub graphs obtained by eliminating the edge (ℓ_i,ℓ_j) if and only if (ℓ_i,ℓ_j) is a bridge of neutrosophic labeling graph $G^* = (v,\sigma_1,\mu_1)$.*

Proof. Let $G^* = (v, \sigma_1, \mu_1)$ and $H^* = G^* - (\ell_i, \ell_j)$ be the sub graph of G^* obtained by eliminating the edge (ℓ_i, ℓ_j). Assume that $C_{H^*} < C_{G^*}$. This implies that the value $C_s(\ell_i, \ell_j)$ has been neglected from the connectivity index C_{G^*}, which implies that the edge (ℓ_i, ℓ_j) has maximum component values. Hence, (ℓ_i, ℓ_j) is a bridge of *neutrosophic labeling graph* (NLG) $G^* = (v, \sigma_1, \mu_1)$. □

4. Wiener Index for Neutrosophic Labeling Graphs

In this section, we proposed a geodesic path and weiner index for neutrosophic labeling graph are investigated.

Definition 4.1. A geodesic path between two vertices in a graph is defined by two main attributes:

Strong Path: This means the path itself is robust, composed of edges with high membership values or certainty.

Minimality: It is the shortest path between the vertices, ensuring there is no other shorter path with higher membership values.

In this framework, the weights associated with a geodesic path are determined by summing the truth, indeterminacy, and falsity membership values for all arcs that make up the path. Specifically:

NL-T-weight: The summation of the neutosophic labelling truth membership values.

NL-I-weight: The summation of the neutosophic labelling indeterminacy membership values.

NL-F-weight: The summation of the neutosophic labelling falsity membership values.

These weights collectively reflect the overall strength and uncertainty inherent in the geodesic path.

Definition 4.2. The WI of an NLG (*Neutrosophic Labeling graph*), $G = (Z, Y)$ is defined as follows:

$$NLWI(G) = \sum_{\ell,\ell' \in Z^*} T_Z(\ell)T_Z(\ell')d_{sT}(\ell,\ell') + \sum_{\ell,\ell' \in Z^*} T_{ZL}(\ell)T_Z(\ell')d_{sI}(\ell,\ell') + \sum_{\ell,\ell' \in Z^*} T_Z(\ell)T_{ZL}(\ell')d_{sF}(\ell,\ell'),$$

when $d_{sT}(\ell,\ell')$ is the minimum T weight sum, $d_{sI}(\ell,\ell')$ is the minimum I weight sum, $d_{sF}(\ell,\ell')$ is the minimum F weight sum of the geodesics from a to b.

Definition 4.3. We have an *Neutrosophic labeling graph* (NLG) $G = (Z, Y)$. The average T-Wiener index is defined as,

$$ATWI(G) = \frac{1}{\binom{n}{2}} \left(\sum_{(\ell,\ell') \in Y^{*+}} T_Z(\ell)T_Z(\ell')d_T(\ell,\ell') \right)$$

the average I-Wiener index of G is defined as,

$$AIWI(G) = \frac{1}{\binom{n}{2}} \left(\sum_{(\ell,\ell') \in Y^{*+}} T_Z(\ell)T_Z(\ell')d_i(\ell,\ell') \right).$$

The average F-Wiener index of G is defined as,

$$AFWI(G) = \frac{1}{\binom{n}{2}}\left(\sum_{(\ell,\ell')\in Y^{*+}} T_Z(\ell)T_Z(\ell')d_F(\ell,\ell')\right).$$

Definition 4.4. Let $G = (Z,Y)$ be an neutrosophic labeling graph. The average Wiener index of G is defined to be the sum of average T-Wiener index, I-Weiner index and average F-Wiener index of G. Mathematically, it is shown as

$$\begin{aligned}AWI(G) &= \frac{1}{\binom{n}{2}}\left(\sum_{(\ell,\ell')\in Y^{*+}} T_Z(\ell)T_Z(\ell')d_T(\ell,\ell')\right) + \frac{1}{\binom{n}{2}}\left(\sum_{(\ell,\ell')\in Y^{*+}} T_Z(\ell)T_Z(\ell')d_i(\ell,\ell')\right)\\ &\quad + \frac{1}{\binom{n}{2}}\left(\sum_{(\ell,\ell')\in Y^{*+}} T_Z(\ell)T_Z(\ell')d_F(\ell,\ell')\right)AWI(G)\\ &= ATWI(G) + AIWI(G) + AFWI(G).\end{aligned}$$

Example 4.1.

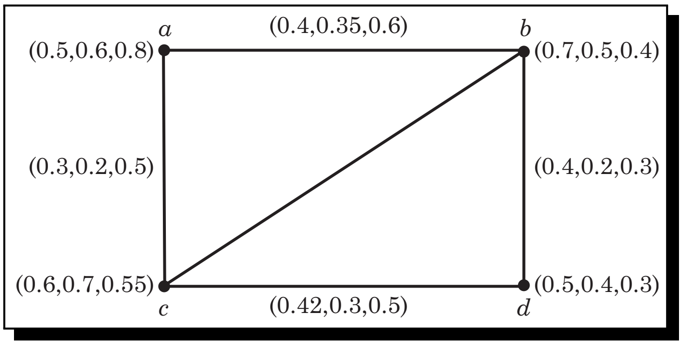

Figure 4. Neutrosophic labeling graph

Table 1 represents the summation of T-weight, I-weight, F-weight of the Neutrosophic labeling edges between each pair of vertices (ℓ,ℓ').

Table 1. Weiner index of Figure 4

	$d_{sT}(\ell,\ell')$	$d_{sI}(\ell,\ell')$	$d_{sF}(\ell,\ell')$
A,B	$0.4+0.5=0.9$	0.2	0.5
A,C	$0.4+0.5+0.42=1.32$	$0.2+0.3=0.5$	$0.5+0.5=1$
A,D	0.4	$0.2+0.3+0.2=0.7$	$0.5+0.5=1$
B,C	0.42	0.3	0.5
B,D	0.5	$0.3+0.2=0.5$	0.5
C,D	$0.42+0.5=0.92$	$0.3+0.35=0.65$	0.3

Using the above table, we can find

$$\begin{aligned}NLWI_T(G) &= \sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sT}(\ell,\ell')\\ &= (0.5)(0.6)(0.9) + (0.5)(0.5)(1.32) + (0.5)(0.7)(0.4)\\ &\quad + (0.6)(0.5)(0.42) + (0.6)(0.7)(0.5) + (0.7)(0.5)(0.92)\end{aligned}$$

$$
\begin{aligned}
&= 1.398,\\
NLWI_i(G) &= \sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sI}(\ell,\ell')\\
&= (0.6)(0.7)(0.2)+(0.6)(0.4)(0.5)+(0.6)(0.5)(0.7)\\
&\quad +(0.7)(0.4)(0.3)+(0.7)(0.5)(0.5)+(0.4)(0.5)(0.65)\\
&= 0.852,\\
NLWI_F(G) &= \sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sF}(\ell,\ell')\\
&= (0.6)(0.55)(0.5)+(0.6)(0.3)(1)+(0.6)(0.4)(1)\\
&\quad +(0.55)(0.3)(0.5)+(0.55)(0.4)(0.5)+(0.3)(0.4)(0.3)\\
&= 0.8135,\\
NLWI(G) &= 1.398+0.852+0.813 = 3.063,\\
ANLWI_T(G) &= \frac{1}{\binom{n}{2}}\left(\sum_{(\ell,\ell')\in Y^{*+}} T_Z(\ell)T_Z(\ell')d_T(\ell,\ell')\right)\\
&= \frac{1}{6}(1.398)\\
&= 0.233,\\
ANLWI_i(G) &= \frac{1}{\binom{n}{2}}\left(\sum_{(\ell,\ell')\in Y^{*+}} T_Z(\ell)T_Z(\ell')d_i(\ell,\ell')\right)\\
&= \frac{1}{6}(0.852)\\
&= 0.142,\\
ANLWI_F(G) &= \frac{1}{\binom{n}{2}}\left(\sum_{(\ell,\ell')\in Y^{*+}} T_Z(\ell)T_Z(\ell')d_F(\ell,\ell')\right)\\
&= \frac{1}{6}(0.813)\\
&= 0.1355,\\
AWI(G) &= ATWI(G)+AIWI(G)+AFWI(G),\\
AWI &= 0.5105.
\end{aligned}
$$

Note 4.1. It may be noted that $NLWI_T(G) > NLWI_i(G) > NLWI_F(G)$ which exhibits a higher level of truth.

Note 4.2. The average value of neutrosophic labeling Weiner index is always between 0 and 1.

Theorem 4.1. *Let $G^* = (Z,Y)$ be an Neutrosophic labeling graph with $|Z^*| = n$ and $(T_n(\ell), I_n(\ell), F_n(\ell)) = (\sigma,\eta,\gamma)$, for all $\ell \in V(\chi)$ with $0 \le \sigma+\eta+\gamma \le 1$. Let $x = \cap\{T_Y(\ell,\ell') \mid (\ell,V\prime) \in Y^*$ and (ℓ,ℓ') is not a $\delta-arc\}$, $y = \cap\{I_Y(\ell,\ell') \mid (\ell,\ell') \in Y^*$ and (ℓ,ℓ') is not a $\delta-arc\}$ and $z = \cap\{F_Y(\ell,\ell') \mid (\ell,\ell') \in Y^*$ and (ℓ,ℓ') is not a $\delta-arc\}$. Then, $NLWI(G^*) \ge ((x\sigma^2+y\eta^2+z\gamma^2))[n(n-1)-|Y^*|]$.*

Proof. Let $G^* = (Z,Y)$ be an NLG. Let $x = \cap\{T_Y(\ell,\ell') \mid (\ell,\ell') \in Y^*$ and (ℓ,ℓ') is not a δ-arc$\}$, $y = \cap\{I_Y(\ell,\ell') \mid (\ell,\ell') \in Y^*$ and (ℓ,ℓ') is not a δ-arc$\}$ and $z = \cap\{F_Y(\ell,\ell') \mid (\ell,\ell') \in Y^*$ and (ℓ,ℓ') is not a δ-arc$\}$. Let $\ell,\ell' \in Z^*$. If $(\ell,\ell') \in Y^*$, then $ds_T(\ell,\ell') \geq x$. The number of unordered pairs (ℓ,ℓ') satisfying $ds_i(\ell,\ell') \geq y$ and $ds_F(\ell,\ell') \geq z$ is $|Y^*|$, and the number of unordered vertex pairs (ℓ,v) satisfying $ds_T(\ell,\ell') \geq 2p$ is $\binom{n}{2} - |Y^*|$. Then,

$$\begin{aligned}
NLWI_T(G) &= \sum_{\ell,\ell' \in Z} T_Z(\ell)T_Z(\ell')d_{sT}(\ell,\ell'') \\
&= \sigma^2 \sum_{\ell,\ell' \in Z} d_{sT}(\ell,\ell') \\
&= \sigma^2 \left[\sum_{\ell,\ell' \in Y^*} d_{sT}(\ell,\ell') + \sum_{\ell,\ell' \in Z*} d_{sT}(v,v') \right] \geq \sigma^2 \left[|Y^*|x + \left\{ \binom{n}{2} - |Y^*| \right\} 2x \right] \\
&= x\sigma^2 \left[|Y^*| + 2\frac{n(n-1)}{2} - 2|Y^*| \right] \\
&= x\sigma^2[n(n-1) - |Y^*|].
\end{aligned}$$

Now, let $y = \cap I_Y(\ell,\ell') \mid (\ell,\ell') \in Y^*$ and (ℓ,ℓ') is not a δ-arc$\}$ and let $z = \cap F_Y(\ell,\ell') \mid (\ell,\ell') \in Y^*$ and (ℓ,ℓ') is not a δ-arc$\}$. Then, as argued before, we have the following:

$$\begin{aligned}
NLWI_i(G^*) &= y\eta^2[n(n-1) - |Y^*|], \\
NLWI_F(G^*) &= z\gamma^2[n(n-1) - |Y^*|].
\end{aligned}$$

Thus, $NLWI(G^*) = NLWI_T(G^*) + NLWI_i(G^*) + NLWI_F(G^*) \geq ((x\sigma^2 + y\eta^2 + z\gamma^2))[n(n-1) - |Y^*|]$. □

Theorem 4.2. *If $G_1^* = (Z,Y)$ and $G_2^* = (Z',Y')$ are isomorphic labeling graph then $NLWI(G_1^*) = NLWI(G_1^*)$.*

Proof. Let $G_1^* = (Z,Y)$ and $G_2^* = (Z',Y')$ be isomorphic neutrosophic labeling graph.

From Z^* to $Z^{'*}$ is a bijective map such that for all $k \in Z^*$ and for all $(\ell,\ell') \in Y^*$. We have

$$T_z(k) = T'_z(m(w)), \quad I_z(k) = I'_z(m(w)), \quad F_z(k) = F'_z(m(w))$$

and

$$T_Y(\ell,\ell') = T'_Y(m(\ell)m(\ell'), \quad I_Y(\ell,\ell') = I'_Y(m(\ell)m(\ell'), \quad F_Y(\ell,\ell') = F'_Y(m(\ell)m(\ell').$$

For all $(\ell,\ell') \in Z^*$. Let $P_{\ell,\ell'}$ be the geodesic path and it serves for $d_T(\ell,\ell')$, $d_i(\ell,\ell')$, $d_F(\ell,\ell')$ corresponding to each edge $(a,b) \in p_{\ell,\ell'}$ there exist an edge $m(a),m(b) \in G_2^*$, such that $T_Y(a,b) = T'_Y$. Then, the path $P'_{\ell,\ell'}$ to $P_{\ell,\ell'}$. Total weight of this path is minimum among all. Thus,

$$\begin{aligned}
NLWI_T(G_1^*) &= \sum_{\ell,\ell' \in Z^*} T_Z(\ell)T_Z(\ell')d_{sT}(\ell,\ell') \\
&= \sum_{\ell,\ell' \in Z^*} T_{Z'}(m(\ell)m(\ell'))d_{sT}(m(\ell)m(\ell')) \\
&= NLWI_T(G_2^*), \\
NLWI_i(G_1^*) &= \sum_{\ell,\ell' \in Z^*} I_Z(\ell)I_Z(\ell')d_{sI}(\ell,\ell')
\end{aligned}$$

$$= \sum_{\ell,\ell' \in Z^*} T_{Z'}(m(\ell)m(\ell'))d_{sT}(m(\ell)m(\ell'))$$

$$= NLWI_i(G_2^*),$$

$$NLWI_F(G_1^*) = \sum_{\ell,\ell' \in Z^*} F_Z(\ell)F_Z(\ell')d_{sF}(\ell,\ell')$$

$$= \sum_{\ell,\ell' \in Z^*} F_{Z'}(m(\ell)m(\ell'))d_{sF}(m(\ell)m(\ell'))$$

$$= NLWI_F(G_2^*).$$

Hence $NLWI_T(G_1^*) + NLWI_i(G_1^*) + NLWI_F(G_1^*) = NLWI_T(G_2^*) + NLWI_i(G_2^*) + NLWI_F(G_2^*)$ which implies $NLWI(G_1^*) = NLWI(G_2^*)$. □

5. Applications

Ecosystem dynamics in river basins are influenced by seasonal changes, including variations in water flow, temperature, and species behavior. This study calculates the Wiener index for neutrosophic graphs representing the river basin ecosystem in different seasons, offering insights into seasonal ecological dynamics. Consider a study on ecosystem dynamics in a river basin over the span of a year. You decide to employ neutrosophic sets to represent the uncertainties and varying conditions within the ecosystem. Dividing the year into four distinct seasons, you create neutrosophic labeling graphs for each time period:

(G_1) (*Winter*): From January 1st, 2023, to March 30th, 2023 (Figure 5), capturing the ecosystem's state during the colder months when water levels might be lower, and certain species exhibit specific behaviors.

(G_2) (*Spring*): From April 1st, 2023, to June 30th, 2023 (Figure 6), representing the transition period from winter to spring, characterized by melting snow, rising water levels, and potential changes in species distribution and activity.

(G_3) (*Summer*): From July 1st, 2023, to September 30th, 2023 (Figure 7), reflecting the ecosystem's condition during the warmer months, with increased biological activity, vegetation growth, and possibly altered water quality dynamics.

(G_4) (*Autumn*): From October 1st, 2023, to October 30th, 2023 (Figure 8), capturing the changes occurring as summer transitions to autumn, such as decreasing temperatures, foliage changes, and potential shifts in species behavior and population dynamics.

By analyzing these neutrosophic labeling graphs across the four time periods, you aim to understand how the river basin ecosystem's structure and interactions evolve seasonally, considering factors like water flow, temperature variations, species composition, and environmental pressures. This approach allows for a nuanced exploration of the ecosystem's dynamics, accounting for uncertainties and fluctuations inherent in natural systems.

We now calculate the neutrosophic labeling graph Wiener index for each of the above time periods.

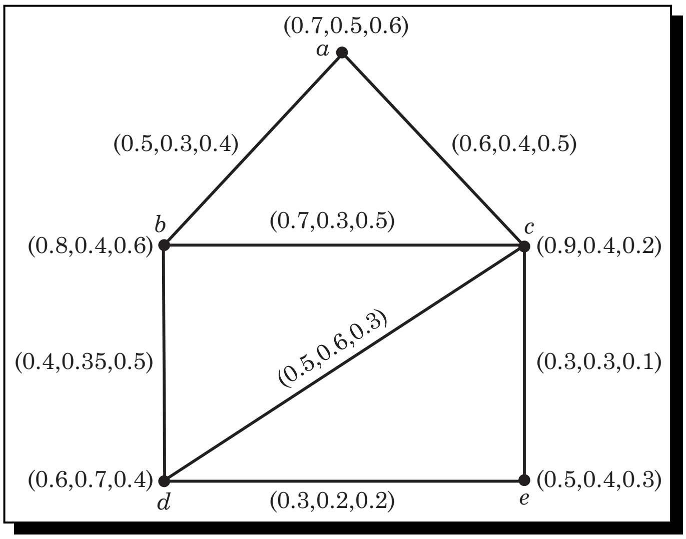

Figure 5. Neutrosophic labeling graph (G_1)

Table 2 represents the sum of T-weight, I-weight, F-weight of the Neutrosophic labeling edges between each pair of vertices (ℓ,ℓ').

Table 2. Weiner index of Figure 5

	$d_{sT}(\ell,\ell')$	$d_{sI}(\ell,\ell')$	$d_{sF}(\ell,\ell')$
a,b	$0.6+0.7=1.3$	0.3	0.4
a,c	0.6	$0.3+0.3=0.6$	0.5
a,d	$0.6+0.5=1.1$	$0.3+0.35=0.65$	$0.4+0.5=0.9$
a,c	$0.6+0.4=1.0$	$0.3+0.3+0.3=0.9$	$0.4+0.5+0.2=1.1$
b,c	0.7	0.3	0.5
b,d	$0.7+0.5=1.2$	$0.3+0.3+0.2=0.8$	0.5
b,e	$0.7+0.4=1.1$	$0.3+0.3=0.6$	$0.5+0.1$
d,e	$0.5+0.4=0.9$	0.2	0.2
c,d	0.5	$0.3+0.2=0.5$	$0.1+0.2=0.3$
c,e	0.4	0.3	0.1

Using the above table, we can find

$$NLWI_T(G_1)=\sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sT}(\ell,\ell')=4.158,$$

$$NLWI_i(G_1)=\sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sI}(\ell,\ell')=1.3775,$$

$$NLWI_F(G_1)=\sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sF}(\ell,\ell')=0.804,$$

$$NLWI(G_1)=\sum_{\ell,\ell'\in NL^*} T_Z(\ell)T_Z(\ell')d_s(\ell,\ell'),$$

$$NLWI(G_1)=\sum_{\ell,\ell'\in NL^*} T_Z(\ell)T_Z(\ell')d_s(\ell,\ell')=6.3395,$$

$$ANLWI(G_1)=0.63395.$$

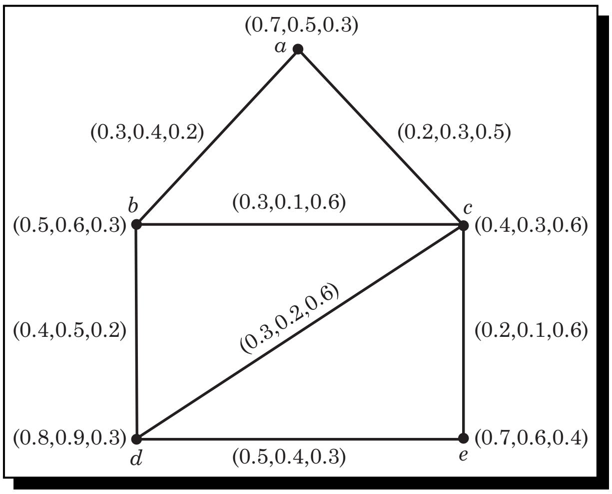

Figure 6. Neutrosophic labeling graph (G_2)

Table 3 represents the sum of T-weight, I-weight, F-weight of the Neutrosophic labeling edges between each pair of vertices (ℓ,ℓ').

Table 3. Weiner index of Figure 6

	$d_{sT}(\ell,\ell')$	$d_{sI}(\ell,\ell')$	$d_{sF}(\ell,\ell')$
a,b	0.3	$0.3+0.1=0.4$	0.2
a,c	$0.3+0.3=0.6$	0.3	0.5
a,d	$0.3+0.4=0.7$	$0.3+0.2=0.5$	$0.2+0.2=0.4$
a,e	$0.3+0.4+0.5=1.2$	$0.3+0.1=0.4$	$0.2+0.2+0.3=0.7$
b,c	0.3	0.1	$0.2+0.5=0.7$
b,d	0.4	$0.1+0.2=0.3$	0.2
b,e	$0.4+0.5=0.9$	$0.1+0.1=0.2$	$0.2+0.3=0.5$
d,e	0.5	$0.2+0.1=0.3$	0.3
c,d	0.3	0.2	$0.5+0.2+0.2=0.9$
c,e	$0.3+0.5=0.8$	0.1	$0.5+0.2+0.2+0.3=1.2$

Using the above table, we can find

$$NLWI_T(G_2)=\sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sT}(\ell,\ell')=2.338,$$

$$NLWI_i(G_2)=\sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sI}(\ell,\ell')=0.996,$$

$$NLWI_F(G_2)=\sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sF}(\ell,\ell')=0.918,$$

$$NLWI(G_2)=\sum_{\ell,\ell'\in NL^*} T_Z(\ell)T_Z(\ell')d_s(\ell,\ell')=4.302,$$

$$ANLWI(G_2)=0.4302.$$

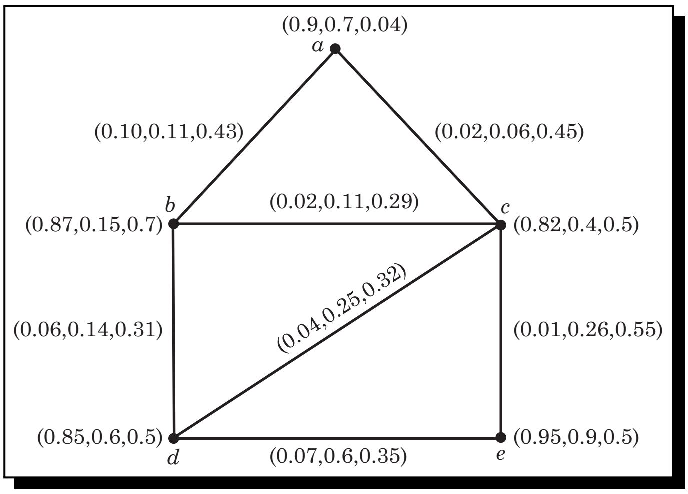

Figure 7. Neutrosophic labeling graph (G_3)

Table 4 represents the sum of T-weight, I-weight, F-weight of the Neutrosophic labeling edges between each pair of vertices (ℓ, ℓ').

Table 4. Weiner index of Figure 7

	$d_{sT}(\ell,\ell')$	$d_{sI}(\ell,\ell')$	$d_{sF}(\ell,\ell')$
a,b	0.10	0.11	0.43
a,c	$0.10+0.06+0.04=0.20$	0.06	$0.43+0.29=0.72$
a,d	$0.10+0.06=0.16$	$0.11+0.14=0.25$	$0.43+0.31=0.74$
a,e	$0.10+0.06+0.07=0.23$	$0.06+0.26=0.32$	$0.43+0.31+0.35=1.09$
b,c	$0.06+0.04=0.10$	0.11	0.29
b,d	0.06	0.14	0.31
b,e	$0.06+0.07=0.13$	$0.11+0.26=0.37$	$0.31+0.35=0.66$
d,e	0.07	$0.25+0.26=0.51$	0.35
c,d	0.04	$0.11+0.14=0.25$	$0.29+0.31=0.60$
c,e	$0.04+0.07=0.11$	0.26	$0.32+0.35$

Using the above table, we can find

$$NLWI_T(G_3) = \sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sT}(\ell,\ell') = 0.954125,$$

$$NLWI_i(G_3) = \sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sI}(\ell,\ell') = 0.87735,$$

$$NLWI_F(G_3) = \sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sF}(\ell,\ell') = 0.83688,$$

$$NLWI(G_3) = \sum_{\ell,\ell'\in NL^*} T_Z(\ell)T_Z(\ell')d_s(\ell,\ell') = 2.668355,$$

$$ANLWI(G_3) = 0.2668355.$$

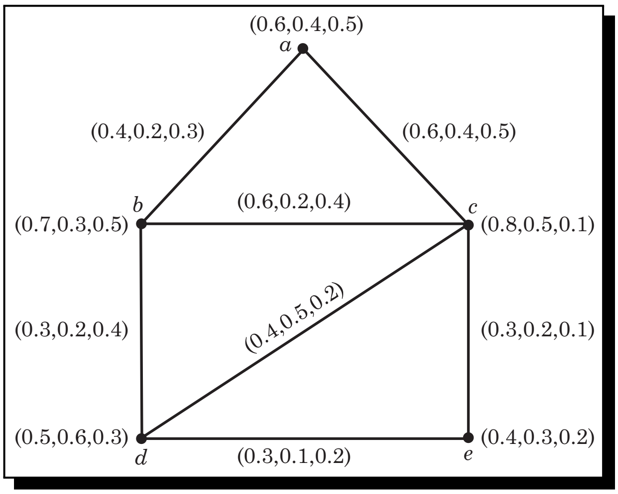

Figure 8. Neutrosophic Labeling Graph (G_4)

Table 5 represents the sum of T-weight, I-weight, F-weight of the Neutrosophic labeling edges between each pair of vertices (ℓ,ℓ').

Table 5. Weiner index of Figure 8

	$d_{sT}(\ell,\ell')$	$d_{sI}(\ell,\ell')$	$d_{sF}(\ell,\ell')$
a,b	$0.5+0.6=1.1$	0.2	0.3
a,c	0.5	$0.2+0.2=0.4$	0.4
a,d	$0.5+0.4=0.9$	$0.2+0.2=0.4$	$0.3+0.4=0.7$
a,c	$0.5+0.3=0.8$	$0.2+0.2+0.1=0.5$	$0.4+0.1=0.5$
b,c	0.6	0.2	0.4
b,d	$0.6+0.4=1$	0.2	0.4
b,e	$0.6+0.3=0.9$	$0.2+0.2=0.4$	$0.4+0.1=0.5$
d,e	0.3	0.1	0.2
c,d	0.4	$0.2+0.1=0.3$	0.2
c,e	0.3	0.2	0.1

Using the above table, we can find

$$NLWI_T(G_4)=\sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sT}(\ell,\ell')=2.418,$$

$$NLWI_i(G_4)=\sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sI}(\ell,\ell')=0.662,$$

$$NLWI_F(G_4)=\sum_{\ell,\ell'\in Z^*} T_Z(\ell)T_Z(\ell')d_{sF}(\ell,\ell')=0.400,$$

$$NLWI(G_4)=\sum_{\ell,\ell'\in NL^*} T_Z(\ell)T_Z(\ell')d_s(\ell,\ell')=3.4800,$$

$$ANLWI(G_4)=0.34800.$$

Annual Average Neutrosophic Labeling Weiner index

$$= \frac{1}{4}[ANLWI(G_1) + ANLWI(G_2) + ANLWI(G_3) + ANLWI(G_4)]$$

$$= \frac{1}{4}[0.63395 + 0.4302 + 0.2668355 + 0.34800] = 0.419.$$

Results Interpretation

Winter (*More*): The Neutrosophic Labeling Wiener index is significantly above the annual average, indicating lower overall interaction levels and more significant uncertainty impacts, likely due to lower water levels and adverse weather conditions.

Spring (*Moderate*): The Neutrosophic Labeling Wiener index is close to the annual average, indicating moderate network efficiency with transitional weather conditions affecting ecological interactions.

Summer (*Lower*): The Neutrosophic Labeling Wiener index is significantly below the annual average, reflecting increased biological activity and possibly greater uncertainty due to altered water quality dynamics and vegetation growth.

Autumn (*Moderate*): The Neutrosophic Labeling Wiener index is slightly below the annual average, indicating moderate levels of interaction with the transition from summer to cooler weather conditions.

6. Advantages

Our study has the following advantages:

- *Randić Index of Neutrosophic Labeling Graph* measures node connections with specific uncertainty labels and gives a detailed measure of connectivity that includes specific levels of uncertainty for each connection but in neutrosophic graph measures node connections with general uncertainty, and provides a broad measure of connectivity under uncertain conditions.
- *Harmonic Index of Neutrosophic Labeling Graph* calculates average distances with detailed uncertainty labels and provides a more nuanced average distance measure, considering specific uncertainty levels but in neutrosophic graph calculates average distances with vague connections and gives a generalized view of average distances in the presence of uncertainty.
- *Zagreb Index of Neutrosophic Labeling Graph* reflects node degrees with specific uncertainty levels for each link and provides a detailed view of node degree, accounting for each link's specific uncertainty but in neutrosophic graph gives only basic measure of node degree under general uncertainty.
- *Wiener Index of neutrosophic Labeling Graph* measures overall distance considering detailed edge uncertainties and provides a precise distance measure, incorporating specific uncertainty levels for each edge but in neutrosophic graph measures overall distance in graph with only general uncertainty and yields an aggregate distance measure under broad uncertainty.

7. Conclusion

The primary objective of this study was to introduce the concept of topological indices in neutrosophic labeling graphs, specifically focusing on degrees characterized by truth membership, indeterminacy, and falsity membership. We proposed definitions for the Randić, harmonic, and Zagreb indices within neutrosophic labeling graphs, accompanied by illustrative examples. We investigated several theorems related to these indices. We discussed the *Neutrosophic Labeling Wiener Index* (*NLWI*), providing examples and related results. We provided the application of these concepts in river ecosystem analysis. In future research, we plan to extend our work to T-spherical fuzzy graphs and picture fuzzy graphs. Furthermore, we will explore interval-valued neutrosophic graphs and their potential applications.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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