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Research Article

A Steady State Behavior of M/M/1 Queue with an Optional Differentiated Working Vacation and Arrival Restriction

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Abstract. In this paper, we aim to study the steady-state behaviour of an M/M/1 queue with an optionally differentiated working vacations I and II. After finishing a busy period, the server takes either an optional working vacation I or an optional working vacation II. Customer arrivals are restricted when the server is either on an optional working vacation I or an optional working vacation II. The model's stable solution is derived using the probability generating function. Additionally, specific expressions are used to discuss several performance measurements for the provided model. For cost optimization analysis, the *Particle Swarm Optimization* (PSO) method is also employed to reduce the total cost. We minimize the total cost of providing the best service by using the PSO optimization method. A few numerical examples are provided to illustrate the effects of different arrival rates, service rates, server vacation times, and customer waiting times.

Keywords. Single server queue, Multiple variants working vacations, Stationary analysis, Optimization and arrival restriction

Mathematics Subject Classification (2020). 60K25, 68M20, 90B22

1. Introduction

The vacation concept was first used in the M/M/1 queueing system by Doshi [5]. This work splits the queueing system into primary and secondary (vacation) queues and provides numerous examples. The customer frustration behaviour of an M/M/1 queueing system with unique server vacations was studied by Sampath and Liu [16]. Unni and Mary [24] handled differentiated vacations using a variety of client strategies, that is, the chance that a consumer will join during both types of vacations. Bouchentouf and Medjehri [3] analysed the system's economic analysis and the effectiveness of various multiple vacation queues in the M/M/1 queue. The finite buffer Markovian queue during varied vacations was discussed by Vadivukarasi *et al*. [26] in terms of optimization. Retrial queueing model with state-dependent arrival rates and different vacations was first proposed by Gupta and Kumar [8]. The idea of varied vacation queueing systems with varying vacations in the M/M/1 queue was addressed by Isijola-Adakeja and Ibe [10]. Ibe and Isijola [9] made a similar suggestion regarding vacation breaks in the M/M/1 queue. The concept of C-servers with two distinct vacation options was put up by Unni and Mary [25]. Ke *et al*. [11] demonstrated a number of current breakthroughs in system communications and flexible manufacturing that have been made in vacation queuing models. To optimize the long-term average value generated, Ata and Shneorson [1] discuss a service station where the arrival and rate of service are dynamically managed by the system management. In the rate-setting problem we first consider, a service station is represented as an M/M/1 queue with parameters. Under dynamic service control, Dimitrakopoulos and Burnetas [4] deal with the methods for client balance in an M/M/1 queue. Depending on the level of system congestion, the service rate alternates between low and high values. Tian *et al*. [21] proposed a fixable M/M/1 retrial queueing paradigm with setup delays. The server will be shut down to save money once the system is completely empty, and only when a new consumer shows up will the system be turned on. Customers that activate servers are placed in the retry orbit while they wait to reapply for service.

Bagyam and Chandrika [2] deal with the state-dependent batch arrival and a two-stage queue for retrials, and acceptance of every customer into the system is based on the server's state. All customers who have been admitted are given the first essential service by the server. The customer can select a second optional service or exit the system when the first essential service is finished. Servi and Finn [19] ultimately adopted working vacations that provided services at a more leisurely pace. Tian *et al*. [23] examined the customers' stability and socially ideal joining-balking behaviour in an M/M/1 queue with Bernoulli interruptions and a working vacation. The sensitivity analysis of limiting arrivals in a single working vacation queue was looked at by Yang *et al*. [31]. Since the customer arrives during the server's vacation, they may disappoint the system, but at that time, the server gives the service at a more leisurely pace, indicating that the customer may wait for the system, which concept is highly beneficial to modern time queueing systems. Tian *et al*. [22] suggested a matrix-geometric solution for a working vacation queue. Ye and Liu [32] reviewed the results of a single working vacation's performance evaluation of a new GI/M/1 type queue. Xu *et al*. [30] analysed the setup times for a working vacation on the M/M/1 queue. The working breakdown approach for the M/M/1/N queue with catastrophe was addressed by Seenivasan and Abinaya [17]. These are the core concepts of one working vacation in the M/M/1 queuing system. The Geo/Geo/1 queue notion of equilibrium customer strategies with single working vacations, as defined by Wang *et al*. [29], says that customers decide whether to utilize or reject the system based on a natural reward-cost structure, the status information provided about the server, and the wait time that is present when they arrive. It was specifically the impatient customer's personal timer that was looked at.

An interesting concept of priority clients with just brief working vacation interruptions was created by Goswami and Selvaraju [7]. On the M/G/1 queue, Gao and Liu [6] talked about the single working vacation. Li and Tian [14] examined the GI/M/1 queue's performance with a single working vacation using the matrix geometric approach. In their discussion on M/M/R queue applications using a single working vacation, Lin and Ke [15] covered a few important concepts. The eager clients in an M/M/1 line were described by Selvaraju and Goswami [18] as having both single and multiple working vacations. A two-stage vacation queueing scheme that fails in M/M/1 queues is explored by Sudesh *et al*. [20]. Customers are getting impatient because of the M/M/1 queueing model's divided working vacation notion, claimed by Vijayashree and Ambika [28]. Lakshmi *et al*. [13] looked into the finite buffer M/M/1 Markovian queue with impatience and working vacations. Brief studies of differential vacations were discussed, and Vijayashree and Janani [27] and Lakshmi *et al*. [12] evaluated methods for preventing impatient customers from abandoning the queue while on vacation, respectively. In the aforementioned instance, both of them managed a unique vacation in the M/M/1 queue.

The new idea of an optional differentiated working vacation with an arrival restriction is currently presented in this study. According to this theory, the server either transitions to an optional working vacation I or II after the busy season has passed. The service rate is lower during optional I working vacation than it is during the busy time, and after optional I working vacation, the server moves into optional II working vacation, often known as the busy time. Given that in this scenario, optional working vacation II is expected to have a lower service rate than optional working vacation I.

2. Model Formulation

Consider a single-server queueing system with an optional differentiated working vacation policy for the server. The notations and assumptions listed below have been used. Customers join the queue by a Poisson process with an arrival rate λ. The customers receive service on a *First-Come, First-Received* (FCFR) service scheme. The service time is expected to follow an exponential distribution with a mean of $\frac{1}{\mu}$. An optional working vacation I or II is selected by the server at the busy period completion time, the server will choose either an optional working vacation I with probability μ_p or an optional working vacation II with probability μ_q. While the server is on both working vacations, the customers are receiving service slowly. The assumption is that the service times will be exponentially distributed, with a mean $\frac{1}{\mu_i}$ if the server is on working vacation of type i $(i = 1,2)$ and $\mu > \mu_1 > \mu_2$. If there are no customers in the system at the end of the optional working vacation I, the server moves to the optional working vacation II. If not, the server initiates providing services. If there are any customers remaining in the system, the server begins providing services after the optional II working vacation ends. In the event that it is not, the server goes on optional working vacation II. The duration of both types of optional working vacations I and II is considered to have an exponential distribution, with means $\frac{1}{\gamma_1}$ and $\frac{1}{\gamma_2}$, respectively. The number of customers who can arrive is restricted while the server is either on optional working vacation I or II, i.e., $\lambda_i \le \lambda$, for $i = 1,2$. Interarrival, vacation, and service times are all independent and identically distributed to each other.

3. The Steady State Analysis

Let $N(t)$ represents the total number of customers at moment t, $S(t)$ represents the service provider's status at time t,

$$S(t) = \begin{cases} 0, & \text{the service provider at busy,} \\ 1, & \text{the service provider at optional working vacation I,} \\ 2, & \text{the service provider at optional working vacation II.} \end{cases}$$

Then $\{S(t), N(t), t \ge 0\}$ is a state space of a Markov processes $\Lambda = \{(i,j), i = 0,1,2\}$.

Define $p_{i,j}$ be the probability that the service provider be in the ith state $(i = 0,1,2)$ with $j(\ge 0)$ number of beneficiaries. Figure 1 depicts the state transition diagram.

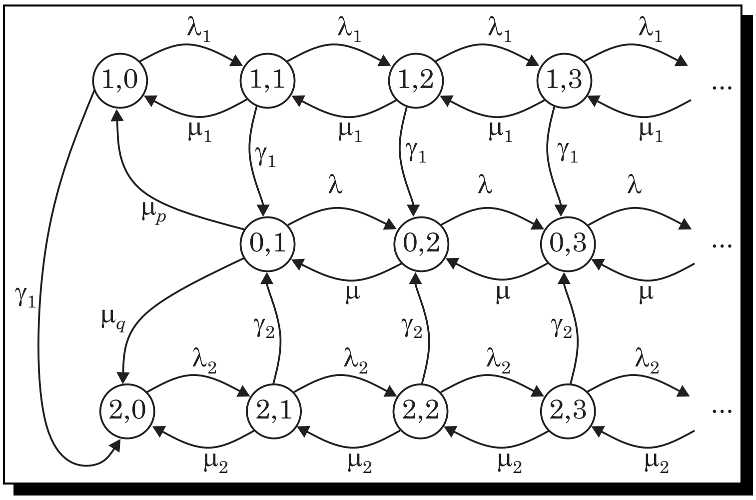

Figure 1. State transition diagram

4. Local Balancing Equation

The considered model's local balance equations are

$$(\lambda+\mu)p_{0,1} = \gamma_1 p_{1,1} + \gamma_2 p_{2,1} + \mu p_{0,2}, \tag{1}$$

$$(\lambda+\mu)p_{0,n} = \gamma_1 p_{1,n} + \gamma_2 p_{2,n} + \mu p_{0,n+1} + \lambda p_{0,n-1}, \quad n \ge 2, \tag{2}$$

$$(\lambda_1+\gamma_1)p_{1,0} = \mu_1 p_{1,1} + \mu p p_{0,1}, \tag{3}$$

$$(\lambda_1+\gamma_1+\mu_1)p_{1,n} = \mu_1 p_{1,n+1} + \lambda_1 p_{1,n-1}, \quad n \ge 1, \tag{4}$$

$$\lambda_2 p_{2,0} = \gamma_1 p_{1,0} + \mu q p_{0,1} + \mu_2 p_{2,1}, \tag{5}$$

$$(\lambda_2+\gamma_2+\mu_2)p_{2,n} = \gamma_2 p_{2,n-1} + \mu_2 p_{2,n+1}, \quad n \ge 1. \tag{6}$$

Equations (1) and (2) are solved by multiplying z^n and yielding

$$P_0(z) = \frac{\gamma_1 z p_{1,0} + \gamma_2 z p_{2,0} + \mu z p_{0,1} - \gamma_1 z P_1(z) - \gamma_2 z P_2(z)}{\lambda z^2 - (\lambda+\mu)z + \mu}. \tag{7}$$

Equations (3) and (4) are solved by multiplying z^n and yielding

$$P_1(z) = \frac{\mu_1 p_{1,0} - \mu_1 z p_{1,0} - \mu p z p_{0,1}}{\lambda_1 z^2 - (\lambda_1 + \mu_1 + \gamma_1)z + \mu_1}. \tag{8}$$

Equations (5) and (6) are solved by multiplying z^n and yielding

$$P_2(z) = \frac{\mu_2 p_{2,0} - z p_{2,0}(\mu_2 + \gamma_2) - \gamma_1 z p_{1,0} - \mu q z p_{0,1}}{\lambda_2 z^2 - (\lambda_2 + \mu_2 + \gamma_2)z + \mu_2}. \tag{9}$$

Equations (7), (8) and (9) are solved by some calculation and yielding

$$P_0(1) = \frac{(\lambda_2\gamma_1\gamma_2)p_{2,0} + (\gamma_2\mu p(\lambda_1 - \mu_1) + \gamma_1\mu q(\lambda_2 - \mu_2))p_{0,1} + (\gamma_1\gamma_2\mu_1 + \gamma_1{}^2(\lambda_2 - \mu_2))p_{1,0}}{\gamma_1\gamma_2(\mu - \lambda)}, \tag{10}$$

$$P_1(1) = \frac{\mu p p_{0,1}}{\gamma_1}, \tag{11}$$

$$P_2(1) = \frac{\gamma_2 p_{2,0} + \gamma_1 p_{1,0} + \mu q p_{0,1}}{\gamma_2}. \tag{12}$$

The law of total probability gives the following:

$$p_{0,1} = \frac{\gamma_1\gamma_2(\mu - \lambda)}{\begin{pmatrix} \gamma_2\mu p(\lambda_1 - \mu_1 + \mu - \lambda) + \gamma_1\mu q(\lambda_2 - \mu_2 + \mu - \lambda) \\ + \alpha(\gamma_1{}^2(\lambda_2 - \mu_2 + \mu - \lambda) + \gamma_1\gamma_2\mu_1) + \beta(\gamma_1\gamma_2(\lambda_2 - \mu_2 + \mu - \lambda)) \end{pmatrix}},$$

where

$$\alpha = \frac{\mu p z_1}{\mu_1(z_1 - 1)}, \quad \beta = \frac{-z_2(\alpha\gamma_1 + \mu q)}{z_2(\gamma_2 + \mu_2) - \mu_2},$$

z_1 is the positive root of $\lambda_1 z^2 - (\lambda_1 + \mu_1 + \gamma_1)z + \mu_1$,

z_2 is the positive root of $\lambda_2 z^2 - (\lambda_2 + \mu_2 + \gamma_2)z + \mu_2$.

5. System Performance Measure

This section covers the important system performance measures for the given model. The mean number of customers in busy period is

$$E(L_B) = \sum_{n=1}^{\infty} n\ p_{n,0}$$
$$= (X + \alpha Y + \beta z)p_{0,1},$$

where

$$X = \frac{\begin{pmatrix} \gamma_1\gamma_2{}^3\mu p[(\mu - \lambda)(\gamma_1\mu_1 + 2\mu_1{}^2 - 4\lambda_1\mu_1 + 2\lambda_1{}^2 + \gamma_1\lambda_1) + 2\lambda\gamma_1(\lambda_1 - \mu_1 + \gamma_1)] \\ + \gamma_2\gamma_1{}^3\mu q[(\mu - \lambda)(\gamma_2\mu_2 + 2\mu_2{}^2 - 4\lambda_2\mu_2 + 2\lambda_2{}^2 + \gamma_2\lambda_2) + 2\lambda\gamma_2(\lambda_2 - \mu_2 + \gamma_2)] \\ -2\lambda\gamma_2{}^3\gamma_1{}^3\mu \end{pmatrix}}{2\gamma_2{}^3\gamma_1{}^3(\mu - \lambda)^2},$$

$$Y = \frac{\begin{pmatrix} (\mu - \lambda)\gamma_2\gamma_1{}^3(\gamma_1\gamma_2\mu_2 + 2\gamma_1\mu_2{}^2 - 4\lambda_2\gamma_1\mu_2 + 2\lambda_2{}^2\gamma_1 + \gamma_1\gamma_2\lambda_2) \\ + (\mu - \lambda)\gamma_2{}^3\gamma_1{}^2\mu_1(2\lambda_1 - 2\mu_1 - \gamma_1) \end{pmatrix}}{2\gamma_2{}^3\gamma_1{}^3(\mu - \lambda)^2},$$

$$Z = \frac{\gamma_2{}^2\gamma_1{}^3\lambda_2[(\mu - \lambda)(2\lambda_2 - 2\mu_2 - \gamma_2) + 2\lambda]}{2\gamma_2{}^3\gamma_1{}^3(\mu - \lambda)^2}.$$

The mean number of customers in optional-I working vacation is

$$E(L_1) = \sum_{n=0}^{\infty} n p_{n,1}$$
$$= \frac{[\mu p(\lambda_1 - \mu_1)]p_{0,1} + \gamma_1 \mu_1 p_{1,0}}{{\gamma_1}^2}.$$

The mean number of customers in optional-II working vacation is

$$E(L_2) = \sum_{n=0}^{\infty} n p_{n,2}$$
$$= \frac{\gamma_2 \lambda_2 p_{2,0} + \gamma_1(\lambda_2 - \mu_2)p_{1,0} + \mu q(\lambda_2 - \mu_2)p_{0,1}}{{\gamma_2}^2}.$$

Total mean number of customers in the system is $= E(L_B) + E(L_1) + E(L_2)$.

The formula determines the estimated waiting period is

$$E(W) = \frac{E(L)}{\lambda}.$$

6. Numerical Illustrations

In this section, we discuss some numerical examples of the mean number of customers in the system and the expected waiting time of the system for the various parameters of λ, μ, μ_1 and μ_2.

Figures 2 to 5 deal with $E(L)$ against various values of λ, μ, μ_1 and μ_2. As shown in Figure 2, as λ increases, will also increase the mean number of customers. Specifically, when service values increase, the mean number of customers decreases, that is, we observed a decrease in the mean number of customers for increased service values of $\mu = 5$, 4.75 and 4.5. Similarly, Figure 3 illustrates that when μ increases, decrease the mean number of customers. Specifically, when arrival rates increase, the mean number of customers also increases. Therefore, we have observed an increase in the figure for the various values of $\lambda = 0.7$, 0.5 and 0.3. Similarly, Figure 4 and Figure 5 says that μ_1 and μ_2 increase, the mean number of customers will decrease. Specifically, when arrival rates increased, correspondingly increased the mean number of customers; that is, we observed that the graph would be increased for different increased arrival rates in Figures 4 and 5.

Figures 6 to 9 shows that $E(W)$ against various values of λ, μ, μ_1 and μ_2. Figure 6 shows that when the value of λ increases, customers are expected to wait more time. Customers' expected waiting times should reduce when service values increase. Specifically, we observed that when service values increased of $\mu = 5$, 4.75 and 4.5, the expected waiting times decreased. Similarly, Figure 7 indicates that when μ increases, the expected waiting time will decrease. Specifically, when arrival rates increase, correspondingly increases the expected waiting time; that is, we observe an increase in the figure for different values of $\lambda = 0.7$, 0.5 and 0.3. Similarly, Figure 8 and Figure 9 shows that the expected waiting time for customers will decrease if parameters μ_1 and μ_2 are increased. Specifically, we observed that when arrival rates increased, correspondingly increased the expected waiting times. The graphs in Figures 8 and 9 will be modified to reflect for different increases in arrival rates.

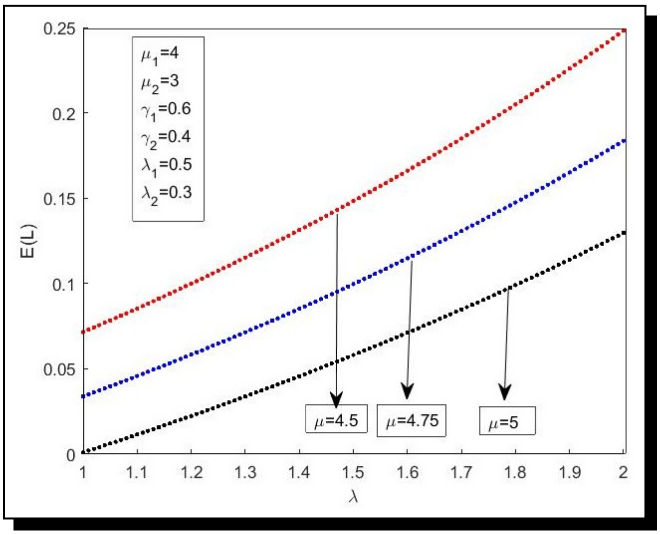

Figure 2. $E(L)$ against λ

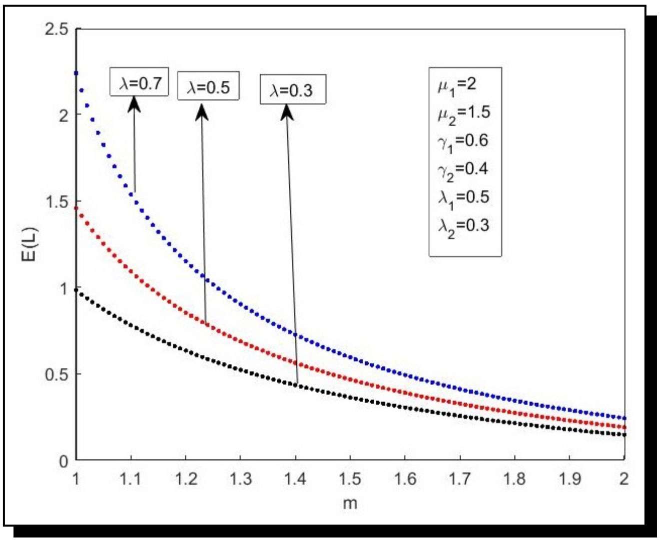

Figure 3. $E(L)$ against μ

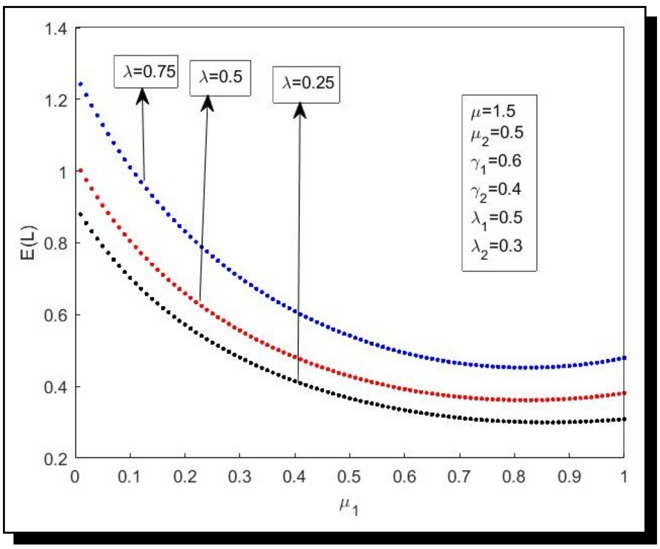

Figure 4. $E(L)$ against μ_1

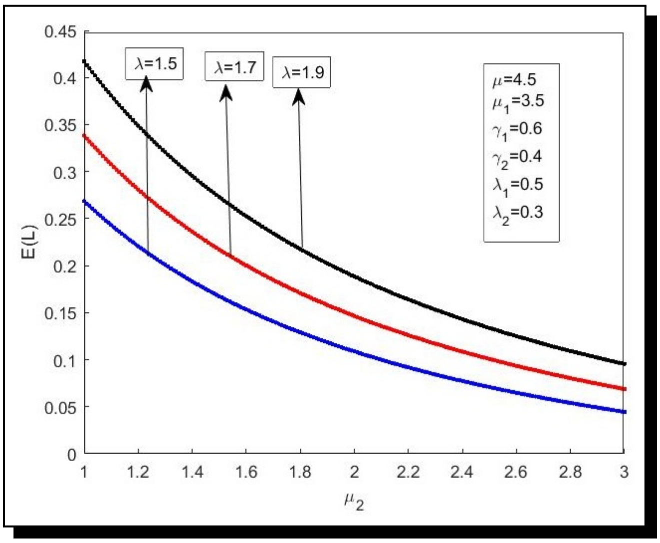

Figure 5. $E(L)$ against μ_2

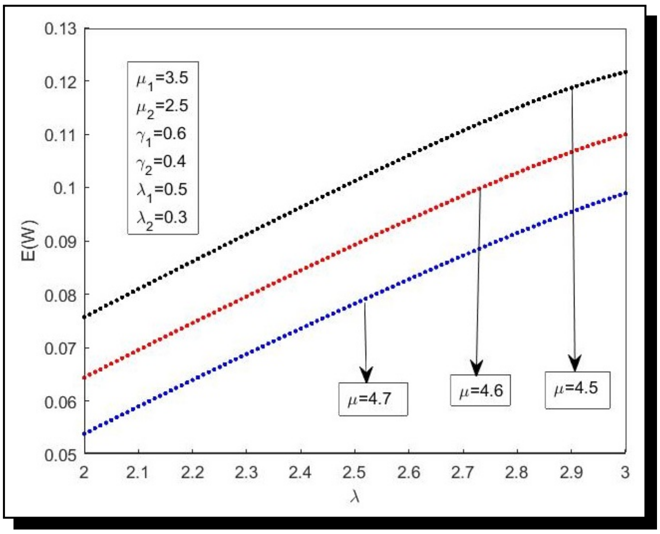

Figure 6. $E(W)$ against λ

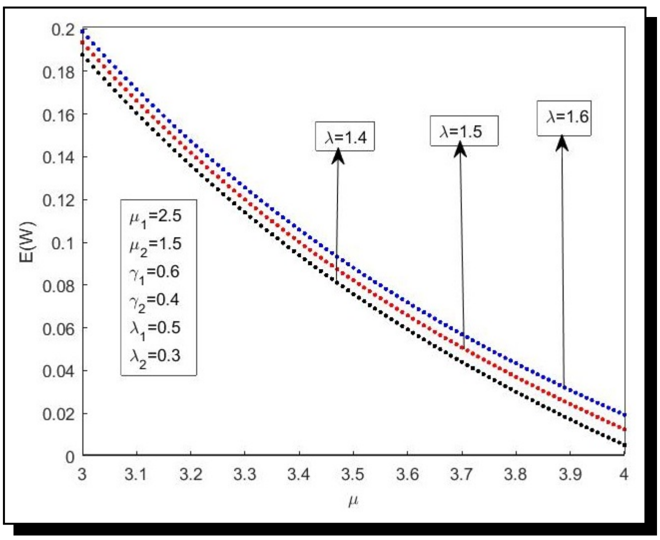

Figure 7. $E(W)$ against μ

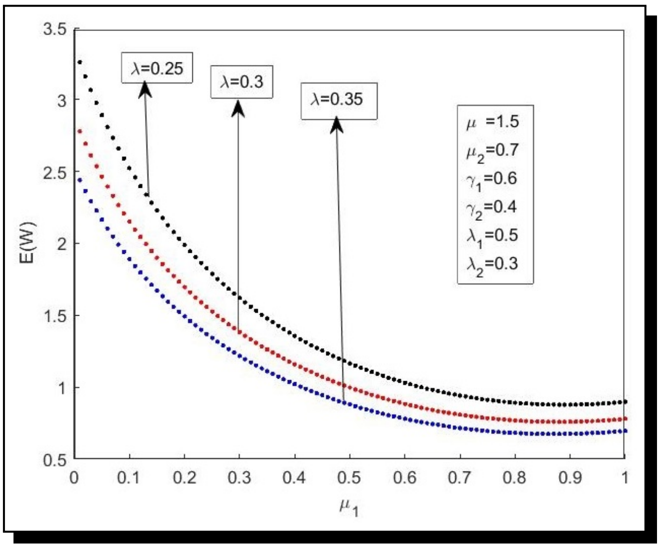

Figure 8. $E(W)$ against μ_1

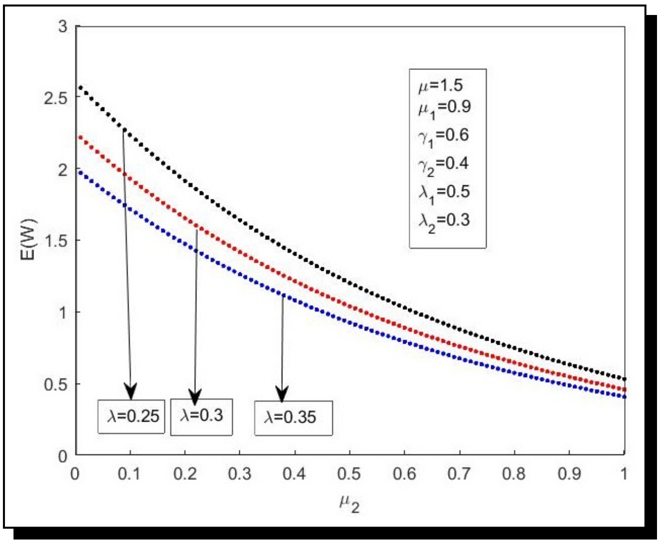

Figure 9. $E(W)$ against μ_2

7. Cost Analysis

In this section, we define the total expense function, which acts as the control variable in an expense model. Our goal is to lower the overall mean expense per quantity by controlling these variables. The following definitions apply to the cost elements:

Define the cost function TC as

$$TC = C_L E(L) + C_W E(W) + C_0 P_0 + \sum_{i=1}^{2} C_i P_i + C_\mu \mu,$$

$$TC_1 = C_L E(L) + C_W E(W) + C_0 P_0 + \sum_{i=1}^{2} C_i P_i + C_\mu \mu_1,$$

$$TC_2 = C_L E(L) + C_W E(W) + C_0 P_0 + \sum_{i=1}^{2} C_i P_i + C_\mu \mu_2,$$

where

$C_L \equiv$ Holding cost for each consumer seen in the system;

$C_W \equiv$ Waiting cost for one consumer requires the service;

$C_0 \equiv$ Cost for the period the server handling service process;

$C_i \equiv$ Cost when the server is on the ith working vacations;

$C_\mu \equiv$ Cost for service;

$P_0 =$ Probability when the server is on busy state;

$P_i =$ Probability when the server is on the ith working vacation;

$E(L) =$ The expected number of beneficiaries in the system;

$E(W) =$ The expected waiting time of a beneficiary in the system, respectively.

Our aim can be displayed mathematically as Minimize TC against μ, μ_1 and μ_2, where μ is optimum service rate for busy period, μ_1 is optimum service rate for optional-I working vacation and μ_2 is optimum service rate for optional-II working vacation.

Figures 10 to 13 shows that total cost TC against various μ, μ_1 and μ_2. It is clear that for different values of μ, μ_1 and μ_2 the total cost curve is concave. Specifically, Figure 10 indicates the optimal service rate for the total cost, and Figure 11 compares the TC to a for different arrival rates; that is, increasing in the arrival rate corresponds to an increase in the TC for different service rates (μ). Figure 12 shows TC against μ_1 for different arrival rates; that is, an increase in arrival rate corresponds to an increase in the total cost TC for different optional I working service rates (μ_1). Figure 13 shows TC against μ_2 for different arrival rates; that is, an increase in arrival rate corresponds to an increase in the total cost TC for different optional-II working service rates (μ_2).

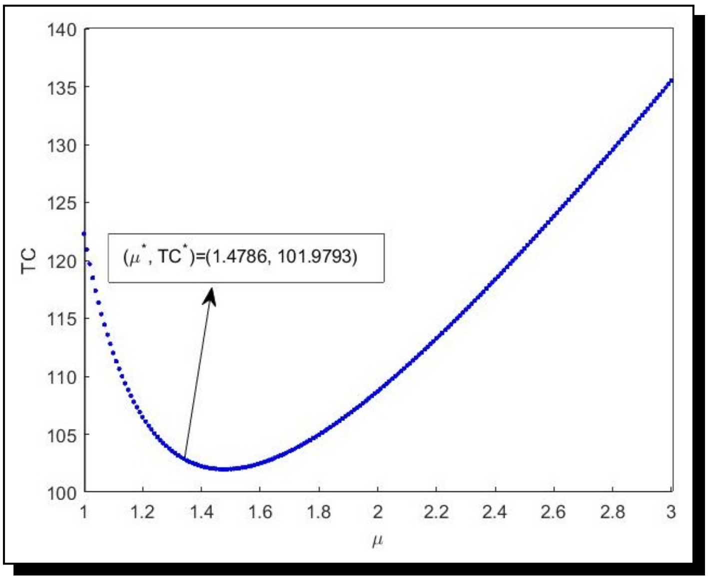

Figure 10. TC against μ

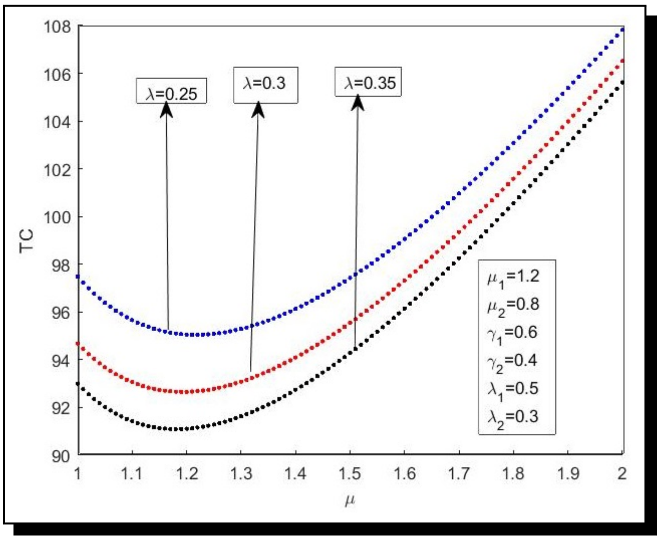

Figure 11. TC against $\mu$for various λ

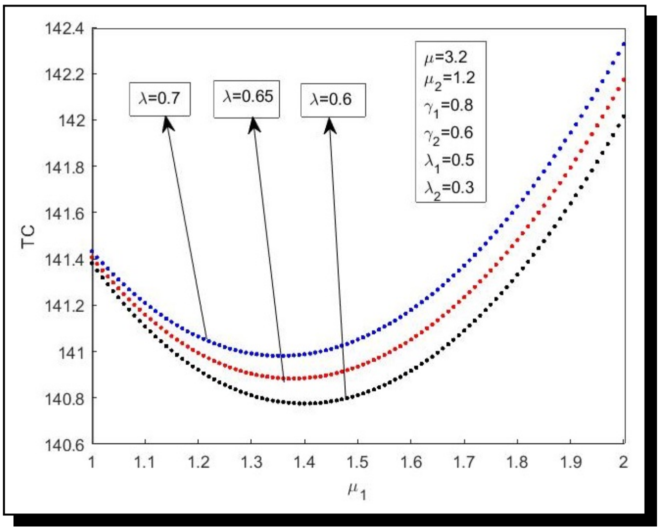

Figure 12. TC against μ_1 for various λ

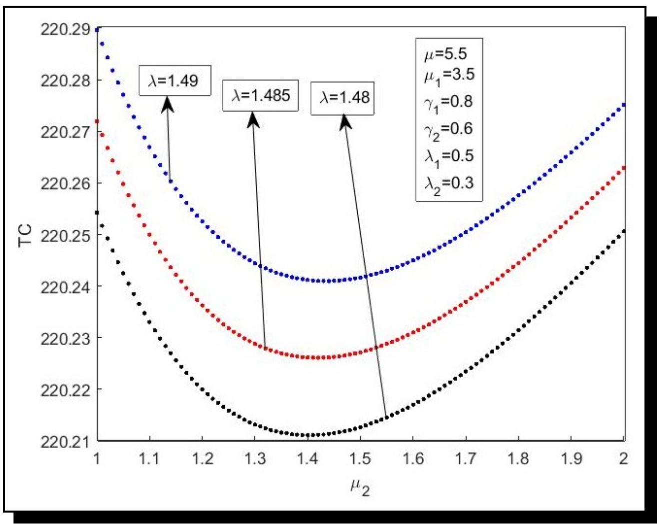

Figure 13. TC against μ_2 for various λ

7.1 Particle Swarm Optimization Algorithm

A population-based strategy approach is used in *Particle Swarm Optimization* (PSO). Many swarming particles are used in it. Every particle suggests for a possible solution. The collection of possible solutions cooperates and coexists concurrently. Every particle in the swarm looks for the best spot to land inside the search region. The set of potential solutions is represented by the search region, and the group (swarm) of flying particles symbolizes the evolving solutions. Every particle records its own optimal solution as well as the best one in the swarm across a number of generations, or iterations. The flying position and velocity settings are then adjusted. Specifically, every particle reacts to its own and its neighbours' flight experiences by dynamically adjusting its speed. Using information about its current position, velocity, distance from personal and swarm optimums, and it attempts to change its location in a manner similar to this. In this case, we minimized the cost of corresponding to the best service by using the PSO optimization method for Table 1.

Table 1. PSO values for various Λ

Λ	$E(L)$	$E(W)$	μ^*	TC
0.50	0.4115	0.8230	1.4786	101.9793
0.55	0.4233	0.7697	1.4904	102.0641
0.60	0.4325	0.7208	1.5053	102.3032
0.65	0.4392	0.6757	1.5224	102.6625
0.70	0.4436	0.6337	1.5409	103.1150

From Table 1 we conclude that, the arrival rates are increasing then the corresponding total cost will be increased. Also, the mean number of customers and expected waiting time of a customer will be increased. In Figure 10, we get the best optimization value $(\mu^*, TC) = (1.4786, 101.9793)$.

8. Practical Application of the Model

Consider a bank teller (server), who occasionally has additional responsibilities in addition to servicing customers.

Optional Working Vacation I: The teller serves customers while handling standard administrative duties (such as document filing), but because they are multitasking, service is delayed.

Optional Working Vacation II: The teller divides their concentration between updating the bank's software system and serving customers, which further slows down operations.

The bank may prevent prospective customers from joining the queue during these working vacations in an effort to better manage workloads and prevent long queues.

Customers might not understand the wait or may decide to leave as a result of this restriction, which could significantly affect the bank's efficiency and level of customer service.

This illustration demonstrates the queuing model's performance in concept but also focuses attention on the difficulties in putting it into practice, particularly in relation to customer happiness and service effectiveness.

9. Conclusion

The present research examines the concept of an M/M/1 queueing system with different optional working vacations I and II. In addition, the arrival is restricted when the server is on vacation. Computing the probability-generating function provided an analysis of the mean number of customers in the system and the expected waiting time for each customer. We analysed a number of numerical examples for arriving rate values, service values for busy and working vacation periods, the mean number of customers, and the expected waiting time of customers during vacation periods for both optional vacations. The total cost for busy and different vacation periods was also determined for different service rate values, the mean number of customers in the system, and the expected waiting times of customers. Next, the PSO technique for reducing expenses was discussed for various arrival rates. If the arrival rates are increasing, then the corresponding total cost will increase. Also, the mean number of customers and expected waiting time of a customer will increase, while the total cost is reduced by the PSO algorithm. The concept's focus is future development using the M^X/M/1 queueing system with optional differentiated working vacation queues with arrival restriction and server breakdown or M/G/1 with optional differentiated working vacation queues with arrival restriction.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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