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Research Article

Impact of Peristalsis and Wall Properties on Diffusion in the Flow of a Chemically Responsive Jeffrey Fluid

G. Ravi Kiran^{*}, K. Rajyalakshmi [®] and B. Ravindar [®]

Department of Mathematics, SR University, Warangal, India *Corresponding author: ravikiran.wgl@gmail.com

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Abstract. The hydrodynamic diffusion of a solute matter in the peristaltic flow of a chemically reactive incompressible Jeffrey fluid is studied as a model of fluid transport in the human intestinal system with wall properties. The long wavelength approximation, Taylor's limiting condition and dynamic boundary conditions at the flexible walls are used to obtain the average effective dispersion coefficient in the presence of combined homogeneous and heterogeneous chemical reactions. The effects of various pertinent parameters on the effective dispersion coefficient are discussed. It is observed that average effective dispersion coefficient increases with amplitude ratio which implies that dispersion is more in the presence of peristalsis. Further, it also increases with the Jeffrey parameter and wall parameters. Conversely, dispersion is found to decreases with homogeneous and heterogeneous chemical reaction rates.

Keywords. Diffusion, Irreversible chemical response, Peristalsis, Fluid-structure interaction: intestine, Jeffrey fluid, Wall properties

Mathematics Subject Classification (2020). 76Z05, 76B15, 92C10, 76A05

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1. Introduction

Dispersion describes the spread of particles through random motion from regions of higher concentration to regions of lower concentration. It plays an important role in physiological systems. For example, the knowledge of substances injected into a blood vessel is useful for many clinical and physiological purposes. Further, it is known to balance material in bioartificial kidney and transporting of oxygen in the human body. The fluid mechanical aspects of hydrodynamic dispersion of a solute in a viscous fluid have received the attention of several investigators (Taylor [22], Aris [3], Rao and Padma [16], and Gupta and Gupta [7]). Subsequently, Chandra and Agarwal [5], Philip and Chandra [14], Alemayehu and Radhakrishnamacharya [1,2] extended the analysis of Taylor [22] to non-Newtonian fluids.

Peristalsis is an inherent property of many tubular organs of the human body. It arises because of progressive waves propagating along the length of a distensible tube containing fluid. In general, a peristaltic pump is a device for pumping fluids by means of a contraction wave traveling along a tube-like structure. In view of its importance, a number of researchers investigated peristaltic transport of Newtonian and non-Newtonian fluids under different conditions (Fung and Yih [6], Shapiro *et al.* [19], Shehawey and Sebaei [20], Takagi and Balmforth [21], Radhakrishnamacharya [15], Rao and Mishra [17], Böhme and Müller [4], and Sankad *et al.* [18]).

One non-Newtonian fluid model that received considerable attention in the recent past is the Jeffrey fluid, which appears to be an appropriate model to describe some physiological and industrial fluids. This model is a relatively simple linear model which uses time derivatives instead of convective derivatives and it accounts for rheological effects of a viscoelastic fluid (Hayat *et al.* [8]). Further, Jeffrey fluid model is significant because Newtonian fluid model can be deduced from this as a special case. (Vajravelu *et al.* [23], Pandey and Tripathi [13], Kothandapani and Srinivas [11], and Ravi Kiran *et al.* [9, 10]).

The effect of combined homogeneous and heterogeneous chemical reactions in the peristaltic motion of a Jeffrey fluid with wall properties has not received any attention. It is envisaged that peristalsis may have significant effect on the hydrodynamic dispersion of a solute in the fluid flow and this may lead to better understanding of the flow situation in physiological systems. Hence, in this paper, the effect of dispersion of a solute in peristaltic flow of a Jeffrey fluid with wall properties is investigated. Using long wavelength approximation and Taylor's approach, analytical expression has been obtained for the average effective dispersion coefficient, in the presence of combined homogeneous and heterogeneous irreversible chemical reactions and the effects of various relevant parameters on it are studied.

2. Mathematical Model

Consider the hydrodynamic diffusion of a solute in peristaltic flow of a Jeffrey fluid in an infinite uniform channel of width 2d. The walls of the channel are flexible and are taken as stretched membrane, on which traveling sinusoidal waves of long wavelength are imposed.

Cartesian coordinate system (x, y) is chosen with x-axis aligned with the center line of the channel. The traveling waves are represented by

$$y = \pm h = \pm \left[d + a \sin \frac{2\pi}{\lambda} (x - ct) \right], \qquad (2.1)$$

where a is the amplitude, c is the speed and λ is the wavelength of the peristaltic wave (Figure 1).



Figure 1. An idealized intestinal geometry

The governing equation of motion of the flexible wall may be expressed as (Mittra and Prasad [12])

$$L(h) = p - p_0, (2.2)$$

where L is an operator which is used to represent the motion of stretched membrane with damping forces such that

$$L = -T\frac{\partial^2}{\partial x^2} + m\frac{\partial^2}{\partial t^2} + C\frac{\partial}{\partial t}.$$
(2.3)

Here T is the tension in the membrane, m is the mass per unit area and C is the coefficient of viscous damping force.

This approach studies the deformability of the conduit wall allowing for fluid-structure interaction in the peristaltic flow.

The equations governing two-dimensional motion of an incompressible Jeffrey fluid for the present problem are given by (Kothandapani and Srinivas [11])

$$\rho\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]u = -\frac{\partial p}{\partial x} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y},$$
(2.4)

$$\rho \left[\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right] v = -\frac{\partial p}{\partial y} + \frac{\partial S_{yy}}{\partial y} + \frac{\partial S_{yx}}{\partial x}$$
(2.5)

and the equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.6}$$

where u, v are the velocity components in the x and y directions respectively, p is the pressure, ρ is the density, S_{xx} , S_{xy} , S_{yx} , S_{yy} are extra stress components and μ is the viscosity coefficient.

Under long wavelength approximation, the governing equations for the present problem reduce to,

$$\frac{\partial p}{\partial x} = \frac{\mu}{1 + \lambda_1} \frac{\partial^2 u}{\partial y^2},$$
(2.7)

$$\frac{\partial p}{\partial y} = 0 \tag{2.8}$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.9}$$

It is assumed that $p_0 = 0$ and the channel walls are inextensible so that only lateral motion takes place and the horizontal displacement of the wall is zero.

The no-slip boundary conditions for the velocity is given by

$$u = 0 \text{ at } y = \pm h.$$
 (2.10)

The dynamic boundary conditions at the flexible walls (Mittra and Prasad [12]) can be written as

$$\frac{\partial}{\partial x}L(h) = \left(\frac{\mu}{1+\lambda_1}\right)\frac{\partial^2 u}{\partial y^2} \quad \text{at } y = \pm h \,, \tag{2.11}$$

where $\frac{\partial}{\partial x}L(h) = \frac{\partial p}{\partial x} = -T\frac{\partial^3 h}{\partial x^3} + m\frac{\partial^3 h}{\partial x \partial t^2} + C\frac{\partial^2 h}{\partial x \partial t}.$

Solving (2.7) and (2.8) under the boundary conditions (2.10) and (2.11), we get

$$u(y) = -\left(\frac{1+\lambda_1}{\mu}\right) \left[\frac{P'}{2} \left(h^2 - y^2\right)\right]$$
(2.12)

where

$$P' = -T\frac{\partial^3 h}{\partial x^3} + m\frac{\partial^3 h}{\partial x \partial t^2} + C\frac{\partial^2 h}{\partial x \partial t}$$
(2.13)

Further, the mean velocity is defined as

$$\bar{u} = \frac{1}{2h} \int_{-h}^{h} u(y) dy$$
(2.14)

Substituting equation (2.12) in equation (2.14), we get

$$\bar{\mu} = -\left(\frac{1+\lambda_1}{\mu}\right) P' \frac{h^2}{3} \tag{2.15}$$

If we now consider convection across a plane moving with the mean speed of the flow, then relative to this plane, the fluid velocity is given by (Gupta and Gupta [7], Alemayehu and Radhakrishnamacharya [1,2])

$$u_x = u - \bar{u} \,. \tag{2.16}$$

Substituting equations (2.12) and (2.15) in equation (2.16), we get

$$u_x = -\frac{1}{2} \left(\frac{1+\lambda_1}{\mu} \right) P' \left[\frac{h^2}{3} - y^2 \right].$$
(2.17)

Diffusion with Shared Homogeneous and Heterogeneous Chemical Responses

It is assumed that a solute diffuses and simultaneously undergoes a first order irreversible chemical reaction in peristaltic transport of micropolar fluid in a channel under isothermal conditions. Using Taylor's approximation, i.e., $\frac{\partial^2 C}{\partial x^2} \ll \frac{\partial^2 C}{\partial y^2}$, the equation for the concentration *C* of the solute is given by (Gupta and Gupta [7])

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial y^2} - k_1 C, \qquad (2.18)$$

where D is the molecular diffusion coefficient and k_1 is the first order reaction rate constant.

For typical values of physiologically relevant parameters of this problem, it is realized that $\bar{u} = c$ (Alemayehu and Radhakrishnamacharya [1,2]). Using this condition and making use of the following dimensionless quantities:

$$\theta = \frac{t}{\bar{t}}, \quad \bar{t} = \frac{\lambda}{\bar{u}}, \quad \eta = \frac{y}{d}, \quad \xi = \frac{(x - \bar{u}t)}{\lambda}, \quad H = \frac{h}{d}, \quad P = \frac{d^2}{\mu c}P'.$$
(2.19)

Equations (2.17) and (2.18) reduce to

$$u_{x} = -\frac{d^{2}}{2\mu}(1+\lambda_{1})P'\left[\frac{H^{2}}{3} - \eta^{2}\right],$$

$$\partial^{2}C = k_{1}d^{2} = d^{2} = \partial C$$
(2.20)

$$\frac{\partial^2 C}{\partial \eta^2} - \frac{k_1 d^2}{D} C = \frac{d^2}{\lambda D} u_x \frac{\partial C}{\partial \xi}, \qquad (2.21)$$

where $P = -\varepsilon \left[(E_1 + E_2)(2\pi)^3 \cos(2\pi\xi) - E_3(2\pi)^2 \sin(2\pi\xi) \right]$, $E_1 \left(= -\frac{Td^3}{\lambda^3 \mu c} \right)$ is the rigidity,

$$E_2 = \left(\frac{mcd^3}{\lambda^3\mu}\right) \text{ is the stiffness and } E_3 = \left(\frac{Cd^3}{\mu\lambda^2}\right) \text{ is the viscous damping force in the wall.}$$
(2.22)

It is assumed that a first order irreversible chemical reaction takes place both in the bulk of the fluid (homogeneous) as well as at the walls (heterogeneous) of the channel which are assumed to be catalytic to chemical reaction. Thus, the corresponding boundary conditions at the walls (Philip and Chandra [14]) are given by

$$\frac{\partial C}{\partial y} + fC = 0, \tag{2.23}$$

$$\frac{\partial C}{\partial y} - fC = 0. \tag{2.24}$$

If we introduce the dimensional variables (2.19), the above boundary conditions become

$$\frac{\partial C}{\partial \eta} + \beta C = 0 \text{ at } \eta = H = [1 + \varepsilon \sin(2\pi\xi)], \qquad (2.25)$$

$$\frac{\partial C}{\partial \eta} - \beta C = 0 \text{ at } \eta = -H = -[1 + \varepsilon \sin(2\pi\xi)], \qquad (2.26)$$

where $\beta = f d$ is the heterogeneous reaction rate parameter corresponding to catalytic reaction

at the walls.

Solving (2.21) under the boundary conditions (2.25) and (2.26) by assuming that $\frac{\partial C}{\partial \xi}$ is independent of η at any cross section, we get the solution for the concentration of the solute *C* as

$$C(\eta) = \left[\frac{d^4}{2\lambda\mu D}\frac{\partial C}{\partial\xi}\right]\frac{P}{\alpha^2}(1+\lambda_1)\left[\frac{\cos h(\alpha\eta)}{L}\left(2H+\beta\left(\frac{2H^2}{3}+\frac{2}{\alpha^2}\right)\right) - \left(\eta^2 - \frac{H^2}{3} + \frac{2}{\alpha^2}\right)\right],\qquad(2.27)$$

where $L = \alpha \sinh(\alpha H) + \beta \cosh(\alpha H)$ and $\alpha = \left(\frac{k_1 d^2}{D}\right)^{1/2}$.

The volumetric rate Q at which the solute is transported across a section of the channel of unit breadth is defined by

$$Q = \int_{-H}^{H} C \, u_x \, d\eta \,. \tag{2.28}$$

Substituting equations (2.20) and (2.27) in equation (2.28), we get the volumetric rate Q as

$$Q = -2\frac{d^6}{\lambda\mu^2 D}\frac{\partial C}{\partial\xi}G(\xi,\alpha,\beta,\varepsilon,E_1,E_2,E_3,\lambda_1),$$
(2.29)

where

$$G(\xi, \alpha, \beta, \varepsilon, E_1, E_2, E_3, \lambda_1) = \frac{(1+\lambda_1)^2}{\alpha^2} \left[\left(\frac{P}{3L\alpha^3} \right) \left(H + \frac{\beta H^2}{3} + \frac{\beta}{\alpha^2} \right) \left(3\alpha H \cos h(\alpha H) - (3 + H^2 \alpha^2) \sin h(\alpha H) \right) + \frac{P H^5}{45} \right].$$

$$(2.30)$$

Now comparing the equation (2.29) with Fick's first law of diffusion, the effective dispersion coefficient D^* with which the solute disperses relative to a plane moving with the mean speed of the flow, is obtained as,

$$D^{*} = 2\frac{d^{6}}{\mu^{2}D}G(\xi, \alpha, \beta, \varepsilon, E_{1}, E_{2}, E_{3}, \lambda_{1}).$$
(2.31)

Let the average of G be \overline{G} , and is defined by

$$\overline{G} = \int_0^1 G(\xi, \alpha, \beta, \varepsilon, E_1, E_2, E_3, \lambda_1) d\xi.$$
(2.32)

3. Numerical Results and Discussion

The equation (2.32) gives the dispersion coefficient D^* through the function \overline{G} , which has been computed numerically using MATHEMATICA software and the results are presented graphically. The key dimensionless quantities involved in the discussion are: the amplitude ratio ε , the homogeneous reaction rate α , the heterogeneous reaction rate β , the Jeffrey parameter λ_1 and the wall parameters E_1, E_2, E_3 . Further, from the equation (2.22) we may note that E_1, E_2 and E_3 cannot be taken as zero simultaneously.

The effect of the rigidity parameter (E_1) on the effective dispersion coefficient is shown in Figures 2-4. It is observed that the dispersion increases with the rigidity parameter in the cases of (i) no stiffness in the wall $(E_2 = 0)$ and perfectly elastic channel wall $(E_3 = 0)$ (Figure 2); (ii) stiffness in the wall $(E_2 \neq 0)$ and perfectly elastic wall $(E_3 = 0)$ (Figure 3); (iii) no stiffness in

the wall $(E_2 = 0)$ and dissipative wall $(E_3 \neq 0)$ (Figure 4). For all values of rigidity parameter in the case of (i) i.e. Figure 2, the dispersion coefficient decreases as the heterogeneous reaction rate β increases from unity to 10. Figure 3 shows that dispersion coefficient increases with amplitude ratio ε . A nonlinear and positive influence of peristalsis on effective dispersion coefficient is therefore evident in Figure 3. In Figure 4, dispersion coefficient, \overline{G} clearly descends as the homogeneous reaction rate α increases, demonstrating an inverse relationship. The rate of descent of the profiles is accentuated with higher values of the rigidity parameter (E_1) . This clearly demonstrates that both parameters exert a non-trivial effect on hydrodynamic dispersion in peristaltic flow and has implications for the effectiveness of dispersion phenomena.



Figure 2. Effect of E_1 on \overline{G} ($\alpha = 0.5$, $\varepsilon = 0.2$, $E_2 = 0.0$, $E_3 = 0.0$, $\lambda_1 = 1$)

Figure 3. Effect of E_1 on \overline{G} ($\alpha = 0.5$, $\beta = 5$, $E_2 = 4.0$, $E_3 = 0.0$, $\lambda_1 = 1$)



Figure 4. Effect of E_1 on \overline{G} ($\varepsilon = 0.2$, $\beta = 5$, $E_2 = 0.0$, $E_3 = 0.06$, $\lambda_1 = 1$)

Figure 5. Effect of E_2 on \overline{G} ($\alpha = 0.5$, $\varepsilon = 0.2$, $E_1 = 0.1$, $E_3 = 0.0$, $\lambda_1 = 1$)

Figures 5-7 show that the dispersion coefficient increases as the stiffness in the wall (E_2) increases for both the cases of perfectly elastic wall $(E_3 = 0)$ (Figure 5) and dissipative wall $(E_3 \neq 0)$ (Figures 6 and 7). In Figure 5, the dispersion coefficient values are plateau-profiles and

remain at consistent distances from one another, as the heterogeneous reaction rate β ascends from 1 to 10. The response to a change in amplitude ratio (Figure 6) and homogeneous reaction rate (Figure 7) is similar to that observed in Figures 3 and 4. Dispersion coefficient sharply ascends with an increase in the former, whereas it decays approximately linearly with a rise in the latter.

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3.6 3.2

2.4

2.0

0.2 0.3 0.4

E_=3

G 2.8



Figure 6. Effect of E_2 on G ($\alpha = 0.5$, $\beta = 5$, $E_1 = 0.1, E_3 = 0.06, \lambda_1 = 1$)



0.8 0.9

E_=4

0.5 0.6 0.7

F = 6

1.0

1.1



Figure 8. Effect of E_3 on \overline{G} ($\alpha = 0.5$, $\varepsilon = 0.2$, $E_1 = 0.1$, $E_2 = 4.0$, $\lambda_1 = 1$)

Figure 9. Effect of E_3 on \overline{G} ($\alpha = 0.5$, $\beta = 5$, $E_1 = 0.1$, $E_2 = 0.0$, $\lambda_1 = 1$)

It is seen from Figures 8-10 that the dispersion increases with viscous damping force (E_3) in the cases of (i) stiffness in the wall $(E_2 \neq 0)$ (Figures 8 and 10) and (ii) no stiffness in the wall $(E_2 = 0)$ (Figure 9). Distinct from earlier graphs, it is observed that a sharp monotonic decay in hydrodynamic dispersion coefficient (Figure 8) accompanies an increase in heterogeneous chemical reaction rate β . It is also observed that the variation of dispersion coefficient with amplitude ratio ε (Figure 9) is similar to that observed in Figure 6. Further, Figure 10 indicates that the effect of homogeneous reaction rate α is less pronounced for all values of viscous damping E_3 , compared with the distributions in Figures 4 and 7.

It is noticed that the effective dispersion coefficient \overline{G} increases with Jeffrey parameter λ_1 (Figures 11-13). This is true for the cases of (i) stiffness in the wall $(E_2 \neq 0)$ and perfectly elastic wall $(E_3 = 0)$ (Figure 11); (ii) no stiffness in the wall $(E_2 = 0)$ and dissipative wall $(E_3 \neq 0)$ (Figures 12 and 13). It is also noticed that the hydrodynamic dispersion values are largely invariant with alteration in the heterogeneous chemical reaction rate β (Figure 11).



Figure 10. Effect of E_3 on \overline{G} ($\varepsilon = 0.2$, $\beta = 5$, $E_1 = 0.1$, $E_2 = 4.0$, $\lambda_1 = 1$)



Figure 11. Effect of λ_1 on \overline{G} ($\alpha = 0.5$, $\varepsilon = 0.2$, $E_1 = 0.1$, $E_2 = 4.0$, $E_3 = 0.0$)



Figure 12. Effect of λ_1 on \overline{G} ($\alpha = 0.5$, $\varepsilon = 0.2$, $E_1 = 0.1, E_2 = 0.0, E_3 = 4.0$)



It is seen from Figures 3, 6, 9 and 12 that the average effective dispersion coefficient increases with amplitude ratio ε . This implies that the peristalsis enhances dispersion of a solute in fluid flow. This result agrees with that of Alemayehu and Radhakrishnamacharya [1,2]. Further, as discussed earlier, the hydrodynamic dispersion decreases with homogeneous chemical reaction rate α (Figures 4, 7, 10 and 13) and heterogeneous chemical reaction rate β (Figures 2, 5, 8 and 11). This result agrees with that of Rao and Padma [16], and Gupta and Gupta [7].

4. Conclusion

The effect of combined homogeneous and heterogeneous chemical reactions on dispersion in peristaltic flow of a Jeffrey fluid with wall properties has been studied analytically under long wavelength approximation and Taylor's limiting condition. The model developed is relevant to fluid transport in the human digestive system. It is observed that peristaltic motion enhances hydrodynamic dispersion and dispersion decreases with both the homogeneous and heterogeneous chemical reaction rates. It is also observed that the effective dispersion coefficient increases with Jeffrey parameter, rigidity, stiffness, viscous damping.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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